The Real Income Shares of Labor, Human and Physical Capital from Micro- and Macro-Data

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Abstract

Micro data are used to separate the wage income of employed workers into components of basic labor and human capital. Further on the wage components of the self employed are determined taking into account their higher qualification and longer working hours. The fractions of these wage components are used to obtain the total income shares of basic labor, human and physical capital from yearly GDP calculations. This procedure provides a yearly information on the development of the factor shares for individual countries, a tool for understanding development and growth.

German census data of the years 1976, 1985, 1995, and 2006 are selected in order to demonstrate the method. As result the factor shares for these years are obtained. The average shares are in agreement with the well known results of Mankiw, Romer and Weil [8] if only employed workers are considered.

If self-employed labor is also taken into account, the share ratios of physical and human capital and labor change to \( s_K : s_H : s_L = 0.21 : 0.25 : 0.54 \). This result differs considerably from the generally expected share ratios for developed countries of \( 1/3 : 1/3 : 1/3 \).

Further on, the development in time is investigated. A considerable variation is observed in the last period: 1995 - 2006. It is contradictory to a constant behavior as expected from Kaldor’s stylized facts. The source could be traced to considerable changes in the qualification structure of the German work force.

Keywords: human capital, Mikrozensus, annual factor income shares, factor share development

JEL classification: D33, E25, J24
1 Introduction

Considering only capital \(K\) and labor \(L\) as production factors their total income shares - \(s_K\) and \(s_L\) - are found to be constant. For the USA one finds \(s_L : s_K \approx 2/3 : 1/3\) [5]. A similar behavior is observed for other industrial countries.

In 1992 Mankiw, Romer and Weil [8] showed that human capital \(H\) is a further important production factor that needs considering. They assumed a Cobb-Douglas production function with production factors \(K, H,\) and \(A\cdot L\) (\(A\) being a Harrods-neutral technical progress that results in an enhanced \(L\)-wage). They analyzed economic data of 98 different countries from 1960 - 1985. They obtained ratios of income shares \(s_K : s_H : s_L\) compatible with equal shares of \(1/3\). Their final conclusion was that \(H\) as production factor is necessary to explain cross country differences.

Human capital is considered to be the dominant driving force for economic growth. According to Le, Gibson and Oxley [7] the determination of \(H\) is essential in order to establish the correlation between \(H\) and growth. Folloni and Vittadini [3] have provided an extensive survey on procedures for human capital measurements. They differentiated 5 methods for obtaining a measure of \(H\):

1. retrospective method:
   measure of \(H\) is the present value of the costs of formation of the current stock of \(H\),

2. prospective method:
   measure of \(H\) is the present value of the lifelong income of the workers,

3. individual investments:
   measure of \(H\) is \(w(s,x)\), the wage as function of the number of schooling years \((s)\) and the number of years with professional experience \((x)\),

4. educational attainment level:
   possible measures of \(H\) are attainment levels, like fractions of population above certain educational level (e.g. reading capability), GDP-fraction of educational costs, ratio of teachers per pupil/student,

5. \(H\) as latent variable:
   there is no direct measure of \(H\); instead it is assumed to be an unknown function of several qualitative and quantitative parameters.

All these different methods aim at determining measures of the aggregate \(H\). The results of empirical studies of a correlation with economic growth are either controversial or insignificant (see e.g.[6]). Folloni and Vittadini state [3](p. 265): “Empirical studies have demonstrated the existence of wide differences between micro analysis (micro data) and those of macro ones. Micro data find substantially positive impact of educational attainment on earnings; macro studies show very controversial results”.

This unconvincing situation can be attributed to the qualitative difference between economic growth (a dynamic variable) and aggregate \(H\) (a rather constant variable w.r.t. growth). Therefore the main mode of operation of \(H\) is believed to work via external effects. These are difficult to determine and to valuate.

A possible alternative has been proposed by the OECD [13]: the income ratio of higher educated to lower educated workers may serve as measure of \(H\) of the higher educated workers. As result one could obtain a value for the human capital stock: “by weighting different segments of the workforce by the ratio of earnings at different levels of education, it is possible to derive an index of the value of average human capital stock” (p.28).

A similar approach for measuring the level of \(H\) has been proposed by Mulligan and Sala-i-Martin [12]. They use the total income ratio of all workers and those without schooling as measure of the aggregate value of \(H\). So the income of workers with no education serves as unit. This is based on the assumption that a person without schooling is the same always and everywhere. They further assume that the such determined \(H\) is a measure of the aggregate \(H\).
B. Jeong [4] developed this method further in order to compare \( H \) across countries, implying, however, more stringent assumptions, like a fixed income share of \( H \) independent of time and country.

However, also these approaches assume to determine a measure of the aggregate value of \( H \).

This paper - in contrast to the above approaches - is aiming at a direct correlation between \( H \) and growth. \( H \) is restricted to its productively used fraction, a small part of the aggregate \( H \). Its factor share can be determined for a single economy and within limited time intervals.

The factor share of \( H \) is a measure of the productively used \( H \) in units of the total income. The size is a measure of its importance for economic growth. The development in time demonstrates a constant, increasing or decreasing importance for growth. These data can become a powerful tool for understanding and improving the development of an economy.

The factor share of the productively used \( H \) for a single economy is obtained in a two step procedure:

step 1: \( H \) used in manufacturing increases the productivity and results in a higher wage. This has been shown by Jacob Mincer [11] (and others later on) who demonstrated a positive relation between wage and the years of education and work time.

In this paper it is assumed that a worker earns a basic wage \((w_o)\) and a surplus wage from the use of human capital in production \((w_h)\). The total wage \((W)\) as well as \(w_o\) are determined from wages of all workers and of those without qualification. The latter did not invest in \( H \). So they do not earn a surplus wage from productively used \( H \). \( w_h \), the surplus wage, is obtained from the difference: \( w_h = W - w_o \).

\( W \) and \( w_o \) are determined from wage distribution which are available from yearly micro data.

The final result of this procedure are the wage fractions \( f_o = w_o / W \) and \( f_h = w_h / W \).

step 2: \( f_o \) and \( f_h \) from step 1 are applied to the total labor share of the NNP (net national product), the total income of the economy \(^1\). As result separated shares of basic labor \((s_L)\) and \( H \) \((s_H)\) are obtained.

This procedure transfers the correlation between \( H \) and income from the micro to the macro level: \( H \)-investment and higher earnings (micro level) to the \( H \)-share of total income \((s_H)\) (macro level).

The above sketched method is presented in section 2.1. Details of the determination of wages and total income shares are described in section 2.2, followed by a discussion of the method (section 2.3).

As an example the German Mikrozensus (MZ) data of 1976, 1985, 1995 and 2006 are used in order to determine the wage parameters \( W, w_o, w_h, f_o \) and \( f_h \) (section 3.1).

First the wage parameters of the employed workers and self-employed are determined (section 3.2). Then the wage parameters are used for the determinations of the NNP shares (section 4). At first only the \( L \)-share of the employed workers \((EL)\) is separated into \( s_L(EL) \) and \( s_H(EL) \). The resulting data (section 4.1) are compared and found to be in agreement with the MRW-results [8].

Further on the labor and human capital shares of the self-employed workers \((SE)\) are determined (section 4.2). In general they are included in the \( K \)-share. A separation from the \( K \)-share does yield the \textit{real} \( s_K \) without admixtures of \( L \) and \( H \). It also yields the real shares \( s_H \) and \( s_L \) (including contributions of \( EL \) and \( SE \))

Significant differences w.r.t. the generally expected ratio of 1/3 : 1/3 : 1/3 are observed and discussed.

The German development from 1976 - 2006 does show a considerable variation in time. This is in contradiction to a constant behavior as expected for developed countries like Germany. This behavior is investigated further using detailed micro data of different groups of qualification (section 4.3).

