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Default Option, Risk-Aversion and Household Borrowing Behaviour

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Abstract

Assuming a risk-neutral bank and assuming household utility to be exponential, we show how under information symmetry the covariance of income and loan repayments may explain higher household borrowings than in the case without default option. Under ex post information asymmetry and positive control costs, the result is less clear-cut. We also make evident that in a situation in which a household without default option would neither borrow nor save, the existence of a default option makes household borrowing behaviour unpredictable.

Keywords: Consumption, exponential utility, certainty equivalent, households, default option, borrowing, risk, risk aversion, risk management

JEL classification: D11, D14, D18, D53, D81

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1 Introduction

Since the ongoing 1990s household liabilities have risen sharply in the Eurozone and in other OECD countries, both in absolute as well as in relative size (Rinaldi et al., 2006). Underlying these developments have been growing competitive pressures within and on the banking sector which resulted in declining interest rates on consumer loans as well as in lower credit constraints (Crook et al. 2006). A further argument explaining the increase in household debt is related to the implementation of consumer bankruptcy laws which offer households the option to default on their loans. These laws fix exemption rates on wealth and income and clarify conditions under which a household may obtain release from its debt. The observed sharp increase in bankruptcy filings in the US during the 1990s has fuelled a debate on whether households use their default option only in the face of adverse shocks which make them unable to reimburse their borrowings or whether household behave strategically. Whereas in particular sociologists advocate the first view (Sullivan et al., 1989), Fay et al. (2002) have found evidence that households use their default option strategically. According to their findings, households' bankruptcy filing rates are positively and significantly correlated with the exemption rate, i.e. that part of wealth and income which a bank is not allowed to pledge. To understand household behaviour under a default option is important because this helps to explain whether the observed absolute and relative increase in debt may also enhance households' financial fragility which – in particular due to households' limited liability – may spill over to the banking sector. A positive correlation between household borrowings and household financial fragility is less the case if low interest rates or higher incomes can be made out as a determining factor of the credit boom than if high exemption rates set households incentives to borrow more than otherwise.

In the following we analyse whether and how such strategical behaviour fits into the expected utility framework. Households maximizing expected utility choose a time path for consumption which maximizes an increasing and strictly concave utility function. According to the first property a household prefers more consumption to less, whereas the second property makes a household risk-averse in the sense that it prefers a safe over a random level of consumption. The assumption of a planning horizon extending one period moreover requires from a household to strike a balance between present and future utility and hence between present and future consumption. In this respect borrowing as well as saving allow to shift consumption between periods, and the degree of risk-aversion then determines the extent to which a household prefers these shifts due to changing interest rates. In the literature dealing with optimal household behaviour it is predominantly assumed that households have to repay their liabilities at the end of the examined planning horizon, i.e., households do not possess a default option. Credit constraints in these models indeed appear to lack any reasoning since default and furthermore information asymmetry are typically excluded from these approaches.(Lawrance, 1995).[1] An exception has been provided by Lawrance (1995) who concludes that risk-averse households are more inclined to borrow when they have a default option than this would be the case without such a possibility. In particular she finds that in a situation in which a household would neither borrow nor save in the absence of a default option, this same household will borrow once it can declare bankruptcy. She explains this result by a household's risk-aversion which implies that the non-seizable income, which indeed constitutes a safe income, is always preferred to random revenues. However, her model is rather special concerning the assumed Bernoulli probability distribution of future incomes and the size of the non-seizable income which is assumed to be fixed at the lowest possible income level. Due to this last restriction implications of the size of non-seizable income remain beyond discussion. Moreover the use of a Bernoulli distribution for future incomes makes it necessary to assume that default coercively occurs once a low income situation is realized. This in turn implies that households with a preference for borrowing always borrow at least as much as this lowest income level. Finally, using a CRRA utility function, her conclusions are more or less restricted to a graphical analysis.

In the following we generalize Lawrance (1995) by assuming a continuous probability distribution for future incomes and furthermore leave the exemption ratio of income unspecified which allows us to discuss its role for optimal borrowing. Contrary to Lawrance (1995) we assume utility to be exponential. This allows us to introduce the certainty equivalent into expected utility which makes it possible to analyse explicitly the role of risk and risk-aversion. The remainder of the paper is structured as follows: In the next section the optimization model of a risk-averse household is presented where as a first step the loan interest rate is taken as given Then optimal bank behaviour with regard to a quantitative credit constraint and the loan interest rate will be specified, where as a first step symmetric information between borrower and lender will be assumed. Taking the bank's credit policy into account, the household's optimization problem will be solved and results discussed. As a further step ex post information asymmetry will be introduced, which motivates the bank to charge control costs from its borrower. The paper concludes with a summary of relevant results and a suggestion concerning extensions.

[1] There is indeed a large literature on the impact of credit constraints on household behaviour. Examples are Zeldes (1989); Deaton (1991); Carroll et al. (2001); Cox et al. (1993), Ludvigson (1999) to name but a few.

2 A Model of Risk-Averse Household Behaviour

We consider a household which maximizes expected utility over two periods and in doing so uses borrowings and savings in order to tilt, smooth and stabilize[2] consumption. Uncertainty is related to second-period labour income which is assumed to be a continuous random variable $\widetilde{Y}$[3] with realizations Y_{t+1} falling into the interval $\left[\underline{y}, \overline{y}\right]$, density function $f\left(Y_{t+1}\right)$ and cumulative distribution function $F\left(Y_{t+1}\right)$, where $F\left(\overline{y}\right) = 1$ and $F\left(\underline{y}\right) = 0$. All variables are expressed in real terms, hence income uncertainty in our model is equivalent with uncertainty concerning employment or real wages. If the household decides to save, then it will purchase a riskless asset at the amount D_t yielding a safe gross interest rate R_{t+1} in the second period. Assuming utility to be exponential, allows us to use the certainty equivalent which turns expected utility into[4]

$$U = \left(1 - e^{-\alpha C_t}\right) + \beta\left(1 - e^{-\alpha CE}\right) \tag{1}$$

where C_t stands for current consumption and can be expressed in terms of the first-period budget constraint

$$C_t = Y_t + L_t - D_t \tag{2}$$

with L_t standing for loans taken in the first periods. In our model we have neglected initial wealth, and we do not exclude the possibility that a household will choose to borrow and to save simultaneously. CE denotes the certainty equivalent and is determined by (Größl and Fritsche, 2007)

$$CE = E\left[\widetilde{C}\right] - \frac{\alpha}{2} Var\left[\widetilde{C}\right] \tag{3}$$

where future random consumption $\widetilde{C}$ is given by the second-period budget constraint as follows

$$\widetilde{C} = \widetilde{Y} + D_t R_{t+1} - \widetilde{X} \tag{4}$$

Under a prevailing default option, the repayment of loans is a random variable, too, which we henceforth denote by $\widetilde{X}$.

