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The Store-of-Value-Function of Money as a Component of Household Risk Management

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Abstract

We analyse how money as a store of value affects the decisions of a representative household under diversifiable and non-diversifiable risks. given that the central bank successfully stabilizes the rate of inflation at a low level. Assuming exponential utility allows us to derive an explicit relationship between optimal money holdings, the household's desire to tilt, smooth and stabilize consumption as well as minimize portfolio risk. In this context we also show how the correlation between stochastic labour income and stock returns impact the store-of-value function of money. Finally we prove that the store-of-value benefits of money holdings continue to hold even if we take riskless alternatives into account.

Keywords: Money demand, consumption, CRRA, CARA, exponential utility, households, risk, risk management

JEL classification: D11, E21, E41

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1 Introduction

Household risk management strategies – risk-coping, risk-sharing and risk-mitigating – are increasingly becoming a matter of concern for social policy-makers and economists. Concepts like social risk management – which "... consists of public interventions to assist individuals, households, and communities better manage risk" (Holzmann and Jorgenson 2001) – are gaining widespread interest. How households behave in a risky environment has been studied in particular with respect to precautionary savings (Carroll, 1998; Carroll and Samwick, 1997; Kimball, 1990; Weil, 1993), wealth effects (Carroll et al., 2006; Slacalek, 2006a; Slacalek, 2006b), and household credit (Lawrance, 1995; Patterson, 1993; Rinaldi and Sanchis-Arellano, 2006). However, in these contributions the role of money as a store of value has not been explicitly analysed so far. In our paper we seek to fill this gap.

Point of departure is the evidence that due to increasing competition in the banking industry the nominal opportunity cost of holding even narrow money is decreasing, and that due to financial innovations the role of cash is continuously declining. These developments have been paralleled by a successfully inflation-fighting monetary policy in many OECD countries with the Eurozone as a prominent example, leading to moderate and stable rates of inflation in general. Such an environment purports the role of money as a safe asset not only in nominal but also in real terms. It furthermore strengthens the importance of money as a store of value providing a safe haven against income risks irrespective of whether their source is aggregate or idiosyncratic. To the extent that this is the case, the already heavily criticized second pillar of European monetary policy would gain new importance, though with a possibly different interpretation. In particular observed excessive monetary growth might then signal a higher degree of uncertainty regarding employment, wages or financial market stability. Indeed at least implicitly the found evidence of strong money growth – compared to the self-defined target – has already stimulated several empirical studies which related money demand to several uncertainty measures (Greiber and Lemke, 2005; European Central Bank, 2005; Carstensen, 2006).

In our paper we explore the idea that price stability might increase the significance of money as a store of value in the context of household risk management. In doing so we build on models of intertemporal household optimization under uncertainty and bring together two branches of research. On the one hand we draw upon approaches which explain household consumption in particular under non-diversifiable income risks but in most cases ignore money. On the other hand we draw upon models which explain the role of money in the framework of household optimization. Notably we do not provide the reader with an exhaustive overview of existing approaches (cf Barr and Cuthbertson, 1991; Cuthbertson, 1997; Mizen, 1997, in this respect). Rather we restrict our analysis to those intertemporal household optimization models which have a focus on the explanation of money as a store-of-value. We go beyond this literature in several respects:

1. We analyse household risk management in a stochastic environment which is marked by both diversifiable and non-diversifiable risks.
2. We show how the type of utility function affects the formal capability to obtain a closed-form solution for optimal consumption and money holdings. In this regard we also prove that the introduction of money into the utility function might not be predominantly a matter of econonomic reasoning but rather might turn out as necessary in order to derive a money demand function.
3. By assuming an exponential type of utiliy, we are able to show explicitly how diversifiable and non-diversifiable income risks and money holdings are related in a highly non-linear manner. This allows us to clarify the store-of-value function of money as an important component of household risk management even if any direct utility of money is absent.

For the remainder of this article we proceed as follows: In the next section we discuss the usefulness of the expected utility approach in explaining intertemporal household behaviour under risk and discuss alternative utility functions. We then proceed analysing the store-of-value function of money in an expected utility framework assuming exponential utility. In doing so we start with money as the only asset and then go on discussing the role of liquidity services. As a final step we introduce risky stocks. Section 5 concludes. Throughout our analysis we use a two-period framework. This simplifies and clarifies our argument without impairing the generality of our results.

2 Income Risks and Households' Risk Preferences

Risk can broadly be defined as the possibility of deviations from one's expectations. In terms of households this concerns in particular deviations from expected future income or expenditures. Household risk management in this respect relates to measures which serve to avoid, mitigate or cope with these risks (Holzmann and Jorgenson, 2001). The type of appropriate actions depends in the first place on whether risks have their source in idiosyncratic or aggregate shocks. If shocks were merely idiosyncratic, risk management would be a matter of coordination between the interests of agents who are exposed to losses in specific states of the world and agents who experience gains in the same states. Given financial market completeness or at least perfectness, then idiosyncratic risks could be perfectly diversified away thus providing households with perfect insurance. In spite of ongoing financial innovations, financial markets are far from being complete or perfect, and this applies above all for households. Moreover aggregate shocks hit all agents in the same manner, and hence in this case diversification will not be a feasable option. Against this background diversification remains an important component of household risk management, however, it might not be sufficient.