Finally the results for Germany are discussed in section 5.

\(^1\)Values of GDP and NNP as well as the income shares of labor and capital are published by statistical institutes of governments in yearly intervals.
2 Method for Separating the Wage Components of Basic Labor and Human Capital

2.1 Theoretical Background

In this section a method is described that allows the simultaneous determination of a basic labor wage and the surplus wage that is earned depending on the productive use of individual human capital.

Following Mankiw, Romer and Weil [8] a production function with the the factors K, H, L and an exogenous technology A is considered:

\[ Y = F(K, H, AL) = r \cdot K + r_h \cdot H + Aw \cdot L. \]

It should be noted that \( Y \) stands for the NNP, the total income of the production factors. The difference to the GDP is essentially due to depreciation of \( K \) and the balance of duties and subsidies of the firms. These are not accounted in the NNP in order to put all production factors onto the same footing. (Otherwise depreciation and the balance of duties and subsidies would count as additional rent of \( K \).) In this way the NNP serves as reference value for the shares of the production factors. Further details are described in appendix A.

At the microeconomic level the individual worker earns a basic wage \( w_o \) plus a surplus wage \( w_h \) depending on his individual human capital engagement:

\[ W(i) = w_o + w_h(i) \quad \text{(total wage of indiv. worker i)} \]

So a worker without any human capital engagement will earn only the basic wage \( w_o \). This fact is used to determine \( w_o \); the wage of workers without schooling and professional qualification.

At the macroeconomic level the mean human capital is used:

\[ Y = r \cdot K + \sum_{i=1}^{L} w_h(i) + w_o \cdot L \]

\[ Y(s_K + s_H + s_L), \]

where \( w_h \) is the mean of the individual \( w_h(i) \), and \( s_K \), \( s_H \) and \( s_L \) are the NNP shares. The sum in the equation above is the rent obtained from productively used \( H \).

A simultaneous determination of the total wage \( W \) and \( w_o \) yields \( w_h = W - w_o \). Also the wage fractions \( f_o = w_o / W \) and \( f_h = w_h / W \) are determined. They are used in separating the labor income into contributions from \( H \) and \( L \).

It should be noted that the assumed Harrods neutral technology implies factor neutrality. In case of non neutrality there will be efficiency differences between all production factors. In this case the different factor shares may have technology enhanced return rates.

2.2 Determination of Wages and Income Shares

Fixed values of the wages \( W \) and \( w_o \) are assumed in the theoretical considerations above. However, in reality wage distributions have to be evaluated in order to obtain representative values of \( W \) and \( w_o \).

In the micro data incomes of individual workers are available. They have to be converted to gross hourly wages. This will yield comparable results independent of varying working times that are dependent on individual choice or general agreements.

The resulting distributions of wages/hour of all workers and those with no qualification have to be converted into representative values of \( W \) and \( w_o \). For this purpose different methods can be used, e.g.
the mean, the median, and a truncated mean, where the tails of the distributions are ignored. Also logarithmic wage distributions can be evaluated. These different methods have to be investigated. As result $W$ and $w_o$ are determined using the data of the EL (employed worker) only. This will yield the EL factor shares.

The stated incomes of the SE (self-employed) comprises their labor income as well as the rent of their investments. Therefore in general only the EL-incomes and corresponding factor shares are considered. However the obtained results depend on the decision of workers to work as employed or self-employed. This might create a bias in comparisons of different economies or years. In order to overcome this bias the SE-labor income is estimated from the EL-wages taking into account a different qualification structure and longer working times of the SE. The basic assumption is that $EL$ and $SE$ of the same qualification earn the same wage/hour: $W_q(SE) = W_q(EL)$. In detail the distributions of EL of different qualification levels ($q$) are used: \{w_i\}_q(EL).

In case of a minimum qualification the assumption $W_{q,\text{min}}(EL) = w_o(EL) = w_o(SE) = w_o$, so the basic wages of EL and SE are identical. It is taken from the EL-data. $W(SE)$ is determined from the weighted EL-wage distributions of different qualification levels:

$$W_q(SE) = \sum_q \{w_i\}_q(EL) \cdot \frac{N_q(SE)}{N_q(EL)}$$

with $N_q$ being the number of worked hours of SE and EL. The wage of the SE is determined from the such obtained distribution.

After this procedures $W$ and $w_o$ and the wage ratio $f_o = w_o/W$ are available for EL and SE. They are used as input data in the calculation of the NNP shares of K, H and L.

Further input data are the published annual income share of the EL. These are part of the annual economic summary data provided by statistical institutes, e.g. for Germany the VGR (Volkswirtschaftliche Gesamtrechnung) of the “Statistisches Bundesamt” [15],[17]. Part of these data are the NNP and its shares of labor and capital: $s_L(\text{VGR})$ and $s_K(\text{VGR})$, where $s_L(\text{VGR})$comprises the sum of $s_L(EL)$ and $s_H(EL)$ and $s_K(\text{VGR})$comprises $s_K$ and the sum of $s_L(SE)$ and $s_H(SE)$.

The shares of the EL are obtained with

$$s_L(EL) = s_L(\text{VGR}) \cdot f_o(EL) \quad \text{and} \quad s_H(EL) = s_L(\text{VGR}) \cdot (1 - f_o(EL))$$

so $s_L(\text{VGR})$ is divided up by using the wage ratios of the EL.

The labor income shares of the SE are determined as follows

$$s_L(SE) = s_L(EL) \cdot \frac{N(SE) \cdot h_o(SE)}{N(EL) \cdot h_o(EL)}$$
$$s_H(SE) = s_L(SE) \cdot \frac{w_h(SE)}{w_o}$$

First the basic labor share $s_L(SE)$ is determined by scaling $s_L(EL)$ with the ratio of numbers (N) and weekly working hours ($h_o$) of SE to EL.

Second the human capital share $s_H(SE)$ is determined from $s_L(SE)$ by using the ratio of $w_h / w_o$.

Finally the real NNP shares of the production factors L, H, K are then obtained by

$$s_L = s_L(EL) + s_L(SE)$$
$$s_H = s_H(EL) + s_H(SE)$$
$$s_K = 1 - s_L - s_H$$

They are called real because they do not contain admixtures from the other two production factors.
2.3 Discussion of the Method

The assumptions are:

- there are 3 production factors K, H, L that determine the total output of a country,
- a basic labor income $w_o$ is earned by persons of minimal qualification: they did not invest in H (education or professional qualification),
- persons that have invested in H do earn a surplus wage $w_h$ that is proportional to their productively used H
- the wage ratios $f_o$ and $f_h$ of the EL can be applied to the total labor share yielding $s_L(EL)$ and $s_H(EL)$. It is assumed that this procedure can be followed in spite of differences in the data accumulation between the GDP/NNP calculations and the micro data. In consequence this procedure transfers directly the well established correlation between H-investment and income at the micro level to the factor shares at the macro level.

The presented method allows a separation of the NNP into contribution of the production factors K, H and L. The resulting real NNP shares can be determined for a single country using annual micro data. These shares are called “real” in order to distinguish the results from the generally quoted results that either refer to $s_H$ and $s_L$ of the EL only and to a $s_K$ that comprises admixtures of $s_H$ and $s_L$ of the SE.

Important features of the presented method are:

- the real shares do represent the economic status of a country,
- their size demonstrates the importance of the productions factors for size and growth of total income,
- this can be considered as prove of the correlation between H and growth,
- the income shares can be considered as measures of the production factors K, H, and L using the NNP of the economy as unit,
- time series of the real factor shares become available with frequencies of the micro data. They are expected to improve the understanding of the relative importance of the production factors for the development of a country,
- they allow a yearly evaluation of costs and benefits of H-investments for the general public. Surplus taxes obtained from $w_h$ can be compared to the costs of education, universities, etc,
- they allow a verification of the constancy of the real factor shares. This fact has been stated in Kaldor’s stylized facts. It is a basic assumption in numerous theoretical considerations,
- investigations of cross country differences of annual factor incomes should improve the understanding of development differences.