The certainty equivalent expresses a risk-averse household's willingness to pay a premium in order to enjoy expected future consumption with certainty. By help of (4), we can rearrange (3) to become

$$\begin{aligned} CE \quad = \quad & E\left[\widetilde{Y}\right] + D_t R_{t+1} - E\left[\widetilde{X}\right] - \frac{\alpha}{2} Var\left[\widetilde{Y}\right] - \frac{\alpha}{2} Var\left[\widetilde{X}\right] + \\ & \alpha Cov\left[\widetilde{X}, \widetilde{Y}\right] \end{aligned} \tag{5}$$

[2]As in Größl and Fritsche (2007) we use the term "consumption-stabilizing" to explain savings from a precautionary motive.

[3]Since it is clear that random variables relate to the second period we omit the time index henceforth.

[4]A formal derivation is presented in Größl and Fritsche, 2007.

Notice that the expected utility framework specifies risk by the variance. Hence a risk-averse household dislikes both positive as well as negative deviations from long-run averages. This in turn explains why squared deviations of debt repayments from a long-run average $Var\left[\widetilde{X}\right]$, are *added* and not subtracted from the expected loan repayment. Note also that in contrast a positive covariance of income and loan repayments *augments* the certainty equivalent. In order to understand this, assume that income deviates negatively from its long-run average. Then, given a positive correlation coefficient, loan repayments will fall short of their long-run average, too. Whereas the first effect reduces the certainty equivalent, the second effect does the opposite.

In accordance with the probability distribution of income specified above, the household can expect as an average labour income over the long run

$$E\left[\widetilde{Y}\right] = \int_{\underline{y}}^{\overline{y}} Y_{t+1} f\left(Y_{t+1}\right) dY_{t+1} = \overline{y} - \int_{\underline{y}}^{\overline{y}} F\left(Y_{t+1}\right) dY_{t+1} \tag{6}$$

and as an average of squared deviations from this expectation

$$Var\left[\widetilde{Y}\right] = \int_{\underline{y}}^{\overline{y}} \left(Y_{t+1}\right)^2 f\left(Y_{t+1}\right) dY_{t+1} - \left(E\left[\widetilde{Y}\right]\right)^2 \tag{7}$$

Since loans are repaid out of income, debt repayment $\widetilde{X}$, is continuous, too with identical density and cumulative distribution. Since the household possesses a default option, realized loan repayments are determined piecewise, i.e.,

$$X_{t+1} = \begin{cases} L_t R_t^L \text{ if } Y_{t+1} \geq L_t R_t^L + \underline{C} - D_t R_{t+1} \equiv \widehat{y} \\ Y_{t+1} + D_t R_{t+1} - \underline{C} \text{ if } Y_{t+1} < L_t R_t^L + \underline{C} - D_t R_{t+1} \equiv \widehat{y} \end{cases} \tag{8}$$

where R_t^L denotes the loan interest charged by the bank in t, and $\underline{C}$ the exempted part of household income which we take to be guaranteed by the prevailing bankruptcy law. Henceforth we assume that riskless assets plus interest income serve as collateral. Taking expectations from (8), we obtain

$$\begin{aligned} E\left[\widetilde{X}\right] &= \int_{\underline{y}}^{\widehat{y}} \left(Y_{t+1} - \underline{C} + D_t R_{t+1}\right) f\left(Y_{t+1}\right) dY_{t+1} + \\ \int_{\widehat{y}}^{\overline{y}} L_t R_t^L f\left(Y_{t+1}\right) dY_{t+1} &= L_t R_t^L - \int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right) dY_{t+1} \end{aligned} \tag{9}$$

From (9) we see that the existence of a default option limits a household's liability with respect to its debt, which implies that on average a borrowing household repays less than the contractually agreed repayment. The amount of this difference is determined by expected default $\int_{\underline{y}}^{\widehat{y}} F(Y_{t+1})\, dY_{t+1}$. Limited liability would motivate a *risk-neutral* household to borrow more than without default option. However, a risk-averse household will also take the variance of debt repayment and the covariance of income and debt repayment into account:

$$\begin{aligned} Var\left[\widetilde{X}\right] &= \int_{\underline{y}}^{\widehat{y}} \left[\left(Y_{t+1} - \underline{C} + D_t R_{t+1}\right)\right]^2 f(Y_{t+1})\, dY_{t+1} + \\ &\quad \int_{\widehat{y}}^{\overline{y}} \left(L_t R_t^L\right)^2 f(Y_{t+1})\, dY_{t+1} - \left(E\left[\widetilde{X}\right]\right)^2 \end{aligned} \tag{10}$$

$$\begin{aligned} Cov\left[\widetilde{X}, \widetilde{Y}\right] &= \int_{\underline{y}}^{\widehat{y}} \left[\left(Y_{t+1} - \underline{C} + D_t R_{t+1}\right)\right] Y_{t+1} f(Y_{t+1})\, dY_{t+1} + \\ &\quad \int_{\widehat{y}}^{\overline{y}} \left(L_t R_t^L\right) Y_{t+1} f(Y_{t+1})\, dY_{t+1} - E\left[\widetilde{X}\right] E\left[\widetilde{Y}\right] \end{aligned} \tag{11}$$

Considering only the variance (10), we see that a risk-averse household's incentive to borrow more under a default option is reduced by its aversion against deviations of repayments from their long-run average. Concerning positive deviations, this is immediately intuitive: In good years the household has to pay for a default risk which in fact does not realize. Concerning negative deviations, this might appear implausible, however, we have to recall that risk-aversion in the expected utility framework is associated with the household's dislike of income fluctuations as such and not a dislike of incomes which are too low compared to some target level. We also observe, however, that a positive covariance between income and debt repayments increases a household's appetite for credit, because any undesired labour income volatility is at least partly compensated by a volatility of debt repayments. Since the loan interest rate is not exogenous in our model but rather follows from the bank's interest rate policy, we now turn to specifing bank behaviour.