Irrespective of available strategies which serve to handle risks, households are willing to exploit these options only if they dislike income risks which in its turn is a matter of their risk preferences. In this context the distinction between diversifiable and non-diversifiable risks plays an important role. Whereas aversion against diversifiable risk requires a household's utility function to be strictly concave, this is not sufficient in order to describe aversion against non-diversifiable risks. Indeed for quite a time intertemporal household optimization models assumed household utility to be quadratic (cf. Blanchard and Mankiw, 1988, for a critical assessment). This allowed to deriving explicit functions for optimal consumption as well as optimal asset holdings directly from the Euler equations. The obtained functions explain optimal household decisions in terms of consumption-smoothing, consumption-tilting, and they reveal how a risk-averse households seeks protection against diversifiable risks through the choice of an appropriate portfolio for its wealth. However, no such relationship could be derived in terms of non-diversifiable income risk. Differently put, a household with a quadratic utility function is risk-averse concerning the volatility of its income but this does not necessarily imply that this risk-averse household takes appropriate actions in order to reduce this risk. If utility is quadratic this is only the case for diversifiable risk whereas non-diversifiable risk enters utility as a constant thus dropping out during the process of optimization. Hence households with a quadratic utility function will experience disutility from non-diversifiable income risks without taking suitable actions in order to mitigate their undesired impact. This has been considered as an unsatisfactory modelling strategy in particular by Leland (1968) and Sandmo (1970) because it contradicts the very notion of risk-aversion. Both authors derive conditions under which a household that seeks protection against undiversifiable income risks enhances its savings beyond the amount which follows from consumption-smoothing and consumption-tilting. They show that the existence of this so-called *precautionary saving* depends crucially on the existence of a *third* derivative of the household's utility function. Given that the second derivative is negative which implies aversion against diversifiable risks, the third derivative is positive for a household which also dislikes non-diversifiable risks. In order to distinguish this behaviour from "classical" risk-aversion, a household with a negative second and a positive third derivative of its utility function has henceforth been called "prudent". To understand their point consider a household maximizing the following two-period expected utility function[1]:

$$U_t = u\left(C_t\right) + E_t\left[u\left(\widetilde{C}_{t+1}\right)\right] \tag{1}$$

where C_t denotes current real consumption and E_t stands for the expectation operator. Future real consumption is modelled as a continuous random variable which we henceforth denote by a tilde. Since precautionary savings do not derive from some discrepancy between the discount factor and real interest rates we set both equal to zero. The budget constraints for the two periods are then given by

$$Y_t = C_t + S_t \tag{2}$$

$$\widetilde{Y}_{t+1} + S_t = \widetilde{C}_{t+1} \tag{3}$$

whereY_t, $\widetilde{Y}_{t+1}$ are current and future stochastic income, respectively, and S_t stands for real savings. Assuming a strictly concave utility function, we receive the following Euler equation as necessary and sufficient condition for a utility maximum:

$$u'\left(Y_t - S_t\right) = E_t\left[u'\left(\widetilde{Y}_{t+1} + S_t\right)\right] \tag{4}$$

If the household does not discount future consumption, this is equivalent to saying that the household is interested in enjoying the same marginal utility in both periods. In order to derive explicit results we assume that future income fluctuates randomly around some trend value $\overline{Y}$ according to

$$\widetilde{Y}_{t+1} = \overline{Y} + \widetilde{\varepsilon}, \qquad \widetilde{\varepsilon} \sim N\left(0, \sigma_y^2\right) \tag{5}$$

with

$$Y_t = \overline{Y} \tag{6}$$

[1]For a similar procedure see Aizenman (1998).

(6) rules out consumption-smoothing and renders future income uncertainty as the only motive to save. In the next step we linearize the optimality condition (4) around trend values using second order Taylor expansions. Starting with the left-hand side of (4) we obtain

$$u'\left(\overline{Y}-S_t\right)\approx u'\left(\overline{Y}\right)-u''\left(\overline{Y}\right)S_t+\frac{1}{2}u'''\left(\overline{Y}\right)S_t^2 \tag{7}$$

Applying the same procedure to the right-hand side of (4) and assuming that $Cov(S,\widetilde{\varepsilon})=0$, we get

$$E_t\left[u'\left(\overline{Y}+\widetilde{\varepsilon}+S_t\right)\right]\approx u'\left(\overline{Y}\right)+u''\left(\overline{Y}\right)S_t+\frac{1}{2}u'''\left(\overline{Y}\right)S_t^2+\frac{1}{2}u'''\left(\overline{Y}\right)\sigma_y \tag{8}$$

Using (8) and (7), we can reformulate (4) to become

$$S_t\approx-\frac{u'''\left(\overline{Y}\right)}{2u''\left(\overline{Y}\right)}\sigma_y^2 \tag{9}$$

According to equation (9) the existence of precautionary savings depends on the ratio between the second and the third derivative of the household's utility function. Assuming risk-aversion in the "classical" sense (i.e. $u''<0$), we observe that precautionary savings are positively correlated with undiversifiable income riks provided that the third derivative of household utility is positive. The ratio $-\frac{u'''\left(\overline{Y}\right)}{2u''\left(\overline{Y}\right)}$ is usually used as a measure of prudence (Barucci, 2003). Whereas risk-aversion motivates a household to diversify its wealth, prudence gives rise to precautionary savings.

In our paper we are interested in the role of money as a component of household risk management given that the household is risk-averse and prudent. In particular we aim to derive *explicit and plausible* relationships between money demand and income risks. In the following we show that this imposes restrictions upon our choice among available and appropriate utility functions.

3 Choice of the Utility Function

Risk-aversion as well as prudence are compatible with utility functions based on constant *relative* risk-aversion (CRRA) with

$$u=\frac{c^{1-\Theta}}{1-\Theta},\qquad\Theta>0 \tag{10}$$

as a widely used example, as well as with utility functions based on constant *absolute* risk-aversion (CARA) with the exponential utility function

$$u = 1 - e^{-\alpha c}, \quad \alpha > 0 \tag{11}$$

as a prominent representative. In the following we show that the widely used CRRA utility function (10) does not allow us to derive a fully-fledged solution to a household's optimization problem thus also ruling out the possibility to derive an explicit relationship between the demand for money and income risks. This in turn will qualify exponential utility (11) as the appropriate candidate for our analysis.