Concerning the comparison of different countries: it has to be taken into account that the used technologies are different in general. This will result in different technology enhanced wages esp. of $w_o$. Further differences do originate from differences of the minimal qualification: e.g. in countries with obligatory 8 years of schooling, there will be no persons without schooling at all, while in some development countries there can be quite some.

Concerning technology: it is considered to be freely available, however, not necessarily its use. It might not be available for some countries if the amount of H is not available that is required for its use in production. The above mentioned obligatory years of schooling establish already a kind of minimal H that will result in a higher $w_o$ as compared to countries with a considerable amount of persons without any schooling.
3 Wage Components of Basic Labor and Human Capital for Germany

The wages are determined from wage distributions of workers of different qualification levels. These distributions are obtained from German MZ data. The present analysis is based on data of the years 1976, 1985, 1995, 2006.

The German MZ data contain the results from annual interviews of a representative 1% sample of the population [9]. In this work data of individuals are evaluated. These are the weight for scaling to the total population, the type of place of living, the occupation, the position in employment, the dominant source of income, the highest level of education and professional training, the net monthly income and the usual working hours per week.

3.1 Micro Data Analysis

In the following details of the data analysis are described.

- **Selection Criteria**
  The following selection criteria are applied:
  1. only working persons at the main place of living are taken into account,
  2. the statements to all the used data have to be complete,
  3. the dominant source of income is labor (for EL only),
  4. employed or self-employed persons are selected.

Further and more explicit details are described in appendix B

- **Conversion of Data**
  The monthly net income values need some conversions for use in the further analysis:
  1. conversion of the net income to the gross income; including income tax, social and health insurances as well as contributions of the employers,
  2. conversion of the monthly gross income to the wage/hour:
     \[(\text{wage/hour} = \frac{\text{monthly income}}{4\times \text{hours/week})}\],
  3. correction for inflation and conversion into Euro. The deflator of different years are obtained from the nominal and inflation corrected GDP data [18]. Reference year is 1991. For this year there exist 2 sets of GDP-data: for the Federal Republic alone and for the unified Germany.

Further and more explicit details are described in appendix C

- **Unified Scale of Qualifications**
  The different levels of attainment in education and professional training have changed over time in the yearly data.

  A unified definition of the lowest qualification that is valid for all datasets is needed for the determination of \(w_0\). A general scale of qualification levels for all datasets is required for the determination of the SE-wage. Therefore a unified scale of qualification levels is defined for the different years data. The resulting 7 levels of qualification are shown in table 5.1 as well as their frequency for the 2006-data. The mean wages in the different years are shown as well. The correlation between wage and qualification level resp. H-investment is quite evident.

Further and more explicit details are described in appendix D
• Evaluation of $w_o$

$w_o$ is the wage of workers with minimal qualification. According to the unified scale these are the workers of level $q_0$. However, a separate class of $q_0$-workers exists only for the 2006-data. The other yearly data have only information of the combined classes $q_0$ and $q_1$.

For the 2006-data there is a difference of 7% between the wages of class $q_0$ and and the combined class $q_01$. It is used as correction factor for the years with combined $q_01$ classes:

$$w_o = w(q_0, q_1) = \frac{w(q_0)_{2006}}{w(q_0, q_1)_{2006}}$$

Further and more explicit details are described in appendix E.

• $W$ and $w_o$ from Wage Distributions

Wage distributions of all and unqualified workers are obtained from the MZ-data. From these representative values of $W$ and $w_o$ have to be determined. The simplest way would be the use of mean values. However the shapes of the two distributions differ considerably. So a simple mean will have a bias from the different tails of the distributions: the wage distribution of $W$ has a long tail towards high values. Therefore the mean of this distribution is systematically biased to higher values. It can be avoided by taking the mean of the logarithmic distribution. However in this case the $w_o$-distribution has a longer tail towards low values. This will result in a systematically lower values of the mean.

The essential parameter for the determination of the income factor shares is the wage ratio $f_o = w_o / W$. The effect of different biases from the tails of the distributions are studied by evaluating the results of 5 different methods (m1 - m5): The standard mean of the linear(m1) and logarithmic(m2) wage distributions, the means of the 10-90-percentiles of linear (m3) and logarithmic (m4) wage distributions, and the median of the distributions(m5). For methods m3 - m5 no bias is expected from the tails of the distributions.

The resulting values are shown in figure 5.1 a. Considerable differences in size are visible. Esp. the standard mean of $W$(m1) is considerably higher as expected from the influence of the high tail of the linear wage distribution. The wage results of the other methods are close together.

The wage ratio $f_o = w_o / W$ is essential for the determination of the factor shares of basic labor and human capital. The development in time is shown in figure 5.1 b. All methods show the same behavior: a slightly fluctuating rise in the first two periods followed by a rather steep fall in the last period. This will be discussed when factor shares are determined (section 4.3). The observed $f_o$-variation in size of the different methods can be interpreted as a systematic error. The development in time is rather similar independent of the used method.

In the further analysis method m3 is selected, ascribing the means of the 10-90-percentiles linear wage distributions to $W$ and $w_o$. Further and more explicit details are described in appendix F.

• Statistical Errors

Statistical errors are also determined. The distributions of hourly wages contain entries with different weights: the product of the individual weight for the representative population and the individual working hours/week. These have to be taken into account in the calculation of average and variance. As example table 5.2 shows the results from the EL-wage distributions including the statistical errors (in brackets). They come out rather small ranging from about 0.1% for $W$ to 2-3% for $w_h$ and $f_h$. They can be neglected if the results from the different methods are considered. Further and more explicit details are described in appendix G.

3.2 Labor Wages of Employed and Self-Employed

EL wages and wage fractions are obtained from the EL income distributions. These have been obtained following the procedures of the previous section 3.1 for the MZ-data of 1976, 1985, 1995, and 2006. The
resulting wages and the basic wage, $W$ and \( w_o \), have been used to determine \( w_h \), and the fractions of basic wage and human capital:

\[
\begin{align*}
  w_h &= W - w_o, \\
  f_o &= \frac{w_o}{W}, \\
  f_h &= 1 - f_o.
\end{align*}
\]

The results are shown in table 5.2.

SE wages and wage fractions are obtained following the procedures described in section 2.2. \( w_o \) is the same for EL and SE. \( W \) of the SE is not available from the micro data. It is estimated from income distributions of different EL-qualification levels. The weighted sum of these yields the SE wage distribution. \( W(SE) \) is determined from it. The results are shown in table 5.3.

A comparison of EL and SE wages demonstrates the better qualification structure of the SE: e.g. \( f_h(SE) \) is considerably higher than \( f_h(EL) \). It should be further noted that the wages are stated per working hour. Comparing yearly incomes the SE do earn even more because of a \( \approx 30\% \) higher number of working hours per week.

### 3.3 Development of Labor Wages

The development of the hourly wages of employed and self-employed are shown in figure 5.2 a. The total wage \( W \) shows a rather constant growth as expected from a growing NNP resp. GDP.

The wage component of human capital \( w_h \) shows for the SE considerably higher values as compared to the EL. This originates from the higher qualification of the SE.

The basic labor wage \( w_o \) is the same for EL and SE. In the first two periods it shows a similar growth as the total wage. However in the last period a flattening is visible. This is compensated or taken over by an increase of \( w_h \). So human capital seems to gain more importance in the last period.