3 Symmetric Information

3.1 A Model of Bank Behaviour under the Existence of a Default Option When Information is Symmetric

To begin with, we assume that the distribution of information between the bank and the borrower is symmetric. In accordance with Lawrance (1995), we assume furthermore that the bank is risk neutral and hence is willing to grant loans, if it can expect to earn on average the rate of return of the riskless alternative. This requires that the expected repayment from lending $E\left[\widetilde{X}_{t+1}\right]$ is identical to the gross income from a riskless asset, i.e.

$$E\left[\widetilde{X}_{t+1}\right] = L_t R_{t+1} \tag{12}$$

Substituting (9) into (12) leads to

$$L_t R_t^L = L_t R_{t+1} + \int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right) dY_{t+1} \tag{13}$$

Obviously the bank loan interest rate R_t^L exceeds the riskless rate by the amount of expected default. By claiming at least on average the riskless interest rate, the bank obviously removes any incentive on the part of a *risk-neutral* debtor to borrow more when a default option is available than would be optimal without such a possibility. The bank's interest rate policy indeed implies that a borrower pays on average an interest rate on loans which coincides with the riskless rate. The question now arises whether in addition the bank will have a motive to impose quantitative credit constraints. If information is symmetric and if furthermore the bank is risk-neutral, then the bank will grant any loan provided it does not exceed the maximum of income possible plus collateral and minus the non-seizable income. This is exactly the Lawrance case. Of course further arguments in favour of tighter credit constraints, like aversion against losses above some target level may be conceived of, but if we want to examine consequences of an empirically observed lowering of credit constraints, the Lawrance approach appears to be a good approximation. This gives rise to an upper bound to lending determined by

$$L_t^{\max} = \frac{\overline{y} + D_t R_{t+1} - \underline{C}}{R_{t+1}^L} \tag{14}$$

where $\overline{y}$ describes the maximum of income which the household may receive. Considering (6), (13) and taking into account that

$$\widehat{y}_{|L=L^{\max}} = \overline{y} \tag{15}$$

we can rewrite (14) to become

$$L_t^{\max} = \frac{E\left[\widetilde{Y}_{t+1}\right] + D_t R_{t+1} - \underline{C}}{R_{t+1}} \tag{16}$$

3.2 Determining Optimal Household Behaviour under Information Symmetry

Taking (13) into account, the certainty equivalent changes to

$$CE = E\left[\widetilde{Y}\right] - L_t R_{t+1} - \frac{\alpha}{2} Var\left[\widetilde{Y}\right] - \alpha Z, \tag{17}$$

$$Z \equiv \frac{1}{2} Var\left[\widetilde{X}\right] - Cov\left[\widetilde{X}, \widetilde{Y}\right] \tag{18}$$

With

$$\begin{aligned} Var\left[\widetilde{X}\right] &= 2L_tR_{t+1}\int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right)dY_{t+1} + \left(\int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right)dY_{t+1}\right)^2 - \\ & 2\int_{\underline{y}}^{\widehat{y}} \left(Y_{t+1} - \underline{C} + D_tR_{t+1}\right)F\left(Y_{t+1}\right)dY_{t+1} \end{aligned} \tag{19}$$

and

$$\begin{aligned} Cov\left[\widetilde{X}, \widetilde{Y}\right] &= \left(E\left[\widetilde{Y}\right] + \int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right)dY_{t+1}\right)\int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right)dY_{t+1} + \\ & \left(L_tR_{t+1} - D_tR_{t+1} + \underline{C}\right)\int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right)dY_{t+1} - \\ & 2\int_{\underline{y}}^{\widehat{y}} Y_{t+1}F\left(Y_{t+1}\right)dY_{t+1} \end{aligned} \tag{20}$$

we obtain

$$\begin{aligned} Z &= \int_{\underline{y}}^{\widehat{y}} Y_{t+1}F\left(Y_{t+1}\right)dY_{t+1} - E\left[\widetilde{Y}\right]\int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right)dY_{t+1} - \\ & \frac{1}{2}\left(\int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right)dY_{t+1}\right)^2 \end{aligned} \tag{21}$$

Z can be interpreted as the risk cost of borrowing. Obviously its sign is ambiguous depending on the size of $\widehat{y}$, which denotes a level of income that is sufficient for a household in order to be able to serve its debt. If the household borrows the maximum loan size, then $\widehat{y} = \overline{y}$ and the variance of credit repayments as well as the covariance of income and repayments are identical. Since the weight of the covariance in Z is twice as large as that of the variance, Z is negative in this case. Stated differently, the fact that undesired fluctuations of income around its long-run average are - depending on the loans size- partly or even fully compensated by corresponding fluctuations of loan repayments around their long-run average, leads to lower cost of borrowing compared to the no-default-option-case. This and not the prospect of a safe non-seizable income in the second period fuels the household's appetite for loans.

The household maximizes (1) subject to (16), the nonnegativity constraints

$$D_t \geq 0 \tag{22}$$
$$L_t \geq 0 \tag{23}$$

and the credit constraint

$$L_t^{\max} \geq L_t \tag{24}$$

In order to solve the optimization problem, we compute first-order conditions from the Lagrangian

$$\mathcal{L} = \left(1 - e^{-\alpha C_t}\right) + \beta \left(1 - e^{-\alpha CE}\right) + \lambda_1 D_t + \lambda_2 L_t + \lambda_3 \left(L_t^{\max} - L_t\right) \tag{25}$$

where we use (2), (16), (17) and (21). Differentiating the Lagrangian with respect to the riskless asset and loans, we obtain as first-order conditions

$$-\alpha e^{-\alpha C_t} + \beta \alpha e^{-\alpha CE} \left(\frac{\partial CE}{\partial D_t}\right) + \lambda_1 + \lambda_3 = 0 \tag{26}$$

$$\alpha e^{-\alpha C_t} + \beta \alpha e^{-\alpha CE} \left(\frac{\partial CE}{\partial L_t}\right) + \lambda_2 + \lambda_3 = 0 \tag{27}$$

where

$$\frac{\partial CE}{\partial D_t} = R_{t+1} + \frac{\alpha R_{t+1} F(\widehat{y})}{1 - F(\widehat{y})} \left(L_t R_{t+1} + \underline{C} - D_t R_{t+1} - E\left[\widetilde{Y}\right]\right) \tag{28}$$

$$\frac{\partial CE}{\partial L_t} = -R_{t+1} - \frac{\alpha R_{t+1} F(\widehat{y})}{1 - F(\widehat{y})} \left(L_t R_{t+1} + \underline{C} - D_t R_{t+1} - E\left[\widetilde{Y}\right]\right) \tag{29}$$

$$D_t \geq 0;\ \lambda_1 \geq 0;\ D_t \lambda_1 = 0 \tag{30}$$

$$L_t \geq 0;\ \lambda_2 \geq 0;\ L_t \lambda_2 = 0 \tag{31}$$

$$L_t^{\max} \geq L_t;\ \lambda_3 \geq 0;\ \left(L_t^{\max} R_{t+1} - L_t R_{t+1}\right) \lambda_3 = 0 \tag{32}$$

Since obviously $\frac{\partial CE}{\partial D_t}$ and $\left|\frac{\partial CE}{\partial L_t}\right|$ are identical, a rational household will never prefer to borrow and save simultaneously because this would not allow to obtain any additional utility from this transaction. We may therefore follow that a household either borrows or saves.