Utility Functions Based on CRRA In models of intertemporal household optimization it has become common practice to assume utility functions of the CRRA-type with (10) as the most frequently used example. Constant relative risk-aversion has been found to be compatible with a macroeconomic steady state and sometimes, too, the implied degree of decreasing absolute risk-aversion has been legitimated with a high degree of economic plausibility. On the other hand, (10) does not allow to deriving explicit functions for optimal consumption and optimal asset holdings from the Euler equations which has motivated economists to resort to log-linearization techniques. However, in the case of (10) log-linearization does not deliver a complete solution to the optimization problem either, since this would require to take logarithms of budget constraints which of course is not possible. Related to this is the problem that the derivation of optimal money holdings requires money to enter the utility function which may appear as too restrictive. To show these drawbacks we consider a household who maximizes expected utility over two periods.[2] The household starts with initial wealth and current income in the first period. One component of wealth is money which yields a safe nominal and, due to stable inflationary expectations, also a safe real interest rate. The second wealth component represents a risky asset, for example stocks. The household has to decide under uncertainty about the size of future labour income and the future real rate of return on stocks which we both model as normally distributed random variables. Expected utility is given by

$$U_t = \frac{C_t^{1-\Theta}}{1-\Theta} + \beta E_t \left[\frac{\widetilde{C}_{t+1}^{1-\Theta}}{1-\Theta} \right] + \frac{M_t^{1-\gamma}}{1-\gamma} \tag{12}$$

$$\Theta, \gamma > 0 \tag{13}$$

[2]A similar framework has been used by Pétursson (2000) and Stracca (2003).

M stands for real money, and β for the discount factor. Money yields direct utility through its liquidity services. Beyond that it is also used as a store of value yielding a safe real rate of return. In the following we denote by $R^m_{t+1}$the *gross* real rate of return on money holdings in the second period which is related to the nominal safe interest rate on money i^m and inflation, as follows:

$$R^m_{t+1} \equiv \frac{1 + i^m_{t+1}}{1 + \overline{\pi}} \tag{14}$$

where by $\overline{\pi}$ we denote expected inflation which in our approach is assumed to have zero variance. Since we assume an environment marked not only by zero inflationary variance but also by small rates of inflation, (14) can be approximated by

$$R^m_{t+1} \approx 1 + i^m_{t+1} - \overline{\pi}$$

We denote the stochastic gross real rate of return on stocks in the second period by

$$\widetilde{D}_{t+1} \equiv 1 + \widetilde{d}_{t+1}$$

with $\widetilde{d}_{t+1}$ as the real interest rate on stocks. The household maximizes expected utility subject to the following budget constraints:

$$Y_t + M_{t-1}R^m_t + A_{t-1}D_t = C_t + M_t + A_t \tag{15}$$

$$\widetilde{Y}_{t+1} + A_t\widetilde{D}_{t+1} + M_tR^m_{t+1} = \widetilde{C}_{t+1} \tag{16}$$

where now Y_t and $\widetilde{Y}_{t+1}$ should be interpreted as current and future *labour* income, respectively. A_t stands for the real value of stocks. As a consolidated budget constraint we obtain

$$Y_{t+1} - M_t\left(\widetilde{D}_{t+1} - R^m_{t+1}\right) + (W_t - C_t)\,\widetilde{D}_{t+1} = \widetilde{C}_{t+1} \tag{17}$$

$$W_t \equiv Y_t + M_{t-1}R^m_t + A_{t-1}D_t \tag{18}$$

For the sake of analytical tractability we define

$$\widetilde{\Delta}_{t+1} \equiv \widetilde{D}_{t+1} - R^m_{t+1}, \tag{19}$$

Maximizing the utility function (12) subject to (17) and (19) yields the following system of Euler equations:

$$\beta E_t\left[\widetilde{C}^{-\Theta}_{t+1}\widetilde{D}_{t+1}\right] = C^{-\Theta}_t \tag{20}$$

$$\beta E_t\left[\widetilde{C}^{-\Theta}_{t+1}\widetilde{\Delta}_{t+1}\right] = M^{-\gamma}_t \tag{21}$$

In order to obtain an explicit solution it is usually assumed that $ln\left(\widetilde{C}_{t+1}^{-\Theta}\widetilde{D}_{t+1}\right)$ as well as $\ln\left(\widetilde{C}_{t+1}^{-\Theta}\widetilde{\Delta}_{t+1}\right)$ are normally distributed. Using these assumptions and taking logs, the Euler equations can be reformulated to become:

$$\begin{aligned}-\Theta\ln C_t &= \ln\beta - \Theta E_t\left[\ln\widetilde{C}_{t+1}\right] + E_t\left[\ln\widetilde{D}_{t+1}\right] + \frac{1}{2}\Theta^2 Var\left[\ln\widetilde{C}_{t+1}\right] \\ &+ \frac{1}{2}Var\left[\ln\widetilde{D}_{t+1}\right] - \Theta Cov\left(\ln\widetilde{C}_{t+1},\ln\widetilde{D}_{t+1}\right)\end{aligned} \tag{22}$$

$$\begin{aligned}-\gamma\ln M_t &= \ln\beta - \Theta E_t\left[\ln\widetilde{C}_{t+1}\right] + E_t\left[\ln\widetilde{\Delta}_{t+1}\right] + \frac{1}{2}\Theta^2 Var\left[\ln\widetilde{C}_{t+1}\right] \\ &\frac{1}{2}Var\left[\ln\widetilde{\Delta}_{t+1}\right] - \Theta Cov\left(\ln\widetilde{C}_{t+1},\ln\widetilde{\Delta}_{t+1}\right)\end{aligned} \tag{23}$$