The development in the last period from 1995 - 2006 is also visible in the behavior of the fractions of basic labor (\( f_o \)) and human capital (\( f_h \)) as shown in figure 5.2 b. Here a change of the fractions of 0.053 is observed for the EL (see also tables 5.2). This is a change by \(+19\% \) for \( f_h \), resp \(-7\% \) for \( f_o \); a considerable change for a period of 11 years. This strong change will enter directly into \( s_L \) and \( s_H \). It will be further discussed in section 4.3.

### 4 Total Income Shares of Basic Labor, Human and Physical Capital for Germany

Important macroeconomic parameters are the total income shares of the production factors. They are the rent of the engaged factors and can be considered as measure of its size and of its importance for the total output of an economy. The temporal development of the income shares demonstrates the importance of the respective factors for growth.

The above determined wage components of basic labor and human capital are used to obtain the total income shares. The important assumption is that the relation of basic labor to human capital - \( f_o \) and \( f_h \) - are also valid for the VGR-data, esp. for the wage of all employed workers.

A direct verification of this assumption is not possible because the Mikozensus data sample is smaller. This is due to the selection process (section 3.1). Further on the determination of number and income of employed workers in the VGR originates from different sources, e.g. tax-offices and statements from interrogations of representative firms. In addition the employed workers sample of the VGR comprises also apprentices, persons doing military service, working pensioners and students. These add up the number of employed workers but the increase of total income is minor.
4.1 The Income Shares of Employed Labor

The wage fractions $f_o$ and $f_h$ are stated in table 5.2. Following the description of section 2.2 the shares $s_H(EL)$ and $s_L(EL)$ are determined. The average factor shares are shown in table 5.4 together with the results of Mankiw, Romer and Weil[8] and the reanalysis of Acemoglu [1]. It should be noted that for this comparison $s_K$ in the table still comprises the admixture of $H$ and $L$ of the SE.

The obtained shares are in agreement with the published ones. However, it has to be taken into account that the result of this work refers to Germany alone while the published results imply a different model with stringent assumptions and an average of about 100 countries.

In table 5.4 are also stated the results obtained with the 5 different methods of wage determination (see section 3.1). If the variations of $s_H$ and $s_L$ are interpreted as systematic error, it amounts to about ±0.030 w.r.t. the result of method 3.

4.2 The Income Shares Including Self-Employed Labor

Following the description of section 2.2 the real shares $s_K$, $s_H$ and $s_L$ are determined, using the wages and wage fractions of EL and SE (see tables 5.2 and 5.3).

The resulting average real shares have been added to table 5.4. Contrary to the published results remarkable differences show up: esp. $s_K$ without the labor of the SE is considerably reduced from 0.31 to 0.21.

The German real factor shares demonstrate their relative importance for the growth of the country. Most important seems to be $s_L$ with more than 50%. However, it has to be noted that this is a combined effect of basic labor and technology which enhances productivity and the basic wage. Further on $s_H$ is an important factor that contributes about 25%. It is even higher than $s_K$ which contributes 21% to the total income and its growth.

4.3 Development of the Income Shares

The analyzed German data cover 30 years from 1976 - 2006 with 3 periods of about 10 years. In the total time interval GDP and NNP have increased by 98%.

The real shares have been determined for the different years separately. In total the income per worker has increased by 33% [16]. The yearly results of NNP per person and per worker as well as the factor shares are shown in table 5.5 as well as the changes within the 10 year periods. Statistical errors are given in brackets. The time development of the factor shares is shown in figure 5.3.

A comparison of the developments of NNP and income share shows different behaviors in the 3 periods:

1. 1976 - 1985
   This period has been considered as reference period. The NNP/worker grows by 12.7% corresponding to a yearly growth of 1.3%. The factor shares stay nearly constant.

2. 1985 - 1995
   In this period the German unification took place. Overcoming the problems should dominate the economic behavior. In fact the NNP/worker grows only by 2.9% corresponding to a negligible yearly growth of 0.3%. The factor shares change only slightly: $s_K$ and $s_H$ decrease by about 0.02 while $s_L$ increases correspondingly by 0.04.

3. 1995 - 2006
   In this last period one should expect that the problems of the unification have been overcome. The NNP/worker grows again by 14.7% corresponding to a yearly growth of 1.3%. So in this period the same growth rate as in the first period is observed. However the income shares show considerable changes: $s_L$ suffers a big loss of 0.085 while $s_K$ profits enormously from a gain of 0.063 while $s_H$ gains 0.022.
According to Kaldor’s findings [5] one should expect constant income shares in the time development. The lower average value of $s_K \approx 0.21$ w.r.t. Kaldor’s 1/3 is due to the separation of the self-employed labor from the K-share.

However, the data show a somewhat different behavior. While period 1 and 2 seem to be consistent with a constant value for $s_K$, the 3. period, 1995 - 2006, shows a large redistributions of factor shares, mainly from basic labor to physical capital: the basic labor share $s_L$ looses a large amount of 0.085, a smaller fraction of 0.022 is gained by $s_H$, while $s_K$ could take the largest amount of 0.063, an increase of more than 30% in a period of 11 years.

This behavior is in contradiction to the expected constancy of the income shares. It seems to violate a basic principle of the economic understanding of growth: developed countries, like Germany, are expected to be in a steady state. In this state the total income shares of the production factors are constant. Growth should take place only by progress in technology without changes of the factor shares. A considerable redistribution of them, as observed here for Germany, does not fit into the generally accepted picture.

However, if a progress in technology results in large structural changes, stronger disturbances of the factor shares might occur. This could have happened in the last period when globalization and the so-called IT-revolution cause substantial economic changes. Also a massive increase of capital investments would result in a higher share. However, a corresponding increase of capital investments by 30% in the 3. period seems to be too high.

Further redistribution effects are observed also between $s_H$ and $s_L$. They are smaller than the one observed for $s_K$ but notable.

The above redistribution phenomena are further investigated by an analysis of employed workers with different grades of qualification: low (q0-q3), medium (q4), high (q5) and highest (q6) (see table 5.1). The details of this analysis are described in appendix H.

The remarkable results are:

- the total labor volume (worked hours per year and per inhabitants) has stayed nearly constant over the total 30 years period (-1%),
- the lowest qualification group suffers from a loss of 53% of its labor volume,
- the highest qualification group more than doubles its labor volume (+130%), but it suffers from a 25% loss of $w_h$.

A considerable increase of the qualification of the labor force is demonstrated by these facts. Further on a gain of $s_H$ at the cost of $s_L$ is explained. However the high gain of $s_K$ cannot be explained by these facts.

The consequences for the low qualified labor force is the well known high unemployment rate.

The loss in labor volume of the lower qualified workers has been taken over by the q6-group. However the expected large effect on $s_H$ is considerably reduced by a the decrease of $w_h$. This might be due to an incomplete representation of this group by trade unions. So sufficient higher wages could not be enforced. This reduces the effect on $s_H$.

As a consequence the employers profit from this situation: their income share has increased by 33%.

5 Discussion of the Results for Germany

The method of section 2 has been successfully applied to German data of the years 1976, 1985, 1995, 2006. The German MZ-data of these years have been used to obtain $W$ and $w_o$ of EL and SE. Data of the German "Volkswirtschaftliche Gesamtrechnung" (VGR) of the Statistisches Bundesamt are then evaluated to obtain the real total income shares of the production factors of physical and human capital and labor. The averaged shares (1976 - 2006) are

$$s_K : s_{H_o} : s_{L_o} = 0.21 : 0.25 : 0.54.$$
These factor shares demonstrate their relative importance for the growth of the economy. Remarkable is the considerably higher share of $H$ w.r.t. $K$. The very high value of $s_L$ demonstrates the importance of technology which contributes to this share in form of technology enhanced wages.