Next we investigate the case $D_t > 0$ and $L_t = 0$, which implies $\lambda_1 = \lambda_3 = 0$ and $\lambda_2 > 0$. In this case (26) reduces to

$$-\alpha e^{-\alpha C_t} + \beta \alpha e^{-\alpha CE} \left(\frac{\partial CE}{\partial D_t}\right) = 0 \tag{33}$$

where

$$\frac{\partial CE}{\partial D_{t|L_t=0}} = R_{t+1} \tag{34}$$

which now delivers as the optimal amount of riskless assets

$$D_t = \frac{Y_t - E\left[\widetilde{Y}_{t+1}\right] + \frac{\alpha}{2} Var\left[Y_{t+1}\right] + \frac{1}{\alpha} \ln\left(\beta R_{t+1}\right)}{R_{t+1}} \tag{35}$$

The demand for riskless assets, which in our model coincides with the optimal amount of saving, is determined by three motives: Consumption-tilting suggests the purchase of assets if the gross real interest rate exceeds the discount factor, i.e. $\ln(\beta R_{t+1}) > 0$. Consumption-smoothing requires savings if current income is higher than expected income, and finally consumption-stabilizing requires precautionary savings in order to compensate for labour income risk (Größl and Fritsche, 2007).

Equation (35) also reveals that a household prefers borrowing over saving whenever

$$Y_t - E\left[\widetilde{Y}_{t+1}\right] + \frac{\alpha}{2} Var\left[Y_{t+1}\right] + \frac{1}{\alpha}\ln(\beta R_{t+1}) < 0$$

If the household did not have a default option, then the size of optimal borrowing would be just the inverse of optimal saving. In the presence of a default option, however, the household's aversion against positive and negative deviations of loan repayments from their long-run average has to be considered. To understand what this means, we ignore the covariance of income and loan repayments for the moment. Whether the household borrows more or less than in the absence of a default option in this case depends on how the variance varies with loan size. For illustrative purposes assume that $Var\left[\widetilde{X}_{t+1}\right] = 2\sigma L_t$. In this case we would obtain as first-order conditions

$$e^{\alpha(CE - C_t)} = \beta\left(R_{t+1} + \alpha\sigma\right)$$

or in logs

$$CE - C_t = \frac{1}{\alpha}\ln\left(\beta\left(R_{t+1} + \alpha\sigma\right)\right)$$

delivering

$$L_t = \frac{E\left[\widetilde{Y}_{t+1}\right] - Y_t - \frac{\alpha}{2} Var\left[Y_{t+1}\right] - \frac{1}{\alpha}\ln\left(\beta R_{t+1} + \alpha\sigma\right)}{1 + R_{t+1} + \alpha\sigma}$$

Obviously in this special case, the household's aversion against volatility acts as an additional marginal cost of borrowing thus reducing the optimal loan size below the no-default-option-case. In our model we have

$$\frac{\partial Var\left[\widetilde{X}\right]}{\partial L_t} = 2R_{t+1}\int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right) dY_{t+1} \tag{36}$$

which is positive and which is furthermore positively correlated with the loan size itself. In the absence of the covariance of income and loan repayments, this would support the result of a lower optimal size of borrowings compared to the no-default-option-case even further. However, the variance of debt repayments offers only a partial explanation for marginal risk costs of borrowing. In addition we have to account for the reaction of the covariance, which is given by

$$\frac{\partial Cov\left[\widetilde{X},\widetilde{Y}\right]}{\partial L_t} = R_{t+1}\int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right)dY_{t+1} - \frac{R_{t+1}F\left(\widehat{y}\right)}{1-F\left(\widehat{y}\right)}\left(L_tR_{t+1}+\underline{C}-E\left[\widetilde{Y}_{t+1}\right]\right) \tag{37}$$

If the household prefers the maximum loan level $L^{\max}$ as an optimum, then the second term of the right-hand side of (37) is zero leading to a positive correlation between the covariance and the amount of borrowings, thus *decreasing* marginal risk costs of borrowing. In this case any income volatility is perfectly neutralized by the volatility of debt repayments. If for example the household receives an income below average, which reduces the certainty equivalent, this is offset by a lower debt repayment. If the household chooses less than $L^{\max}$, then the the reaction of the covariance to varying loan size is ambiguous.

Of relevance to the household's borrowing decision are total marginal risk costs of borrowing which amount to

$$\frac{\partial Z}{\partial L_t} = \frac{R_{t+1}F\left(\widehat{y}\right)}{1-F\left(\widehat{y}\right)}\left(L_tR_{t+1}+\underline{C}-E\left[\widetilde{Y}_{t+1}\right]\right) \tag{38}$$

We observe that marginal risk cost of borrowing depend on the optimal loan size. In particular they reduce to zero if the household wants to borrow $L^{\max}$. This is hardly surprising because as we have already noticed, in that case the covariance and the variance of debt repayments coincide with the variance of income and thus do not respond to varying loan amounts.

As first order conditions we obtain

$$e^{\alpha(CE-C_t)} = \beta R_{t+1}\left(1+\frac{\alpha F\left(\widehat{y}\right)}{1-F\left(\widehat{y}\right)}\left(L_tR_{t+1}+\underline{C}-E\left[\widetilde{Y}_{t+1}\right]\right)\right) \tag{39}$$

With the right-hand side of (39) being a function of the loan volume itself, a closed-form solution is not straightforward. However, recalling that $e^{\alpha(CE-C_t)}$ is always positive, we know that the right-hand side must assume a positive value, too. This requires a lower bound for the optimal loan size specified by

$$L_tR_{t+1} > E\left[\widetilde{Y}_{t+1}\right]-\underline{C}-\frac{1-F\left(\widehat{y}\right)}{\alpha F\left(\widehat{y}\right)} \tag{40}$$

As we have already learned, marginal risk cost of borrowing are zero whenever the household takes its maximum loan size. Does therefore $L^{\max}$ represent an optimum? A glimpse at (39) shows us that in this case marginal cost of borrowing coincide with the no-default-case, i.e. are constant and equal to the riskless rate. In order to find out whether $L^{\max}$ satisfies (39) take logs from both sides of (39) delivering

$$(CE - C_t) = \frac{1}{\alpha} \ln (\beta R_{t+1}) \tag{41}$$

where we have used (16). If the household borrows $L^{\max}$, the certainty equivalent and expected consumption for the second period coincide[5] and amount to the level of the household's non-seizable income.