Substituting (22) into (23) and considering that $\widetilde{\Delta}_{t+1}$and $\widetilde{D}_{t+1}$are identically distributed since by assumption money yields a safe real rate of return, i.e.,

$$Var\left[\ln\widetilde{\Delta}_{t+1}\right] = Var\left[\ln\widetilde{D}_{t+1}\right]$$

and

$$Cov\left(\ln\widetilde{C}_{t+1},\ln\widetilde{\Delta}_{t+1}\right) = \left(\ln\widetilde{C}_{t+1},\ln\widetilde{D}_{t+1}\right)$$

we get

$$\ln M_t = \frac{\Theta}{\gamma}\ln C_t - \frac{E_t\left[\ln\left(\widetilde{D}_{t+1} - R_{t+1}^m\right)\right]}{\gamma} + \frac{E_t\left[\ln\widetilde{D}_{t+1}\right]}{\gamma} \tag{24}$$

Recalling that $\widetilde{\Delta}_{t+1} \equiv \widetilde{D}_{t+1} - R_{t+1}^m$, equation (24) explains optimal money holdings as a function of the expected real rate of return on money and on consumption – all expressed in logarithmic terms. Usually (24) is used for econometric analyses, where current consumption is frequently replaced by current income (Stracca, 2003). This, however, ignores that the system of equations (22) and (23) do not provide the complete solution of the optimization problem. Rather such an equation explains how a given wealth position to be expected over the planning horizon is optimally *allocated* between money and current consumption. In order to calculate the complete solution we would have to take the *consolidated budget constraint* into account. This of course is not possible in our approach because we would have to take logs of a sum – which is of course impossible. It also implies that this framework does neither allow to derive a relationship between optimal money holdings and diversifiable risks nor between money and undiversifiable risks. The reason for this result is that both types of risk will leave the optimal structure between *current* consumption and money unaffected. To overcome this problem, Choi and Seonghwan (2003) have introduced an equation that projects current consumption on future output. However, without offering a suitable theoretical foundation, this approach is rather ad hoc. We may therefore conclude that CRRA utility functions are not helpful in explaining a store-of-value function of money. In the following we show that exponential utility functions allow us to overcome these problems. Of course exponential utility has not gone uncriticized. In particular it is incompatible with a long-run steady state. However, since our approach does not intend to draw long-run macroeconomic implications, we do not see this as a pivotal drawback.

In the next section we derive expected utility from an exponential utility function using the certainty equivalent. We then go on studying the special role of money compared to other riskless assets under this setting. Finally we derive an explicit money demand function which explains optimal money holdings in terms of consumption-smoothing, consumption-tilting as well as precautionary and portfolio motives.

Deriving Expected Utility from the Exponential Utility Function: the Return of the Certainty Equivalent If the utility function is assumed to be quadratic, expected utility can be expressed in terms of the certainty equivalent which simplifies the optimization process and allows the derivation of explicit functions for optimal consumption and assets. Frequently therefore the quadratic utility model to household optimization is referred to as *the* certainty equivalent approach (Blanchard and Mankiw, 1988) This statement ignores that exponential utility, too, yields the certainty equivalent as the crucial component of expected utility. However, whereas the certainty equivalent enters expected quadratic utility in a linear manner, the relationship between expected utility and the certainty equivalent is non-linear in the exponential case. This non-linearity follows from the property of exponential utility to have a third (positive) derivative, and indeed it is exactly this property which explains the existence of precautionary savings. To see how the certainty equivalent and the expectation of an exponential utility function are related, we use a result from portfolio theory saying that a risk-averse agent is willing to pay a risk premium in order to convert a lottery into a save income (Barucci, 2003). The lottery in our model relates to second period income which in its turn renders second period consumption an uncertain event. The size of the risk premium ρ which the household is willing to pay, is given by the equality of the household's utility from the certainty equivalent CE and expected utility from future consumption $\overline{C}$, i.e.

$$U\left(CE_{t+1}\right) = E\left[U\left(\widetilde{C}_{t+1}\right)\right] \tag{25}$$

where

$$CE_{t+1} = \overline{C} - \rho \tag{26}$$

In order to derive the risk-premium we assume that future consumption fluctuates randomly around its expectation according to

$$\widetilde{C}_{t+1} = \overline{C} + \widetilde{\varepsilon}, \qquad \approx N\left(0, \sigma_{\varepsilon}^{2}\right) \tag{27}$$

where

$$\sigma_{\varepsilon}^{2} \equiv Var\left[\widetilde{C}\right]$$

Next we linearize the left-hand side of (25) by a Taylor expansion of first order which makes sense if we assume that the risk premium remains sufficiently small. In this case we obtain

$$U\left(CE_{t+1}\right) = U\left(\overline{C}\right) - U'\left(\overline{C}\right)\rho \tag{28}$$

In linearizing the right-hand side of (25), we start with $U\left(\widetilde{C}_{t+1}\right)$ yielding

$$U\left(\widetilde{C}_{t+1}\right) = U\left(\overline{C}\right) + U'\left(\overline{C}\right)\widetilde{\varepsilon} + \frac{1}{2}U''\left(\overline{C}\right)\widetilde{\varepsilon}^{2} \tag{29}$$

Since we have assumed that $\widetilde{\varepsilon}$ describes a random variable, we are not allowed to neglect the third term of the Taylor expansion since this would imply a variance of zero turning the income disturbance into a deterministic variable (Neftci, 2000). Moreover recall that even if $\widetilde{\varepsilon}$ has zero expectation, its variance may be significant. Taking expectations of (29), we obtain

$$E\left[U\left(\widetilde{C}_{t+1}\right)\right] = U\left(\overline{C}\right) + \frac{1}{2}U''\left(\overline{C}\right)Var\left[\widetilde{C}\right] \tag{30}$$