The above numbers differ considerably from the generally assumed equal shares of $1/3$. It is caused by the separation of the self-employed labor from the original capital share of the VGR. Without this separation

$$s_K : s_{H_p} : s_{L_o} = 0.31 : 0.21 : 0.48$$

is obtained. This is in agreement with the published results of Mankiw, Romer and Weil and Acemoglu:

$$s_K : s_{H_p} : s_{L_o} = 0.30 : 0.28 : 0.42 \quad 98 \text{ countries (1960-1985)} \ [8]$$
$$= 0.36 : 0.26 : 0.38 \quad 107 \text{ countries (1960-2000)} \ [1]$$

The difference to the result of this paper can be attributed to the different model, more stringent assumptions and the evaluation of about 100 different countries.

The development of the income shares from 1976 to 2006 does not show a constant behavior as expected from Kaldor’s stylized fact for developed countries. Instead there is a strong increase of the capital share in the 3. period from 1995 to 2006 by 0.063, a 33% increase. The labor share decreases accordingly. This is in complete contradiction with an expected constant behavior.

The presumed strong structural changes could be partly verified by an analysis of workers with different grades of qualification (see section 4.3): the most striking effect is a growth of the labor volume of workers of highest qualification by 130%. However a corresponding increase of $s_H$ is very much reduced because of a simultaneous decrease of $w_H$. Instead an unexplained increase of $s_K$ seems to have taken over. This highly questionable effect is certainly no incentive for investments in higher education.

The markable changes of income shares within the last 10 years, and its probable origin, the strong structural changes, is a source of further questions:

- to government: how to handle and overcome the resulting unwanted consequences,
- to growth theory: how to implement effects like the IT-revolution, or globalization effects,
- to economic science: is there a method to identify the origin and estimate its effect on economy and population, and are there early indicators?

These questions can be addressed in future investigations that would involve more detailed analysis of micro data as well as detailed time series of the income shares, labor participation, etc.

Further on the results for Germany should be compared to those of other countries, e.g. in Europe. The analysis of differences and similarities might improve the understanding of different developments.

In total this first realization of the method with German data shows promising perspectives for future understanding of $H$-influence on economic development.
Figure 5.1: Wages from different methods: $W$, $w_o$ (a) and $f_o$ (b).
Methods: $m_1$: standard mean (full circle), $m_2$: log-mean (open circle), $m_3$: 10-90 percentiles, linear mean (full square), $m_4$: 10-90 percentiles, log mean (open square), $m_5$: median (star)

Figure 5.2: Wages (a) and wage fractions (b) of employed (EL) and self-employed (SE) for the years 1976, 1985, 1995 and 2006
Figure 5.3: Development (1976 - 2006) of total income shares of basic labor ($s_L$), human ($s_H$) and physical capital ($s_K$). For comparison $s_H$ and $s_L$ of the EL alone are also displayed. Also shown is the labor share as provided by the VGR: $s_L(\text{VGR}) = s_L(\text{EL}) + s_H(\text{EL})$. 

\[ s_L(\text{EL+SE}) + s_H(\text{EL+SE}) = 1 - s_K \]
<table>
<thead>
<tr>
<th>level</th>
<th>qualification</th>
<th>persons [1000] in 2006</th>
<th>wages/hour [Euro]</th>
</tr>
</thead>
<tbody>
<tr>
<td>q0</td>
<td>w/o qualification</td>
<td>483</td>
<td>0.0 0.0 0.0 11.5</td>
</tr>
<tr>
<td>q1</td>
<td>basic schooling finished</td>
<td>1821</td>
<td>9.7 10.9 11.9 12.5</td>
</tr>
<tr>
<td>q2</td>
<td>higher schooling finished</td>
<td>1527</td>
<td>13.4 12.0 11.1 12.5</td>
</tr>
<tr>
<td>q3</td>
<td>learning by doing education</td>
<td>361</td>
<td>13.5 15.6 14.7 13.6</td>
</tr>
<tr>
<td>q4</td>
<td>apprenticeship</td>
<td>16109</td>
<td>12.6 13.5 13.7 15.3</td>
</tr>
<tr>
<td>q5</td>
<td>technician, ”Meister”</td>
<td>1899</td>
<td>18.6 20.8 19.5 20.0</td>
</tr>
<tr>
<td>q6</td>
<td>degree of university or ”Fachhochschule”</td>
<td>4612</td>
<td>31.3 29.7 27.1 28.3</td>
</tr>
</tbody>
</table>

Table 5.1: Unified levels of qualification and professional training, frequencies (2006-data), and wage per hour in different years. Wages are corrected for inflation and converted to Euro. The wages of minimal qualification q0 is included in q1 for the years 1976 - 1995.

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W</td>
<td>12.988</td>
<td>14.722</td>
<td>15.270</td>
<td>17.082</td>
<td></td>
</tr>
<tr>
<td>w_o</td>
<td>9.030</td>
<td>10.149</td>
<td>11.105</td>
<td>11.480</td>
<td></td>
</tr>
<tr>
<td>w_h</td>
<td>3.958</td>
<td>4.573</td>
<td>4.165</td>
<td>5.602</td>
<td></td>
</tr>
<tr>
<td>f_o</td>
<td>0.695</td>
<td>0.689</td>
<td>0.727</td>
<td>0.672</td>
<td></td>
</tr>
<tr>
<td>f_h</td>
<td>0.305</td>
<td>0.311</td>
<td>0.273</td>
<td>0.328</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2: Wages and wage fractions of the employed workers. Statistical errors are given in brackets. Results are corrected for inflation and converted to Euro.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>14.734</td>
<td>17.394</td>
<td>17.688</td>
<td>19.038</td>
</tr>
<tr>
<td>(w_o)</td>
<td>9.030</td>
<td>10.149</td>
<td>11.105</td>
<td>11.480</td>
</tr>
<tr>
<td>(w_h)</td>
<td>5.704</td>
<td>7.245</td>
<td>6.584</td>
<td>7.558</td>
</tr>
<tr>
<td>(f_o)</td>
<td>0.613</td>
<td>0.583</td>
<td>0.628</td>
<td>0.603</td>
</tr>
<tr>
<td>(f_h)</td>
<td>0.387</td>
<td>0.417</td>
<td>0.372</td>
<td>0.397</td>
</tr>
<tr>
<td>wage/hour [Euro]</td>
<td>(0.054)</td>
<td>(0.076)</td>
<td>(0.068)</td>
<td>(0.070)</td>
</tr>
<tr>
<td>(w_o)</td>
<td>(0.100)</td>
<td>(0.136)</td>
<td>(0.125)</td>
<td>(0.118)</td>
</tr>
<tr>
<td>(w_h)</td>
<td>(0.114)</td>
<td>(0.136)</td>
<td>(0.142)</td>
<td>(0.137)</td>
</tr>
<tr>
<td>(f_o)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>(f_h)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.007)</td>
</tr>
</tbody>
</table>

Table 5.3: Wages and wage fractions of the self-employed. Statistical errors are given in brackets. Wages are corrected for inflation and converted to Euro.