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$$CE = E\left[\widetilde{C}\right] = \underline{C} \tag{42}$$

Hence in this case the household can expect to enjoy a safe level of consumption. In order to find out whether this constitutes an optimum we start by setting

$$\beta R_{t+1} = 1 \tag{43}$$

In this case (41) is equivalent to

$$\underline{C} = Y_t + \frac{E\left[\widetilde{Y}_{t+1}\right] - \underline{C}}{R_{t+1}}$$

or

$$\underline{C} = \frac{Y_t R_{t+1} + E\left[\widetilde{Y}_{t+1}\right]}{1 + R_{t+1}} \tag{44}$$

Hence given current and expected income, a household borrows $L^{\max}$, whenever its non-seizable income is sufficiently high as determined by (44). For example if current income equals its long-run average, than a household uses its default option, whenever the exemption level of its income equals expected income. This indicates that lower income groups are more prone to risking default than higher income groups. Whenever current income exceeds its long-run average, a household's non-seizable income has to be higher than this long-run average. Should current income fall below its long-run average, then a lower non-seizable income suffices for the household to choose default. This also implies that the household is more inclined to risk default in recessions than this is the case in boom situations. Whenever $\beta R_{t+1} > 1$ and hence $\ln \beta R_{t+1} > 0$, the non-seizable level of household income has to be higher than specified in (44) and lower in the opposite case, according to

$$\underline{C} = \frac{Y_t R_{t+1} + E\left[\widetilde{Y}_{t+1}\right] + \frac{R_{t+1}}{\alpha} \ln (\beta R_{t+1})}{1 + R_{t+1}} \tag{45}$$

[5]Note that in this case $Var\left[\widetilde{C}\right] = \frac{\alpha}{2}\left(Var\left[\widetilde{Y}\right] + Var\left[\widetilde{X}\right] - Cov\left[\widetilde{Y}, \widetilde{X}\right]\right) = 0$ due to $Var\left[\widetilde{X}\right] = Var\left[\widetilde{Y}\right] = Cov\left[\widetilde{Y}, \widetilde{X}\right]$

Since real interest rates are lower in recessions, this supports the argument that a rational household is more inclined to risk default in recessions than in boom situations.

Another interesting point concerns the question whether a household which chooses $L^{\max}$ as its optimum, borrows more than in the no-default-case. To find this out, we substitute (45) into (16),which leads to

$$L_t = \frac{E\left[\widetilde{Y}_{t+1}\right] - Y_t - \frac{1}{\alpha}\ln\left(\beta R_{t+1}\right)}{R_{t+1}} \tag{46}$$

This amount of debt is indeed higher than in the no-default-case, and the difference is determined by income risk and the household's absolute degree of risk aversion. This result is due to the fact that income volatility in the case of $L_t = L^{\max}$ together with the volatility of debt repayments are perfectly offset by the covariance of income and debt reimbursement.

We now return to the Lawrance argument which says that under a default option a household will be willing to borrow even if

$$E\left[\widetilde{Y}_{t+1}\right] - \frac{\alpha}{2}Var\left[\widetilde{Y}\right] - Y_t - \frac{1}{\alpha}\ln\left(\beta R_{t+1}\right) = 0 \tag{47}$$

describing a situation in which a household without default option would neither save nor borrow. She explains this result by risk-aversion which motivates a household to prefer the safe exemption level to a random level of income. We first check whether $L_t = 0$ given $D_t = 0$, is incompatible with the first-order condition (39). If the household does not borrow at all, the right-hand side of (39) boils down to βR_{t+1}. Moreover we obtain

$$CE - C_t = E\left[\widetilde{Y}_{t+1}\right] - \frac{\alpha}{2}Var\left[\widetilde{Y}\right] - Y_t$$

which according to (47) is identical to $\frac{1}{\alpha}\ln\left(\beta R_{t+1}\right)$. Hence the availability of a default option is also compatible with a zero loan volume in the case that (47) holds. A zero optimal loan volume, however, is not coercive. To see this, assume again $L_t = L^{\max}$ as an optimal choice. Again we receive (41). Substituting Y_t by (47), we obtain for (41)

$$\underline{C}\left(1 + R_{t+1}\right) - E\left[\widetilde{Y}_{t+1}\right]\left(1 + R_{t+1}\right) + \frac{\alpha}{2}Var\left[\widetilde{Y}\right] - \frac{1}{\alpha}\ln\left(\beta R_{t+1}\right) = R_{t+1}\frac{1}{\alpha}\ln\left(\beta R_{t+1}\right) \tag{48}$$

Hence whether $L^{\max}$ qualifies as an optimum, too, depends again on the size of the non-seizable income which now has to amount to

$$\underline{C} = E\left[\widetilde{Y}_{t+1}\right] - \left(\frac{\frac{\alpha}{2}Var\left[\widetilde{Y}\right] - \frac{1}{\alpha}\ln\left(\beta R_{t+1}\right)}{1 + R_{t+1}}\right) \tag{49}$$

We may follow from (49) that it is not the mere prospect of a safe income in the second period which may motivate the household to borrow even if the condition (47) is met, rather, the size of this safe income has to be sufficiently high. The reason for this is that a household which maximizes expected utility does not only want to minimize risk but also to maximize consumption, and its optimal choice reflects a compromise of both motives.

We may therefore conclude that it is by no means certain that a household which prefers neither to borrow nor to save in the no-default-case will choose to borrow under default. Rather, households' borrowing behaviour may become unpredictable in situations in which households without such a default option would make a clear decision in favour of consuming all of their current incomes.

4 Implications of Information Asymmetry

Information asymmetry may enter the model before a contract has been concluded.[6] Then the bank is imperfectly informed about the borrower's quality thus risking adverse selection. Another type of information asymmetry which becomes relevant after the contract has been concluded concerns the bank's inability to observe how the borrower uses the money thus risking moral hazard. Finally ex post information asymmetry implies that the bank is able to verify the household's realized income situation only after undertaking costly controlling activities. Unlike Lawrance (1995), we analyse this last type of information asymmetry because we think that it has more relevance than moral hazard which is due to the evidence that nowadays at least consumer loans are not bound to special purposes. It may also be more important than ex ante information asymmetry since modern screening devices typically claim from borrowers to present income statements. Like in Townsend (1979), Diamond (1984) and Williamson (1987) let us assume that the bank undertakes costly controlling activities once the borrower ceases to repay the loan according to the contract. For simplicity we take these control costs as independent of loan size and equal to some positive fixed amount Q.