Equating (28) and (30) delivers as the risk premium

$$\rho = -\frac{u''\left(\overline{C}\right)}{2u'\left(\overline{C}\right)}Var\left[\widetilde{C}\right] \tag{31}$$

where $-\frac{u''\left(\overline{C}\right)}{2u'\left(\overline{C}\right)}$ reflects the absolute degree of risk aversion. Using (25) together with (26) and (31), the utility function can be reformulated to become

$$U_t = u\left(C_t\right) + \beta u\left(CE_{t+1}\right) \tag{32}$$

In case of exponential utility as specified by (11), the absolute degree of risk aversion equals $-\frac{\alpha}{2}$ and the household two-period expected utility function reads as

$$U_t = \left(1 - e^{-\alpha C_t}\right) + \beta \left(1 - e^{\alpha CE_{tt+1}}\right) \tag{33}$$

where the certainty equivalent is given by

$$CE_{t+1} = E_t \left[\widetilde{C}_{t+1}\right] - \frac{\alpha}{2} Var \left[\widetilde{C}_{t+1}\right] \tag{34}$$

In the next section we explore the store-of-value-function of money in intertemporal household optimization using (33) together with (34). We start with the simple case in which wealth is exclusively composed of money. As a next step we investigate how money can be distinguished from other riskless assets in the framework of household optimization. Finally we analyse the relationship between optimal money holdings, diversifiable and non-diversifiable income risks.

4 Household Risk Management and Optimal Money Holdings Under Exponential Utility

Money As the Only Store of Value In the following we continue to assume that inflationary expectations are low and in particular stable which turns money into a safe asset. For the moment we ignore a non-negativity constraint on money holdings but take up this issue when we discuss the optimal results. To simplify the notation we omit the time index for second period variables whenever any confusion with first period values can safely be ruled out. Our household solves the following optimization problem:

$$\max_{M_t} U_t = \left(1 - e^{\alpha C_t}\right) + \beta \left(1 - e^{\alpha CE}\right) \tag{35}$$

with

$$C_t = Y_t + M_{t-1} R_t^m - M_t \tag{36}$$

and

$$CE = E_t \left[\widetilde{C}\right] - \frac{\alpha}{2} Var \left[\widetilde{C}\right] \tag{37}$$

where

$$\widetilde{C} = \widetilde{Y} + M_t R_{t+1}^m \tag{38}$$

and hence

$$CE = E_t\left[\widetilde{Y}\right] + M_t R^m_{t+1} - \frac{\alpha}{2} Var\left[\widetilde{Y}\right] \tag{39}$$

As the first-order condition we obtain

$$-e^{-\alpha C_t} + \beta e^{-\alpha CE}\left(R^m_{t+1}\right) = 0 \tag{40}$$

or equivalently

$$e^{\alpha(CE - C_t)} = \beta R^m_{t+1} \tag{41}$$

Taking logs on both sides of (41) yields

$$CE - C_t = \frac{\ln\left(\beta R^m_{t+1}\right)}{\alpha}. \tag{42}$$

βR^m_{t+1} denotes the ratio between the gross rate of return on money and the gross rate of time preference. If $\beta R^m_{t+1} > 1$, then $\ln\left(\beta R^m_{t+1}\right) > 0$, and the optimality condition (42) requires that the certainty equivalent exceeds current consumption. This implies that the household shifts current consumption into the future i.e. the household saves from out of the consumption-tilting motive. Hence given that money yields a positive (real) rate of return exceeding the rate of time preference, one component of money demand relates to the consumption-tilting motive. However, money will be held even if the real rate of return on money is not higher than necessary to compensate for the household's impatience. To see this, we set $\beta R^m_{t+1} = 1$, and hence $.\ln\left(\beta R^m_{t+1}\right) = 0$ Obviously in this case (42) requires $C_t = CE$. For the moment we ignore income volatility. In this case the certainty equivalent reduces to expected consumption. A coincidence between current and expected consumption expresses the household's desire to enjoy at least on average the same level of consumption in each period. In order to realize this, the household will want to save whenever its future expected income falls short of current income. This is an expression for the classical consumption-smoothing motive. In the face of income volatility, however, a household will want to save even if current and expected income coincide. In this case savings serve to finance the risk premium which the household is willing to pay in order to receive expected consumption with certainty instead of stochastic consumption. Our arguments can be given a formal representation by inserting (36) and (39) into (42). By rearranging terms we obtain as an explicit expression for the demand for money

$$M_t = \frac{1}{1 + R^m_{t+1}}\left[\frac{\ln\left(\beta R^m_{t+1}\right)}{\alpha} + \left(Y_t - E_t\left[\widetilde{Y}\right]\right) + \frac{\alpha}{2} Var\left[\widetilde{Y}\right] + M_{t-1} R^m_t\right] \tag{43}$$

with optimal savings being determined by

$$\begin{aligned} M_t - M_{t-1} &= \frac{1}{1+R^m_{t+1}} \left[\frac{\ln\left(\beta R^m_{t+1}\right)}{\alpha} + \left(Y_t - E_t\left[\widetilde{Y}\right]\right) \right] \\ &+ \frac{1}{1+R^m_{t+1}} \left[\frac{\alpha}{2} Var\left[\widetilde{Y}\right] - \left(1 + \Delta R^m_{t+1}\right) M_{t-1} \right] \\ \Delta R^m_{t+1} &= R^m_{t+1} - R^m_t \end{aligned} \tag{44}$$

Ignoring initial wealth, we see from (43) that money is held out of three motives which all account for a store-of-value function, namely consumption-tilting, consumption-smoothing and the protection from undiversifiable income risks which we call henceforth the "consumption-stabilizing motive". The consumption-stabilizing motive serves as an explanation for positive money holdings even if in the absence of initial money holdings, the real rate of return on money coincides with the rate of subjective time preference, and if current and average income are the same.

A final remark concerns the issue of *negative* money holdings which in our model so far have not been excluded. From (43) we may conclude that optimal money money holdings will certainly be negative if the rate of return on money falls short of the subjective rate of time preference and if this is not compensated by an excess of current over risk-adjusted future income plus positive initial money holdings. Likewise we can expect negative money holdings if the household expects a risk-adjusted income which is higher than the current level and if this is not compensated by an excess of the real rate of return on money over time preference and positive initial money holdings. On the other hand we also observe that the presence of income volatility makes negative optimal money holdings less likely.

