Universität Hamburg

Department Socioeconomics

A Microfounded Model of Money Demand Under Uncertainty, and its Empirical Validation Using Cointegration and Rolling-Window Dynamic Multiplier Analysis

Ingrid Größl

Artur Tarassow

DEP (Socioeconomics) Discussion Papers
Macroeconomics and Finance Series
4/2015

Hamburg, 2015

A Microfounded Model of Money Demand Under Uncertainty, and its Empirical Validation Using Cointegration and Rolling-Window Dynamic Multiplier Analysis.*

Ingrid Grössl
Faculty of Economics and Social Sciences
University of Hamburg
ingrid.groessl@wiso.uni-hamburg.de
+49-40-42838-8729

Artur Tarassow
Faculty of Economics and Social Sciences
University of Hamburg
artur.tarassow@wiso.uni-hamburg.de
+49-40-42838-8683

May 22, 2015

Abstract

In this article we derive a microfounded model of money demand under uncertainty built on intertemporally optimizing risk-averse households. Deriving a complete solution of the optimization problem taking the intertemporal budget constraint into account leads to ambiguous effects w.r.t. to the impact of capital as well as inflation risk, thus contradicting standard results. We estimate both the long- and short-run model dynamics as well as potential time-variation by means of a rolling-window dynamic multiplier analysis using the error-correction framework for the U.S. economy between 1978q1 to 2013q4. The results reveal that U.S. households increase their demand for money in response to positive changes in inflation and stock market risks.

JEL Classifications: C22, E41, E51, E58, G11

Key Words: Money Demand, Uncertainty, Inflation Risk, Stock Market Risk, Monetary Policy, ARDL Model, Cointegration, Dynamic Multiplier, Rolling-Window.

*Address for correspondence: Faculty of Economics and Social Sciences, University of Hamburg, Welckerstraße 8, Hamburg, 20354, Germany. Email: artur.tarassow@wiso.uni-hamburg.de. Tel: +49-40-42838-8683. We are grateful for the helpful discussion of Ulrich Fritsche. Any remaining errors or omissions are strictly our own. This article is part of Artur Tarassow's PhD dissertation project.

Contents

1 Introduction

The rise of monetarism and New Classical Economics in the 1970s and 1980s fueled an ongoing debate about the stability of money demand and its prominent role for the effectiveness of monetary policy (Barnett et al., 1992). In the course of the 1990s the attention shifted away from money as a guide for monetary policy to interest rates. However, in the aftermath of the financial crisis, interest in private actors' liquidity preference has regained academic interest. One line of argument points to quantitative easing policies exercised by central banks leading to growth rates of money which are seen as incompatible with real growth rates thus raising concerns about future inflation. A second line of argument emphasizes the risk of protracted periods of secular stagnation for the world economy (Eggertsson and Mehrotra, 2014) with a high preference for liquidity as a major cause (Bossone, 2014). Both concerns suggest that theoretical as well as empirical research on the determinants of money demand should be resumed. However, different from the debate of the 1970s and 1980s which had a focus on the issue of a stable relationship between money demand and income, now fears of future inflation due to excessive monetary growth directs the attention to whether and how expected inflation as well as its volatility affect money holdings in the non-bank sector. A negative correlation between both variables and money demand implies that the non-bank sector wants to rid itself from high money holdings thus boosting purchases of goods and assets, and accompanied with that, prices. On the other hand, worries about secular stagnation also advocate an interest in how risks might affect actors' liquidity preference. The most prominent fear in this regard is that people do not believe in inflation but are instead afraid of lasting deflationary forces. In this case, a negative correlation between desired money holdings and expected inflation, too, would aggravate the situation whereas a negative correlation between deflationary risks and money demand could act as a stabilizer. Finally, as a consequence of recent financial regulations, central banks will play a more active role in the process of financial supervision. This extension of authority has not gone uncriticized for reasons which point to a possible conflict of interest between financial and price stability. In this respect the demand for money, too, gains importance, where this time reactions to higher financial risk as compared to inflationary risk gain importance. All these arguments suggest that forecasts of monetary demand will play a pivotal role for both the assessment of the future macroeconomic development as well as for the effectiveness of monetary policy.

There is indeed an increasing number of publications examining the impact of diverse risks on money demand. Overwhelmingly, these studies are empirical basing their estimations on either plausibility or on Euler equations. With our paper we aim to contribute to this line of research. In accordance with these last-mentioned studies, we base our estimations on a comprehensive theoretical framework built on intertemporally optimizing households. Contrary to the literature, however, we do not content ourselves with the Euler equations but rather propose a complete solution of the optimization problem taking the intertemporal budget constraint into account. As one main difference between models using the Euler equation as a monetary demand function and our approach, we do not only consider substitution effects but in addition possibly countervailing income effects. As a second difference, in our approach expectations about future income and not only current consumption determine households' money holdings. Finally, we consider different types of liquid assets which bear the major characteristics of money, by taking interest-bearing bank deposits into account. Unlike most empirical applications we estimate both the long-run money demand relationship as well as its short-run dynamics. Additionally, we study the potential time-varying dynamics during the recent financial crisis.