Ex post information asymmetry changes the structure of bank gross revenues from lending X_{t+1}^B as follows

$$X_{t+1}^B = \begin{cases} L_t R_t^L \text{ if } Y_{t+1} \geq \widehat{y} \\ Y_{t+1} - \underline{C} - Q, \text{ if } Y_{t+1} < \widehat{y} \end{cases} \tag{50}$$

Hence in case of default the bank receives less than under symmetric information since it has to take costly controlling activities in order to verify

[6] This case is investigated in Lawrance (1995).

whether the borrower acts strategically or is really unable to service his loan. The bank continues to be willing to grant the loan if it can expect to receive the riskless interest rate at least on average, i.e.

$$\begin{aligned} E\left[\widetilde{X}^B\right] &= L_t R_t^L + \int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right) dY_{t+1} + QF\left(\widehat{y}\right) = L_t R_{t+1} \qquad (51) \\ QF\left(\widehat{y}\right) &= Q \int_{\underline{y}}^{\widehat{y}} f\left(Y_{t+1}\right) dY_{t+1} \end{aligned}$$

with $\widetilde{X}^B$ denoting the bank's revenues from lending. (51) requires that the bank charges a loan interest rate which exceeds the riskless interest rate not only by the amount of expected default but also by the amount of expected control cost $QF(\widehat{y})$. In due consequence the maximum loan volume then changes to

$$L_t^{\max} = \frac{E\left[\widetilde{Y}\right] - \underline{C} - QF\left(\widehat{y}\right)}{R_{t+1}} \qquad (52)$$

which is definitely lower than in the case of information symmetry.

According to the bank's interest rate policy, the borrower has to pay for these expected control costs and since now his expected repayment amounts to

$$E\left[\widetilde{X}\right] = L_t R_{t+1} + QF\left(\widehat{y}\right) \qquad (53)$$

he does so not only when he decides against declaring default but also as an average over the long run. Expected control costs thus drive a wedge between the rate of return on borrowing and lending. The variance and covariance change to

$$\begin{aligned} Var\left[\widetilde{X}\right] &= \left(\int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right) dY_{t+1}\right)^2 + 2\left(L_t R_{t+1} + QF\left(\widehat{y}\right)\right) \int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right) dY_{t+1} - \\ & 2\int_{\underline{y}}^{\widehat{y}} \left(Y_{t+1} - \underline{C}\right) F\left(Y_{t+1}\right) dY_{t+1} \end{aligned}$$

Obviously, the variance of debt repayments increases if the bank charges control costs from the borrower, which depend on the probability of default $F(\widehat{y})$. As the covariance of income and debt repayment we now get

$$\begin{aligned} Cov\left[\widetilde{X}, \widetilde{Y}\right] &= \int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right) dY_{t+1} E\left[\widetilde{Y}\right] + \left(L_t R_{t+1} + QF\left(\widehat{y}\right)\right) \int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right) dY_{t+1} \qquad (54) \\ & \left(\int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right) dY_{t+1}\right)^2 - \int_{\underline{y}}^{\widehat{y}} \left(2Y_{t+1} - \underline{C}\right) F\left(Y_{t+1}\right) dY_{t+1} \end{aligned}$$

leading to risk-borrowing cost Z, of

$$\begin{aligned} Z &= \int_{\underline{y}}^{\widehat{y}} Y_{t+1} F\left(Y_{t+1}\right) - \frac{1}{2}\left(\int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right) dY_{t+1}\right)^2 - \\ & E\left[\widetilde{Y}\right] \int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right) dY_{t+1} \end{aligned} \tag{55}$$

with

$$\frac{\partial Z}{\partial L} = \frac{R_{t+1} F\left(\widehat{y}\right)}{1 - F\left(\widehat{y}\right) - Qf\left(\widehat{y}\right)} \left(L_t R_{t+1} + \underline{C} + QF\left(\widehat{y}\right) - E\left[\widetilde{Y}\right]\right)$$

Note that expected marginal borrowing costs are now higher than without control cost.

As first-order conditions we obtain

$$e^{\alpha(CE - C_t)} = \beta R_{t+1}\left[1 + \frac{\alpha F\left(\widehat{y}\right)}{1 - F\left(\widehat{y}\right) - Qf\left(\widehat{y}\right)}\left(L_t R_{t+1} + \underline{C} + QF\left(\widehat{y}\right) - E\left[\widetilde{Y}\right]\right)\right] \tag{56}$$

Is $L^{\max}$ still an optimal choice in the face of information asymmetry? In answering this question we start analysing the right-hand side of (56). Again $L_t R_{t+1} + \underline{C} + QF\left(\widehat{y}\right) - E\left[\widetilde{Y}\right]$ assumes a zero value in this case. Hence like in the symmetric information case, we obtain

$$-\frac{\partial CE}{\partial L_t}_{|L^{\max}} = \beta R_{t+1}$$

Turning to the left-hand side of (56), note that the certainty equivalent for $L_t = L^{\max}$ is still equal to the non-seizable level of income $\underline{C}$. For $L^{\max}$ to be an optimum we must then have

$$CE - C_t = \frac{1}{\alpha}\ln\left(\beta R_{t+1}\right)$$

or

$$\underline{C} = Y_t + \frac{E\left[\widetilde{Y}\right] - \underline{C} - Q}{R_{t+1}} + \frac{1}{\alpha}\ln\left(\beta R_{t+1}\right) \tag{57}$$

where due to $\widehat{y}_{|L^{\max}} = \overline{y}$, expected and realized contol costs coincide. Hence now $L_t = L^{\max}$ is an optimum if the non-seizable level of income amounts to

$$\underline{C} = \frac{R_{t+1} Y + E\left[\widetilde{Y}\right] - Q + \frac{1}{\alpha} R_{t+1} \ln\left(\beta R_{t+1}\right)}{1 + R_{t+1}} \tag{58}$$

Substituting (58) into 52) yields as an optimal loan volume

$$L_t = L^{\max} = \frac{E\left[\widetilde{Y}\right] - Y_t - \frac{Q}{R_{t+1}} - \frac{1}{\alpha}\left(\ln \beta R_{t+1}\right)}{1 + R_{t+1}} \tag{59}$$

Compared to symmetric information, the non-seizable rate of income can now even be lower in order for $L_t = L^{\max}$. On the other hand, however, the optimal loan size will be lower, too, and it is by no means certain that the household borrows more than in the no-default case. Indeed this would only be the case if $\frac{\alpha}{2} Var\left[\widehat{Y}\right]$ were higher than discounted control cost $\frac{Q}{R_{t+1}}$ Finally note that in spite of control costs, household behaviour remains unpredictable in situations in which a household without default option would neither save nor borrow though the incentive to borrow in this case will be lower due to control costs.