<table>
<thead>
<tr>
<th>income shares</th>
<th>(s_K)</th>
<th>(s_H)</th>
<th>(s_L)</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(1 - \alpha - \beta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD-parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MRW 1992</td>
<td>0.30</td>
<td>0.28</td>
<td>0.42</td>
<td>1960-1985, 98 countries</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Acemoglu 2009</td>
<td>0.36</td>
<td>0.26</td>
<td>0.38</td>
<td>1960-2000, 107 countries</td>
<td></td>
<td></td>
</tr>
<tr>
<td>this work</td>
<td>0.311</td>
<td>0.209</td>
<td>0.480</td>
<td>1976-2006, Germany</td>
<td></td>
<td></td>
</tr>
<tr>
<td>method 1</td>
<td>0.311</td>
<td>0.242</td>
<td>0.447</td>
<td>linear mean</td>
<td></td>
<td></td>
</tr>
<tr>
<td>method 2</td>
<td>0.311</td>
<td>0.244</td>
<td>0.445</td>
<td>log mean</td>
<td></td>
<td></td>
</tr>
<tr>
<td>method 3</td>
<td>0.311</td>
<td>0.209</td>
<td>0.480</td>
<td>linear trunc. mean</td>
<td></td>
<td></td>
</tr>
<tr>
<td>method 4</td>
<td>0.311</td>
<td>0.211</td>
<td>0.478</td>
<td>log trunc.mean</td>
<td></td>
<td></td>
</tr>
<tr>
<td>method 5</td>
<td>0.311</td>
<td>0.184</td>
<td>0.506</td>
<td>median</td>
<td></td>
<td></td>
</tr>
<tr>
<td>real factor shares</td>
<td>0.211</td>
<td>0.248</td>
<td>0.540</td>
<td>1976-2006, Germany</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.4: Factor shares \(s_K\), \(s_H\) and \(s_L\), resp. Cobb-Douglas parameters \(\alpha, \beta, 1 - \alpha - \beta\).

The 3. row contains the results of this work using only EL for the factor shares. In this case \(s_K\) comprises also labor contributions of the SE. This allows a direct comparison to the published results of Mankiw, Romer and Weil [8] and Acemoglu [1].

In the last row the real factor shares are stated. They comprise also the labor and human capital shares of the self-employed.
### Table 5.5: Development of NNP per person and per worker [18] as well as the real shares of physical and human capital and labor from yearly German data of 1976, 1985, 1995 and 2006. NNP-data are in Euro and corrected for inflation. The changes in the time periods are stated as well.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>NNP/person growth</td>
<td>12110.</td>
<td>14532.</td>
<td>15217.</td>
<td>17934.</td>
</tr>
<tr>
<td>NNP/worker growth</td>
<td>28411.</td>
<td>32006.</td>
<td>32935.</td>
<td>37764.</td>
</tr>
<tr>
<td>s_K</td>
<td>0.200</td>
<td>0.207</td>
<td>0.188</td>
<td>0.250</td>
</tr>
<tr>
<td>(\Delta s_K)</td>
<td>0.007</td>
<td>-0.020</td>
<td>0.063</td>
<td>(0.0010)</td>
</tr>
<tr>
<td>s_H</td>
<td>0.251</td>
<td>0.256</td>
<td>0.232</td>
<td>0.253</td>
</tr>
<tr>
<td>(\Delta s_H)</td>
<td>0.005</td>
<td>-0.024</td>
<td>0.022</td>
<td>(0.0051)</td>
</tr>
<tr>
<td>s_L</td>
<td>0.548</td>
<td>0.537</td>
<td>0.581</td>
<td>0.496</td>
</tr>
<tr>
<td>(\Delta s_L)</td>
<td>-0.012</td>
<td>0.044</td>
<td>-0.085</td>
<td>(0.0052)</td>
</tr>
</tbody>
</table>
Appendix

A GDP and Total Income

GDP and total income are closely related. For Germany both are determined by the “Statistisches Bundesamt” in the yearly VGR (“Volkswirtschaftliche Gesamtrechnung”) [14]. The relation is given by

\[
\text{GDP} = 100\% \ (2006) \\
\quad \pm \text{net factor income from abroad} \quad +2.0\% \\
\quad - \text{depreciation} \quad -14.5\% \\
\quad \pm \text{indirect taxes(-) und subsidies(+)} \quad - 9.6\% \\
\quad = \text{NNP} = \text{total income} \\
\quad 78.2\% \\
\quad s_L \text{(VGR): employed labor income share} \quad 63.9\% \ (\text{NNP}) \\
\quad + s_K \text{(VGR): self-employed and firms income shares} \quad 36.1\% \ (\text{NNP})
\]

The NNP (net national product) is the total income of the economy. Its difference to the GDP is rather constant being \(\approx 22\%\): the variation of the difference from 1970 - 2011 is less than \(\pm 0.5\%\). It is the basis for the annual income shares of capital and labor. The VGR separates the NNP into the labor income share \(s_L \text{(VGR)}\) comprising only the labor income of employed workers, and the capital share \(s_K \text{(VGR)}\) comprising the income share of enterprises as well as investment and labor share of self-employed workers.

B Mikrozensus Selection

The method of separating the wage components of basic labor and human capital (see section 2) is based on a simultaneous determinations of incomes of all employed workers and of those with a minimal qualification. This requires identical conditions under which incomes are determined. Therefore the following selection criteria are applied to the individual MZ-data:

1. persons at their main living location (this criterion avoids double counting of people),
2. individual persons who are working (only working persons have a labor income);
3. complete statements w.r.t. income, working hours per week, highest degree in school and professional education (these data are necessary for further evaluation);
4. employed workers: excluded are persons without a labor income, self-employed (their income stems from labor as well as investments), students (they are still in education), apprentices and persons in military or civil duty services (they obtain a subsistence contribution but no labor income);
5. persons whose income stem mainly from labor: no unemployed, no persons with further income from e.g. pensions, rents from capital investments.

Selections 1 and 2 are the same as used by the Statistisches Bundesamt in published income tables [10].

The effect of the selection criteria 3 - 5 on the number of workers is shown in table H.6. The strongest effect is due to criteria 4 and 5. **Criterion 4** eliminates about 15% of the workers: about 10% are self-employed, 5% are apprentices, students or persons in civil or military duty services.
Criterion 5 eliminates further 7.5% of the employed workers: these are persons who have further income sources. Their labor incomes are only a minor source of their total incomes.

In total the selection criteria eliminate 23.5% of the working population. However, the remaining 76.5% comprise the essential fraction of the labor incomes of the employed workers.

It should be noted that for the later estimation of the labor income of the self-employed (section 3.2) the criterion 4 is changed accordingly.

C Details of Income Conversions

Income scale: The income is stated in the MZ data as net monthly income. These statements are given in an ordinal scale that differs for different years. For the 2006-data it is

\[
\{ < 150, -300, -500, -700, -900, ..., -7500, -18000, > 18000 \} \text{ Euro.}
\]

The lowest and the highest bins are unlimited. They have been converted to confined bins by fixing the range to \(50 < \text{income} < 28000\). Income values outside the range are changed to the minimal/maximal value. The lower bound has practical reasons: it allows the evaluation of logarithmic distributions. The upper bound limits extreme incomes in order to reduce their influence on a mean.

Gross income: The net income has to be converted into gross income. This is necessary for a later conversion of wage ratios into shares of the total income which is the sum of gross incomes.

The difference between gross and net income comprises income tax, social insurances and contributions of employers. The conversion of net to gross income is performed by determining the conversion for the different bounds of the ordinal scale. In detail the following procedure is used:

1. the income is multiplied by 12 in order to obtain the yearly income,
2. the income tax is calculated using the tax law of the different years [2],
3. contributions to social insurance as well as contributions to it from employers are obtained from the VGR-tables [19] of the different years,
4. the resulting gross income is divided by 12 in order to obtain the gross monthly income.

The resulting gross scale is used for the evaluation of the income distribution.

Inflation correction: Incomes of different years can be compared only if corrections for inflation are applied. The different deflators are determined from the tables of nominal and deflated GDP values of the years [18]. For Germany there are 2 different tables for the time before and after the unification. They overlap for the year 1991. Therefore 1991 is used as reference year. The obtained deflators are listed in table H.7.

Income per hour: The weekly working hours have changed with the years. In addition the tendency to work part time has increased. This effects will result in a bias when results of different years are compared. Therefore the monthly income is converted to income per hour by using the MZ statements on the weekly working hours and assuming 4 weeks per month.