Considering Liquidity Services How can money be distinguished from other riskless assets? In the literature the two most frequently applied procedures consist of either assigning direct utility to money[3] or of accounting for transaction cost savings in the budget constraint (Saving, 1971; McCallum and Goodfriend, 1987).[4]

[3]This procedure dates back to Patinkin (1965). It has been also applied in Barnett et al. (1992), and it is typically used in New Keynesian models (cf. Mankiw and Romer, eds, 1991, for a survey), see furthermore Dutkowsky and Foote (1992). For an application in real business cycle models cf. Farmer (1997).

[4]An alternative idea has been proposed by Freitas and Veiga (2006) who consider foreign bonds in their model and assume that households have only limited access to bond markets.

Integrating money into the expected utility function (33) is done frequentyly[5] by adding a term which accounts for a direct utility of money. In our case this would mean that the utility function now turns into

$$U_t = \left(1 - e^{\alpha C_t}\right) + \beta \left(1 - e^{\alpha CE}\right) + \left(1 - e^{-\gamma M_t}\right) \tag{45}$$

Maximizing (45) with respect to (36) and (39) yields as first-order conditions

$$\alpha\beta e^{-\alpha CE} = \alpha e^{-\alpha C_t} - \gamma e^{-\gamma M_t} \tag{46}$$

Consider a decline in the certainty equivalent for example due to lower expected income or due to higher income risk, thus increasing its marginal utility. In the absence of a direct utility of money this would have to be compensated exclusively by an increase in the current marginal utility of consumption of the same size, and the only way to achieve this is to save more, i.e., to increase money holdings. On the other hand if money yields direct utility any increase in the marginal utility of the certainty equivalent can also be compensated by a higher marginal utility associated with liquidity services. However, this is not achieved by higher but by *lower* money holdings. Hence if money enters the utility function like in (45), this contributes to mitigating its consumption-smoothing and consumption-stabilizing role, or differently put, the relationship between the store-of-value function and the liquidity function of money is conflicting. We think that this does not have much economic plausibility because if a household wants to smooth or stabilize consumption by accumulating assets, this makes only sense if these assets can be liquidated at no or at least low cost. Hence money as immediate purchasing power and money as a store of value should be complements and not substitutes. This has been considered in Feenstra (1986) who redefined consumption as a variable which both includes physical goods as well as liquidity services, and in Correia and Teles (1999) who redefined leisure appropriately (cf. Wang and Yip, 1992 for a generalization). These authors, too confirm that in this way a qualitative equivalence exists when compared to approaches in which liquidity services are modelled as the saving of transaction costs (de Alencar and Nakane, 2003). For the sake of formal tractability we therefore use this alternative which accounts for liquidity services in the budget constraints. In the literature we find basically two ways how this can be achieved. One way is to model a transaction technology with money as the only input and additional resources due to the avoidance of transaction costs as the output (Végh, 1989; Zhang, 2000). Another way is to assume that the liquidation of assets other than money incurs transaction

[5]This is e.g. a common procedure used in New Keynesian models.

costs (Niehans (1978)). Since the first alternative inflicts upon our approach significant formal difficulties, we have opted for the second alternative and in doing so we take transaction costs to be quadratic. In the following we assume that household wealth is now composed of two risk-free assets, say money and government bonds. For simplicity we ignore sales of bonds in the first period.

Our household then maximizes (33) subject to

$$\begin{aligned} C_t &= Y_t + M_{t-1}R_t^m + B_{t-1}R_t^b - M_t - B_t \qquad (47) \\ W_t &\equiv Y_t + M_{t-1}R_t^m + B_{t-1}R_t^b \\ CE &= E\left[\widetilde{Y}\right] + M_t R_{t+1}^m + B_t R_{t+1}^b - \\ &\quad \nu_b \left(B_t - B_{t-1}\right)^2 - \frac{\alpha}{2} Var\left[\widetilde{Y}\right] \\ \nu_b &> 0 \text{ and constant} \qquad (48) \end{aligned}$$

where B denotes the real value of government bonds which the household holds or wants to hold in its portfolio, and $\nu_b \left(B_t - B_{t-1}\right)^2$ describes convex transaction costs. As first-order conditions for a utility maximum we obtain

$$-e^{-\alpha C_t} + \beta e^{-\alpha CE} R_{t+1}^m = 0 \qquad (49)$$

$$-e^{-\alpha C_t} + \beta e^{-\alpha CE} \left[R_{t+1}^b - \nu_b \left(B_t - B_{t-1}\right)\right] = 0 \qquad (50)$$

(49) and (50) imply

$$R_{t+1}^m = R_{t+1}^b - \nu_b B_t + \nu_b B_{t-1}$$

and hence

$$B_t = \frac{R_{t+1}^b - R_{t+1}^m}{\nu_b} + B_{t-1} \qquad (51)$$

In the face of transaction costs, our household is willing to accumulate government bonds only if they yield a rate of return which exceeds that of money. The size of bond holdings increases with a declining value of ν_b which might be interpreted as an indicator of financial market efficiency.