5 Conclusions and Extensions

The availability of a default option endows a household with limited liability. Hence given that the riskless interest rate and the interest rate on loans coincide, a risk-neutral household will always have an incentive to borrow more than in the no-default-case. Knowing this, a bank which wants to earn the riskless rate at least on average, will charge a sufficiently higher interest rate on loans thus preventing a risk-neutral household from borrowing more than in the absence of a default option. We have shown that this is no longer the case if households maximize a concave utility function and hence dislike fluctuations of consumption around its long-run average. Whereas in the absence of default, these fluctuations are exclusively determined by the volatility and thus variance of household labour income, under a default option, the variance of debt repayments and the covariance of income and debt repayments play a role, too. Were it only for the variance, this would lower a household's appetite for credit compared to the no-default-case. Integrating the covariance of income and debt repayment into the analysis, however, changes the argument significantly. Since income enhances consumption, whereas loan repayments do the opposite, undesired income fluctuations can be partly or - depending on the loan size - even be fully offset by fluctuations in debt repayments. This indeed may explain why households with a default option might be willing to borrow more than without such a possibility. We have also shown that due to this particular relationship between income and debt repayment volatility, a household may be inclined to choose the maximum debt level which implies that this household will choose default. However, in order to be an optimal choice, the non-seizable income has to be sufficiently high because in maximizing expected utility the household does not only want to minimize risk but also to maximize consumption. In particular we have found that lower income groups will be more prone to choosing default than higher income groups, and generally, any household will be more inclined to defaulting in recessions than in boom situations. Finally we have shown that in a situation where a household without default option may neither be willing to save nor to borrow, the behaviour of a household with default option becomes unpredictable in the sense that no borrowings as well as positive - even maximum - borrowings are compatible with an optimum. We also analysed bank and household behaviour under ex post information asymmetry and found that if the bank burdens the borrower with control costs, a household which chooses its maximum loan volume will borrow less than in the symmetric income case and moreover it is not certain that this same household will borrow more than in the no-default-case.

Any appreciation of our results should take into account, however, that in our model a loan contract is one-shot. Hence the household under consideration has no concern about the availability of future loans and thus about its reputation as a borrower. What role relationship banking plays for the behaviour of lenders and borrowers has extensively been investigated in terms of firms. How such a relation impacts household behaviour in the face of a default option thus appears to be a valuable and interesting extension.

References

Carroll, C.D. and Kimball, M.S. (2001): Liquidity Constraints and Precautionary Savings, *NBER Working paper No. W8496.*

Cox, D. and Jappelli, T. (1993): The Effect of Borrowing Constraints on Consumer Liabilities. *Journal of Money, Credit, and Banking, Vol. 25, No.2*, pp. 197-213.

Crook, J. and Hochguertel, S. 2006): Household Debt and Credit Constraints, Evidence from OECD Countries. *Credit Research Centre, University of Edinburgh.*

Deaton, A. (1991): Savings and Liquidity Constraints. *Econometrica, 59,* pp. 1221-1248.

Diamond, D. (1984): Financial Intermediation and Delegated Monitoring. *Review of Economic Studies* 51, pp. 393-414.

Fay, S., Hurst, E., and Michelle J. White (2002): The Household Bankruptcy Decision. *The American Economic Review, Vol. 92, No. 3,* pp. 706-718.

Größl, I. and Fritsche, U. (2007): The Store-of-Value-Function Money as a Component of Household Risk Management. *DEP Discussion Papers, Macroeconomics and Finance Series, No.6, University of Hamburg, Department Economics and Politics, revised version.*

Lawrance, E.C. (1995): Consumer Default and the Life Cycle Model. *Journal of Money, Credit, and Banking, Vol. 27, No. 4 (November, Part I),* pp. 939-954.

Ludvigson, S. (1999): Consumption and Credit: A Model of Time-Varying Liquidity Constraints. *Review of Economics and Statistics,* pp. 434-447.

Rinaldi, L. and Sanchis-Arellano, A. (2006): Household Debt Sustainability. What Explains Household Non-Performing Loans? An Empirical Analysis. *ECB Working paper 670, January.*

Sullivan, T.A., Warren, E. and W., Lawrance, J. (1989): As We Forgive Our Debtors: Bankruptcy and Consumer Credit in America. *New York: Oxford University Press.*

Townsend, R. (1979): Optimal Contracts and Competitive Markets with Costly State Verification. *Journal of Economic Theory* 21, pp. 265-293.

Williamson, S.D. (1986): Costly Monitoring, Financial Intermediation and Equilibrium Credit Rationing,. *Journal of Monetary* Economics, 18, pp. 159-179.

Zeldes, S.P. (1989): Consumption and Liquidity constraints: An Empirical Investigation. *Journal of Political Economy*, 97(2), pp. 305-346.

Appendix

Calculation of $Var\left[\widetilde{X}\right]$ under information symmetry

$$\begin{aligned} Var\left[\widetilde{X}\right] &= \int_{\underline{y}}^{\widehat{y}} \left(Y_{t+1} - \underline{C} + D_t R_{t+1}\right)^2 f\left(Y_{t+1}\right) dY_{t+1} + \\ &\quad \left(L_t R_t^L\right)^2 \int_{\widehat{y}}^{\overline{y}} f\left(Y_{t+1}\right) dY_{t+1} - \left(L_t R_{t+1}\right)^2 \end{aligned} \tag{60}$$

$$\widehat{y} = L_t R_t^L + \underline{C} - D_t R_{t+1} \tag{61}$$

$$L_t R_t^L = L_t R_{t+1} + \int_{\widehat{y}}^{\overline{y}} F\left(Y_{t+1}\right) dY_{t+1} \tag{62}$$