This procedure may result in a range of hourly wage bins that are covered by the original monthly income bin. In this case the wage is attributed to a randomly chosen bin of the covered range. The final distribution of income per hour has equidistant bins with the overflow accumulated in the last bin. Typical distributions are shown in the later discussed figure H.5.
Conversion to Euro: The incomes obtained so far are stated in different units: DM and Euro. All DM-values are converted to Euro using the ratio Euro/DM = 1.95583.

D The Unified Scale of Qualification Levels

When investigating incomes of workers of different qualification a scale of qualification has to be defined. This scale has to be the same for all years.

In the Microzensus data the educational and professional qualification is stated separately as highest degree reached in schooling as well as in professional qualification. As a consequence all persons are counted twice in the 2 qualification scales. For the 2006-data the different degrees are listed in table H.8. There are 6 levels for schooling and 11 levels for professional qualification (university and “Fachhochschul” degrees are listed as professional qualification; schooling degree of “Abitur” or “Fachhochschulreife” are indispensable for them).

Figure H.4 shows the frequency (a) as well as the average net monthly income (b) for different qualifications. The income distribution (b) shows 2 independent qualifications scales: schooling (0 - 6) and professional degree (10 -20). For either scale the income rises with increasing qualification. They have to be combined to a single scale without any double counting. This unified scale which is valid for all years is obtained after the following procedure:

- the degrees of the former GDR are combined with appropriate degrees of the German Federal Republic,
- the professional qualification is given priority over the schooling qualification: it shows a stronger dynamic with the professional degree and minimal schooling degrees are indispensable for the higher professional degrees,
- qualifications with too few persons have been combined in order to avoid qualification groups with too low population.

The final qualification scale consists of the 7 different groups (q0 - q6) shown in table 5.1. The also stated wages demonstrate the considerable increase with the qualification level.

Further on combined groups are specified: the low qualification group q03 comprises q0 - q3, and the group q01 comprising q0 and q1.

The group q01 has been formed in order to overcome a problem of the data from 1976 - 1995. In these data there is no differentiation between q0 and q1. However in the 2006-data q0 and q1 are separate qualification groups that show an income difference of 7%. This difference is used to obtain \( w_0 \) for all years:

\[
w_0 = w(q01) \cdot \frac{W(q0)_{2006}}{W(q01)_{2006}}
\]

E \( w_0 \) Determination

\( w_0 \) is determined as income of workers with minimal qualification. In Germany these are persons who have passed the obligatory years of schooling (about 8). But they finished schooling without a degree.

As shown in section 2.1 a Harrods neutral technology is involved that results in a technology enhanced \( w_0 \). The technology is country specific, in this case specific for Germany.

An alternative for determining \( w_0 \) could be the result of a Mincer regression [11]:

\[
\ln w = \alpha + \rho \cdot s + \beta_0 x + \beta_1 x^2 + \varepsilon,
\]

with the variables s (number of school years) and x (years of working). The value of \( \alpha \), obtained from the regression, is assumed to be the logarithm of the wage for a worker with 0 years of schooling. Arguments against this method are:
1. in many developed countries there are no persons without any schooling; therefore one has to rely on the linearity of the above Mincer equation in a region without data,

2. the type of schools differ considerably in quality; therefore the parameter $\rho$ and with it the extrapolation to 0 school years is determined by the quality of the different types of schools,

3. the German MZ data do not contain statements about the years of schooling; there are only statements on the highest degree reached. An estimate of the years of schooling is not excluded, but it has not been investigated.

Further on the linearity requirement would be an additional rather strong assumption. This is avoided by using the wage of workers with minimal qualification.

F Wage Determination from Income Distributions

The wages $W$ and $w_o$ have to be determined from income distributions of all employed and of the employed with minimal qualification. The further analysis requires the determination of single but representative values for the two parameters.

The distributions of the two groups are shown in figure H.5 using linear (left: a,c) and log. (right: b,d) income scales. Clearly visible are the long tail of high values for $W$ in the linear distribution (H.5a), and the long tail of low values for $w_o$ in the log distribution (H.5d). These tails do result in biased means: an up-shift of the $W$-mean from the linear distribution and a down-shift of the $w_o$-mean from the log distribution. This is also a potential source of a higher volatility of these values in different years.

The determination of the factor shares of basic Labor and $H$ depends on the wage ratio $f_o = w_o/W$. The use of means of linear as well as the log distributions result in down-shifted values of $f_o$ because of the tails of the distributions: the $f_o$-shift of the linear mean is due to the up-shifted $W$, and the $f_o$-shift of the log. mean is due to the down-shifted $w_o$.

Therefore the following methods have been evaluated: the standard mean using the full distributions, the mean of the 10-90-percentiles of the distributions, and the median: the 10-90-percentiles of the distributions are expected to have no tail effects; the median is the same for linear and logarithmic distribution.

The different values of $W$, $w_o$ and $f_o$ from the 5 methods are shown in figure 5.1 for the different years. The results from the different methods show a rather constant shift w.r.t. each other while the shape of the development in time shows nearly the same behavior for all methods.

The $f_o$-results are shown in table H.9. Also an average is given for each year. The results of the different methods are in the range of about ±0.045 corresponding to a 7% systematic error on the absolute value. These shifts can be interpreted as systematic uncertainty of magnitude of $f_o$.

The finally accepted method for the determination of $W$ and $w_o$ is the mean of the 10-90-percentile linear distributions: it has a low dependence on tails to the high or low side (as compared to the standard mean), it coincides with the average of all methods within 0.01 it is preferable to the evaluation of the logarithmic distributions (if subsamples like groups of different qualification level are analyzed: the weighted means of the subsamples do not add up to the total mean in case of a logarithmic distributions).

G Statistical Errors of Average Wages

Average wages are determined from distributions of wages/hour. The distributions have been obtained from the MZ-data using the product of two weights: the individual weight for scaling to the total
population and the individual number of working hours/week for scaling to the labor volume. As a consequence a weighted average is used:

$$W = \frac{\sum w_i \cdot \rho_i}{\sum \rho_i},$$

where $\rho_i$ is the product of the two individual weights and $w_i$ the central value of the wage bin.

The variance (var) of the distribution is

$$\text{var} = \sigma^2 = \frac{\sum w_i^2 \cdot \rho_i}{\sum \rho_i} - W^2,$$

and the std.deviation of the weighted mean is

$$\text{std.dev}(W) = \sigma \cdot \sqrt{\frac{\sum \rho_i^2}{(\sum \rho_i)^2}}.$$

The determination of the factor shares is based on three independent mean wages: $W(\text{EL}), w_o,$ and $W(\text{SE}).$ All other variables are functions of these. The corresponding std.dev. are calculated from the std.dev. of the original wages.

Errors of some scale factors like the ratio of working hours of EL and SE have been neglected. Their contribution is negligible small.

Further on the magnitudes of the yearly VGR data are given w/o errors, e.g. the total income share of EL. They are taken at face value.

The wages and their errors are shown in tables 5.2 and 5.3. The wage errors are less than 1%. The errors of $f_o$ and $f_h$ are less than 2%.

These errors are considerably less than the variations of about 7% that come out from the different methods (appendix F). However the method variations concern only the magnitude of the final factor shares, not the development in time. For the development in time the statistical error seems to be a realistic value. This is demonstrated in table 5.5 where the changes of the factor shares in the time periods are partly compatible with the statistical errors indicating insignificant changes. This stresses the changes in the last period 1995 - 2006 where the change of $s_K$ and $s_L$ are by far larger than the statistical errors.

### H Development of Qualification Groups

The development in time of different parameters has shown indications of redistribution between the production factors in the time period 1995 - 2006. Examples are the total income shares in figure 5.3, or $f_o$ in figure 5.1b.