Returning to to equation (49) we can take logs and obtain

$$\alpha C_t = \alpha CE - \ln\left(\beta R_{t+1}^m\right) \qquad (52)$$

Using (49), (50) and (51), this equation can be rearranged to become

$$M_t = \left(\frac{\ln\left(\beta R^m_{t+1}\right)}{\alpha} + W_t - E\left[\widetilde{Y}\right] + \frac{\alpha}{2} Var\left[\widetilde{Y}\right] \right) - \tag{53}$$

$$\frac{1+R^b_{t+1}}{1+R^m_{t+1}} \left(\frac{R^b_{t+1} - R^m_{t+1}}{\nu_b} + B_{t-1} \right) + \frac{1}{1+R^m_{t+1}} \frac{\left(R^b_{t+1} - R^m_{t+1}\right)^2}{\nu_b}$$

According to the terms in the *first* bracket money continues to serve as a store-of-value according to a consumption-tilting, consumption-smoothing as well as a consumption-stabilizing motive. However, the extent to which this occurs now depends on optimal bond holdings. The terms in the *second* bracket describe the impact of optimal bond holdings due to a discrepancy between the real rate on bonds and money. If this wedge is positive, then the household will want to hold bonds in positive quantity which due to the budget constraint reduces optimal money holdings. The terms in the *third* bracket describe the impact of bonds on optimal money holdings due to adjustment costs which arise in the course of selling government bonds. The existence of these adjustment costs have a positive effect on optimal money holdings. Since we have assumed these costs to be quadratic their value increases with the magnitude of bonds. Recalling that optimal bonds are determined by the wedge between their interest rate and the interest rate on money, this explains why a positive wedge affects optimal money holdings in a positive direction now. In calculating net effects of the interest rate differential between bonds and money, we ignore initial wealth. This assumption simplifies the analysis without affecting our results. Then (53) can be reformulated as follows:

$$M_t = \frac{1}{1+R^m_{t+1}} \left(\frac{\ln\left(\beta R^m_{t+1}\right)}{\alpha} + Y_t - E\left[\widetilde{Y}\right] \right) \tag{54}$$

$$+ \frac{1}{1+R^m_{t+1}} \left(\frac{\alpha}{2} Var\left[\widetilde{Y}\right] - \frac{R^b_{t+1} - R^m_{t+1}}{\nu_b} \right),$$

$$\text{for } M_{t-1} = B_{t-1} = 0$$

We observe that the adjustment cost effect does not dominate, and hence a positive interest rate differential between bonds and money affects optimal money holdings negatively thus increasing the likelihood that our household will not want to hold money at all or even hold it in negative quantities. But still we observe that non-diversifiable income risks lower this probability.

The Store-of-Value-Function of Money in the Face of Undiversifiable and Diversifiable Income Risks Household risk management comprises both: the usage of possible diversification options as well as the accumulation of buffer stocks to protect against non-diversifiable risks. To explore the role of money in this context we now consider a household who faces money and risky stocks as investment alternatives. Since liquidity services are of minor importance in this context, they will henceforth be ignored. Stocks yield a stochastic gross rate of return of $\widetilde{D}$ in t+1 with expectation $E\left[\widetilde{D}\right]$ and variance $Var\left[\widetilde{D}\right]$. The household then maximizes

$$U_t = \left(1 - e^{\alpha C_t}\right) + \beta\left(1 - e^{\alpha CE}\right) \tag{55}$$

subject to

$$C_t = Y_t + A_{t-1}D_t + M_{t-1}R_t^m - M_t - A_t \tag{56}$$

$$Y_t + A_{t-1}D_t + M_{t-1}R_t^m \equiv W_t \tag{57}$$

$$\begin{aligned} CE = E_t\left[\widetilde{Y}\right] + A_t E_t\left[D\right] + M_t R_{t+1}^m - \\ \frac{\alpha}{2}\left(Var\left[\widetilde{Y}\right] + A_t^2 Var\left[\widetilde{D}\right] + 2A_t Cov\left[\widetilde{Y},\widetilde{D}\right]\right) \end{aligned} \tag{58}$$

where A_t again denotes the real value of equity. From (58) we observe that part of the household's income risk follows from its portfolio decisions which concern the optimal structure of its wealth and its savings respectively. The household is now exposed to income risks from two sources. The first source relates to labour, the second source to risky shares. As first-order conditions we obtain

$$e^{-\alpha C_t} = \beta e^{-\alpha CE} R_{t+1}^m \tag{59}$$

$$e^{-\alpha C_t} = \beta e^{-\alpha CE}\left(E_t\left[\widetilde{D}\right] - \alpha A_t Var\left[\widetilde{D}\right] - \alpha Cov\left[\widetilde{Y},\widetilde{D}\right]\right) \tag{60}$$

We can combine (59) and (60) to obtain as optimal stock holdings

$$A_t = \frac{\left(E_t\left[\widetilde{D}\right] - R_{t+1}^m\right) - \alpha Cov\left[\widetilde{Y},\widetilde{D}\right]}{\alpha Var\left[\widetilde{D}\right]} \tag{61}$$

We observe a close similarity to classical portfolio models where the demand for risky assets depends on the wedge between their expected rates of return and the riskless rate, and where in addition stock return volatility measured by its variance plays a crucial role. Furthermore provided that stock returns and labour income are negatively correlated, stock holdings may serve to reduce the negative impact of labour income volatility. By contrast if labour and capital income are positively correlated, then labour income risk propagates capital income risk thus lowering optimal stock holdings. We also observe from (61) that short-selling would always be desirable only if $\left(E_t\left[\widetilde{D}\right]-R^m_{t+1}\right)-\alpha Cov\left[\widetilde{Y},\widetilde{D}\right]\leq 0$. In the following we rule out this case and thus assume that stocks are held in positive quantity.[6] In order to achieve an explicit money demand function for this case, we first take logs of (59) yielding

$$W_t - M_t - A_t + \frac{1}{\alpha}\ln\left(\beta R^m_{t+1}\right) = E_t\left[\widetilde{Y}\right] + A_t E_t\left[\widetilde{D}\right] + M_t R^m_{t+1} - \frac{\alpha}{2}\left(A_t^2 Var\left[\widetilde{D}\right] + Var\left[\widetilde{Y}\right] + 2A_t Cov\left[\widetilde{Y},\widetilde{D}\right]\right) \tag{62}$$