$$\begin{aligned} Var\left[\widetilde{X}\right] &= \left(L_t R_t^L\right)^2 F\left(\widehat{y}\right) - 2\int_{\underline{y}}^{\widehat{y}} \left(Y_{t+1} - \underline{C} + D_t R_{t+1}\right) F\left(Y_{t+1}\right) dY_{t+1} \\ &\quad + \left(L_t R_t^L\right)^2 - \left(L_t R_t^L\right)^2 F\left(\widehat{y}\right) - \left(L_t R_{t+1}\right)^2 \\ &= \left(L_t R_{t+1} + \int_{\widehat{y}}^{\overline{y}} F\left(Y_{t+1}\right) dY_{t+1}\right)^2 - \left(L_t R_{t+1}\right)^2 - \\ &\quad 2\int_{\underline{y}}^{\widehat{y}} \left(Y_{t+1} - \underline{C} + D_t R_{t+1}\right) F\left(Y_{t+1}\right) dY_{t+1} \\ &= 2L_t R_{t+1} \int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right) dY_{t+1} + \left(\int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right) dY_{t+1}\right)^2 - \\ &\quad 2\int_{\underline{y}}^{\widehat{y}} \left(Y_{t+1} - \underline{C} + D_t R_{t+1}\right) F\left(Y_{t+1}\right) dY_{t+1} \end{aligned} \tag{63}$$

Calculation of $Cov\left[\widetilde{X}, \widetilde{Y}\right]$ under Information Symmetry

$$\begin{aligned} Cov\left[\widetilde{X}, \widetilde{Y}\right] &= \int_{\underline{y}}^{\widehat{y}} Y_{t+1}\left(Y_{t+1} - \underline{C} + D_t R_{t+1}\right) f\left(Y_{t+1}\right) dY_{t+1} + \\ &\quad L_t R_t^L \int_{\widehat{y}}^{\overline{y}} Y_{t+1} f\left(Y_{t+1}\right) dY_{t+1} - L_t R_{t+1} E\left[\widetilde{Y}\right] \end{aligned} \tag{64}$$

$$\begin{aligned} Cov\left[\widetilde{X}, \widetilde{Y}\right] &= \widehat{y} L_t R_t^L F\left(\widehat{y}\right) - \int_{\underline{y}}^{\widehat{y}} \left(2Y_{t+1} - \underline{C} + D_t R_{t+1}\right) F\left(Y_{t+1}\right) dY_{t+1} + \\ &\quad L_t R_t^L \overline{y} - \widehat{y} L_t R_t^L F\left(\widehat{y}\right) - L R_t^L \int_{\widehat{y}}^{\overline{y}} F\left(Y_{t+1}\right) dY_{t+1} \\ &\quad - L_t R_{t+1} E\left[\widetilde{Y}\right] \end{aligned} \tag{65}$$

Since

$$\overline{y} - \int_{\widehat{y}}^{\overline{y}} F\left(Y_{t+1}\right) dY_{t+1} = E\left[\widetilde{Y}\right] + \int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right) \tag{66}$$

we obtain

$$Cov\left[\widetilde{X},\widetilde{Y}\right] = -\int_{\underline{y}}^{\widehat{y}} \left(2Y_{t+1} - \underline{C} + D_t R_{t+1}\right) F\left(Y_{t+1}\right) + E\left[\widetilde{Y}\right]\left(L_t R_t^L - L_t R_{t+1}\right) \quad (67)$$

$$L_t R_t^L \int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right) = E\left[\widetilde{Y}\right]\int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right) + \left(L_t R_{t+1} + \underline{C} - D_t R_{t+1}\right)\int_{\underline{y}}^{\widehat{y}} F\left(Y_{t+1}\right) + \left(\int_{\widehat{y}}^{\overline{y}} F\left(Y_{t+1}\right) dY_{t+1}\right)^2 - 2\int_{\underline{y}}^{\widehat{y}} Y_{t+1} F\left(Y_{t+1}\right) dY_{t+1}$$

Calculation of the variance under information asymmetry and $D_t = 0$ From the arguments above we may follow that

$$Var\left[\widetilde{X}\right] = \left(L_t R_t^L\right)^2 - \left(L_t R_{t+1} + QF\left(\widehat{y}\right)\right)^2 - 2\int_{\underline{y}}^{\widehat{y}} \left(Y_{t+1} - \underline{C}\right) F\left(Y_{t+1}\right) dY_{t+1} \quad (68)$$

where now

$$\left(L_t R_t^L\right)^2 = \left(L_t R_{t+1} + \int_{\underline{y}}^{\widehat{y}} Y_{t+1} F\left(Y_{t+1}\right) dY_{t+1} + QF\left(\widehat{y}\right)\right)^2 \quad (69)$$

Substituting (69) into (68), delivers as the variance of debt repayments

$$Var\left[\widetilde{X}\right] = \left(\int_{\underline{y}}^{\widehat{y}} Y_{t+1} F\left(Y_{t+1}\right) dY_{t+1}\right)^2 + 2L_t R_{t+1}\int_{\underline{y}}^{\widehat{y}} Y_{t+1} F\left(Y_{t+1}\right) dY_{t+1} + 2QF\left(\widehat{y}\right)\int_{\underline{y}}^{\widehat{y}} Y_{t+1} F\left(Y_{t+1}\right) dY_{t+1} - 2\int_{\underline{y}}^{\widehat{y}} \left(Y_{t+1} - \underline{C}\right) F\left(Y_{t+1}\right) dY_{t+1} \quad (70)$$

Calculation of the covariance under information asymmetry and $D_t = 0$ Following from (65) and

$$\left(L_t R_t^L\right) = \left(L_t R_{t+1} + \int_{\underline{y}}^{\widehat{y}} Y_{t+1} F\left(Y_{t+1}\right) dY_{t+1} + QF\left(\widehat{y}\right)\right) \quad (71)$$

, we get

$$Cov\left[\widetilde{X},\widetilde{Y}\right] = \left(\int_{\underline{y}}^{\widehat{y}} Y_{t+1} F\left(Y_{t+1}\right) dY_{t+1}\right)^2 + \quad (72)$$

$$E\left[\widetilde{Y}\right]\int_{\underline{y}}^{\widehat{y}} Y_{t+1} F\left(Y_{t+1}\right) dY_{t+1} + \quad (73)$$

$$\left(L_t R_{t+1} + QF\left(\widehat{y}\right)\right)\int_{\underline{y}}^{\widehat{y}} Y_{t+1} F\left(Y_{t+1}\right) dY_{t+1} - \int_{\underline{y}}^{\widehat{y}} \left(2Y_{t+1} - \underline{C}\right) F\left(Y_{t+1}\right) dY_{t+1}$$