In order to obtain a better understanding of these phenomena the EL\(^1\) have been subdivided into 4 groups of different qualification levels: low ($q03 = q0$-$q3$), medium ($q4$), high ($q5$) and highest ($q6$) qualification (see table 5.1). For each of these the time development of labor volume ($V_L$)\(^2\) and the wage components $w_h$ and $w_o$ are investigated. The $V_L$ development represents the behavior of the labor market, described by the available labor positions that are taken by workers of the corresponding qualification level. $w_o$ and $w_h$ development describes the interchange between human capital and basic labor.

The detailed results are shown in table H.10 and figure H.6.

$V_L$ shows the following features (see fig. H.6a):

\(^1\)The inclusion of self-employed would result only in a minor correction because their number is only $\approx 10\%$ of the employed and the wages of the $q$-groups are identical.

\(^2\)The labor volume is the product of labor participation and usual working hours per week; the labor participation is the number of EL per inhabitant.
1. The total $V_L$ (all EL) shows a slight rise of 7% in the 1. period followed by a continuous decrease in the 2 following periods down to nearly the initial value (-1.7%). So it is concluded that the total $V_L$ stays nearly constant in the years until 2006.

2. q5, the worker of high qualification, show a nearly constant $V_L$.

3. q6, the workers of highest qualification, show a continuous rise of $V_L$ over all time periods. It amounts to more than double the initial $V_L$ (+130%).

Considering the nearly unchanged total $V_L$ this is a sign of a massive change of the H-structure of the labor force: workers of highest qualification level have replaced a similar amount of workers of low and medium qualification. This is partly accompanied by a replacement of q03- by q4-worker in the 1. and 2. period.

4. The q03 workers show a dramatic loss in period 1 and 2 (-58%). In the 3. period their labor volume seems to have stabilized. But more than 50% of the jobs for workers of lowest qualification have disappeared in the 30 years until 2006.

5. The q4-worker have taken over some $V_L$ (q03) in periods 1. The enlarged $V_L$ (q4) could stem from q03-workers who improved their professional education.

6. The q4-group is the largest one. It provides about 60% of the total $V_L$. In the last period this group sees a massive loss of their $V_L$ (-11%). So also for this qualification level the lost jobs are no more available.

In summary a large restructuring of the labor force has occurred in the 30 years from 1976 to 2006. The qualification structure of the labor force has considerably improved by a strongly growing group of q6-workers. However the nearly constant total labor volume resulted in an equal loss of jobs of q03- and q4-workers.

The wage development (fig. H.10b) shows one striking feature: the steep decrease of $w_h$ of the q6-group in periods 1 and 2. It goes along with a rising $W$ and $w_o$, and a rather constant $w_h$ for the total work force. So the rising labor volume of q6-workers is compensated by a reduced $w_h$, resulting in a rather constant overall $w_h$. It looks as if the highest qualified workers have pushed out workers of lower qualification level. They have accepted a job, being overqualified for it, but accepting the lower payment of it.

The following slight rise of $w_h$ (q6) in the 3. period is then a considerable break. In this period the further increased number of q6-workers seem to be needed and used in the production process for jobs of adequate qualification. In consequence the $w_h$-payment increases. This does result in an increase of $s_H$ at the cost of $s_L$ as seen in figure 5.3.

The consequences of the above stated facts are:

- the loss of more than 50% of low qualified labor demand resulted in a high unemployment and a decrease of the basic wage $w_o$ as compared to the general trend of growth,
- the q6-group has compensated the loss in labor participation,
- the wage of the q6-group is still quite low as compared to the earliest data. Their probably incomplete representation by trade unions does not allow them to enforce sufficiently higher wages.
- as a consequence the employers profit from this situation: Their income share has increased by 65%.
Figure H.4: Frequency (top) and net monthly income (bottom) of different qualification levels from school and professional education. The numbers of the horizontal scale refer to the qualification groups of table H.8. The number 0 refers to persons having neither a degree from schooling nor from professional training. The number 24 refers to the overall average.
Figure H.5: Income distributions (2006-data) with linear (a,c) and log. (b,d) income scale. (a,b): all employed worker, (c,d): workers with a minimal qualification, q01 and q0 as specified in appendix D.
Figure H.6: Development of labor volume (a), and the wage components $w_h$ and $w_o$ (b). Parameter developments of all and the different qualification groups are marked in the diagrams (for interpretation see text, resp. table 5.1).
### Table H.6: Results for Selections 3 - 5 (2006-data)

<table>
<thead>
<tr>
<th>Selection</th>
<th>Persons Selected [1000]</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 + 2: preselection</td>
<td>35,063</td>
<td>100%</td>
</tr>
<tr>
<td>3. complete statements</td>
<td>34,596</td>
<td>98.7</td>
</tr>
<tr>
<td>4. employed</td>
<td>29,436</td>
<td>84.0</td>
</tr>
<tr>
<td>5. mainly labor income</td>
<td>26,812</td>
<td>76.5</td>
</tr>
</tbody>
</table>

### Table H.7: Inflation correction for the years 1976 - 2006.

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Deflater</td>
<td>1.593</td>
<td>1.163</td>
<td>0.873</td>
<td>0.817</td>
</tr>
</tbody>
</table>

### Table H.8: Qualification groups of 2006 data: highest degree obtained in education and professional training.

<table>
<thead>
<tr>
<th>code</th>
<th>schooling degree</th>
<th>code</th>
<th>professional degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. no schooling level</td>
<td>10. no professional level</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. basic school</td>
<td>11. learning-by-doing training</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. “Realschule”</td>
<td>13. apprenticeship</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. university</td>
<td>19. university</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. PhD</td>
<td>20. PhD</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table H.9: $\ell_p$-results from different methods, as well as the method average and the range of values.

<table>
<thead>
<tr>
<th>method</th>
<th>years</th>
</tr>
</thead>
<tbody>
<tr>
<td>linear mean</td>
<td>1976 0.624</td>
</tr>
<tr>
<td>log mean</td>
<td>0.642</td>
</tr>
<tr>
<td>linear trunc. mean</td>
<td>0.695</td>
</tr>
<tr>
<td>log trunc. mean</td>
<td>0.691</td>
</tr>
<tr>
<td>median</td>
<td>0.721</td>
</tr>
<tr>
<td>average</td>
<td>0.675</td>
</tr>
<tr>
<td>range</td>
<td>-0.050</td>
</tr>
<tr>
<td>average</td>
<td>0.046</td>
</tr>
<tr>
<td>topic</td>
<td>years</td>
</tr>
<tr>
<td>-------------------------</td>
<td>-----------</td>
</tr>
<tr>
<td>mean labor participation</td>
<td>0.301</td>
</tr>
<tr>
<td>mean working hours/week</td>
<td>40.0</td>
</tr>
<tr>
<td>labor volume</td>
<td>12.03</td>
</tr>
<tr>
<td>W</td>
<td>13.0</td>
</tr>
<tr>
<td>w_o</td>
<td>9.0</td>
</tr>
<tr>
<td>w_h</td>
<td>4.0</td>
</tr>
<tr>
<td>q03: labor volume</td>
<td>3.34</td>
</tr>
<tr>
<td>q4</td>
<td>7.00</td>
</tr>
<tr>
<td>q5</td>
<td>0.74</td>
</tr>
<tr>
<td>q6</td>
<td>0.94</td>
</tr>
<tr>
<td>q03: w_h</td>
<td>1.4</td>
</tr>
<tr>
<td>q4</td>
<td>3.6</td>
</tr>
<tr>
<td>q5</td>
<td>9.6</td>
</tr>
<tr>
<td>q6</td>
<td>22.3</td>
</tr>
</tbody>
</table>

Table H.10: Developments of labor volume and wage components for different qualification groups: from low (q03) to the highest (q6) level (see text for further explanation). The last column contains the total change from 1976 to 2006.
I Acknowledgment

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