Using (61), (62) can be rearranged as follows:

$$\begin{aligned} M_t = {} & \frac{1}{1+R^m_{t+1}}\left[\frac{1}{\alpha}\ln\left(\beta R^m_{t+1}\right) + \left(W_t - E_t\left[\widetilde{Y}\right]\right) + \frac{\alpha}{2}Var\left[\widetilde{Y}\right]\right] - \\ & \frac{1}{1+R^m_{t+1}}\left(\frac{\left(E_t\left[\widetilde{D}\right]-R^m_{t+1}\right)-\alpha Cov\left[\widetilde{Y},\widetilde{D}\right]}{\alpha Var\left[\widetilde{D}\right]}\right)\Phi + \\ & \frac{1}{1+R^m_{t+1}}\frac{\left(E_t\left[\widetilde{D}\right]-R^m_{t+1}-\alpha Cov\left[\widetilde{Y},\widetilde{D}\right]\right)^2}{2\alpha Var\left[\widetilde{D}\right]^2}Var\left[\widetilde{D}\right] \\ \Phi = {} & \left(1+E_t\left[\widetilde{D}\right]-\alpha Cov\left[\widetilde{Y},\widetilde{D}\right]\right) \end{aligned} \tag{63}$$

The *first* bracket resumes the results which we have obtained in our simple model with money as the only store of value. In particular money continues to protect our household from undiversifiable income risks. The *second* bracket describes the impact of risky stocks on optimal money holdings due to the fact that stocks and money are competing alternatives and due to the assumption that stocks yield a positive rate of return which may correlate with labour income. We observe that a positive wedge between the rate of return on risky assets and the riskless interest rate affects optimal

[6] Short-selling stocks in our model would not impair the store-of-value function of money but rather augment it. However since this augmenting effect does not have its source in income uncertainty we ignore this possibility henceforth.

money holdings negatively whereas the opposite holds in terms of the variance of stock returns. We also see that due to our assumption of positive stock holdings, $\left(E_t\left[\widetilde{D}\right]-R^m_{t+1}\right)-\alpha Cov\left[\widetilde{Y},\widetilde{D}\right]>0$ and therefore we also have $\left(1+E_t\left[\widetilde{D}\right]-\alpha Cov\left[\widetilde{Y},\widetilde{D}\right]\right)>0$ which implies that a negative correlation between stock returns and labour income dampens the consumption-stabilizing effect of money holdings because in this case shares also qualify as a buffer stock against labour income risks. The *third* bracket describes the impact of stocks due to capital income risk, and hence the effects of stock market volatility on money demand. If this risk goes up, our household will increase its demand for money. Notably capital income risk is positively correlated with optimal stock holdings which explains why now a positive wedge between the rate of return on risky assets and the riskless rate have a positive effect on optimal money holdings. The same applies to a negative correlation between capital and labour income risks. Finally we observe that according to the fourth bracket the impact of the variance of stock returns on money is negative. The reason is that optimal stock holdings are negatively correlated with $Var\left[\widetilde{D}\right]$. Since the variance of capital income is determined by the squared value of optimal stock holdings, this effect dominates. To find out net effects we rearrange (63) accordingly which leads to the following expression for optimal money holdings:

$$M_t=\frac{1}{1+R^m_{t+1}}\left[\frac{1}{\alpha}\ln\left(\beta R^m_{t+1}\right)+\left(W_t-E_t\left[\widetilde{Y}\right]\right)+\frac{\alpha}{2}Var\left[\widetilde{Y}\right]\right]- \tag{64}$$
$$\frac{1}{1+R^m_{t+1}}\left(\frac{\left(E_t\left[\widetilde{D}\right]-R^m_{t+1}\right)-\alpha Cov\left[\widetilde{Y},\widetilde{D}\right]}{\alpha Var\left[\widetilde{D}\right]}\right)\Psi$$

$$\Psi=1+\frac{E_t\left[\widetilde{D}\right]+R^m_{t+1}-\alpha Cov\left[\widetilde{Y},\widetilde{D}\right]}{2} \tag{65}$$

For positive stock holdings Ψ, is positive. Hence we may summarize that in this case optimal money holdings are negatively correlated with a positive wedge between the expected rate of return on stocks and the real rate of return on money which says that a higher interest rate differential motivates a household to restructure its wealth and its saving in favour of stocks. However, given risk-aversion, the variance of stock returns as well as the covariance of capital and labour income come into play. In particular money holdings serve to dampen the impact of risky stock returns and thus serve to lower diversifiable risks. This result also confirms the findings of

Carpenter and Lange (2003) who find empirical evidence for a positive impact of equity market volatiliy on money demand. On the other hand we observe that the role of money in the context of labour income risk is now also influenced by the correlation between labour income and stock returns. In case of a positive correlation, the significance of money holdings as a buffer against labour income risks becomes even more pronounced, whereas the opposite holds in case of a negative covariance. How labour income and stock returns are correlated, is of course a matter of empirical findings. For example Willen and Steven (2000) have found that income distribution plays a role in this respect since a positive correlation appears to be the case in particular for above average income households.

5 Conclusions

In the paper we have made an attempt, to analyse the role of money as a store of value in intertemporal household optimization models under price stability but with diversifiable and undiversifiable income risks – a setting which we found to be a good description of recent trends in industrialized countries. We discussed the appropriateness of several strategies for the theoretical modelling of household utility functions and opted for an exponential utility function in our approach. Using the certainty equivalent, we have been able to derive a non-linear expression for money holdings which allows for an analytical distinction between the classical saving motives – consumption-tilting and consumption-smoothing – on the one hand, and a consumption-stabilisation motive on the other hand, which reflects precautionary savings due to the existence of undiversifiable income risks. We analysed how money can be distinguished from other riskless assets and we furthermore analysed, how savings due to undiversifiable risks might be allocated under the realistic assumption that risks are not independent.

In short, our analysis points to important repercussions of changes in the allocation of risks for important economic relationships – like a structural money demand function. Under the existence of undiversifiable income risks, the interactions of labor income and asset market risks matter for precautionary savings. This in turn implies, that a more careful approach has to be opted for, when introducing more "flexibility" into labor and capital markets which takes into account possible repercussions on important economic relationships as money demand.

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