The remainder of the paper is composed of a theoretical and an empirical part. Each part starts with a brief literature review which helps to clarify the commonalities as well as differences of our approach compared to the state of the art. In the theoretical part we develop a macroeconomic model of money demand using an OLG framework distinguishing between a long-run and short-run perspective. Our analysis is partial in the sense that we do not set up a complete macroeconomic model but concentrate on the demand for alternative assets. The empirical part comprises the solid testing on cointegration and the estimation of error-correction models. The model dynamics are studies by means of (recursive) dynamic multiplier analysis.

2 Theory

2.1 Literature Review

The examination of risk variables as components of the money demand function directs the attention to money as a store of value. That non-interest bearing cash holdings serve to protect investors from capital market risk was emphasized by J.M. Keynes and formally elaborated by James Tobin within a static portfolio framework (Tobin, 1956). On the other hand, a reduction of portfolio risk can be achieved by holding interest-bearing assets provided that they are considered as riskless (Ingersoll, 1987). And indeed, due to numerous financial innovations the supply of interest-bearing assets promising their holders safety, has increased over the years. Hence a further argument is needed to legitimate cash as a store-of-value. In this respect cash as immediate liquidity gained importance, which came to be incorporated into microeconomic models of optimizing behavior either by assigning direct utility to money (based on Patinkin (1965)) or by assuming transaction costs of transforming assets into immediate liquidity (Saving, 1971; McCallum and Goodfriend, 1987). That both approaches are equivalent in terms of their results for optimal cash holdings, was shown for example by Feenstra (1986). Overwhelmingly, in these approaches money is defined as cash thus legitimating its status as immediate liquidity. However, taking into account that due to improved payment technologies, costs of liquidating a broad range of assets have been reduced to a rather negligible quantity, central banks nowadays resort to broad aggregates of money as indicators of the effectiveness of their policies as well as of macroeconomic liquidity preferences. Arguably, this, too, has not gone criticized for reasons which doubt that the components of either monetary aggregate should be considered as perfect substitutes (Barnett et al. (1992) for a review). On the other hand, already the existence of just a few distinct monetary aggregates acknowledges that private actors hold different types of riskless assets reaching from cash to interest-bearing deposits simultaneously, which requires explanation. Macroeconomic theory so far has not taken up this issue (with an exception of Bossone (2014)).

In DSGE models which have come to serve as the workhorse model for monetary policy, cash yields direct utility thus legitimating positive cash holdings even in the presence of a riskless but interest-bearing security of indeterminate maturity. Since this class of models generally exclude the derivation of explicit solutions, log-linearization around the steady state is chosen, which leads to percentage deviations of optimal cash holdings as a function of both deviations of current consumption from steady state values and the riskless nominal rate of interest (Walsh (2003) as one example). Moreover, due to the application of a Taylor expansion of first order, risk variables are excluded from the analysis.

It is finally worth noting at this point that typically intertemporal macroeconomic models do not offer complete solutions for household optimization problems taking the intertemporal budget constraint into account, but derive all types of behavioral functions directly from the Euler equations. This implies that the relationship between money demand and its explanatory variables reflects substitution effects thus telling only half of the story. This procedure is also followed in Choi and Oh (2003), who derive a money demand function from a general equilibrium model focusing on the impact of output as well as monetary uncertainty which has its origins in information deficiencies concerning the money supply process. By assuming that both output and the supply of money are log-normally distributed, they are able to consider risk by including variances and covariances as components of optimal cash holdings. Furthermore they do not need to resort to log-linearization procedures around some equilibrium in order to derive explicit optimality conditions. Money demand here, too, depends on current consumption but furthermore both output shock variances and monetary shock variances play a role though the direction of impact is ambiguous. The authors explain this ambiguity by the coincidence of a substitution and precautionary effect. For example higher monetary uncertainty motivates households to reduce money balances (substitution effect). On the other hand, the authors argue that higher uncertainty as such also motivates higher savings. This last argument is true but its formal derivation requires a complete solution of the household's optimization problem thus resorting to the intertemporal budget constraint. Such a complete solution is missing in the paper and for that reason any ambiguous reaction of money demand to higher monetary uncertainty calls for a different explanation. Rather, the two countervailing effects point to the assumed utility function which departs from the commonly assumed (weak) separability of consumption and money but sees them as complements. Hence if consumption increases due to higher monetary uncertainty, this raises the marginal utility of money thus suggesting higher money holdings, too. Bossone (2014) departs from the standard general equilibrium macroeconomic model by explicitly considering different degrees of liquidity as a distinguishing feature of assets leading to different utilities assigned to them. Of relevance for his results are interactions between rational expectations and market sentiments. Pessimistic market sentiments may be such that households' preferences are directed towards "ultra liquid" assets thus raising money at the expense of expenditures on consumption goods.

2.2 The Theoretical Model

We analyze the role of money holdings in an OLG setting. The economy is inhabited by a young and an old generation. Combined with the assumption of finite life, this allows us to avoid problems following from an infinite series of future incomes when integrating the intertemporal budget constraint into the derivation of a complete solution of the household optimization problem. The young generation lives two periods and plans its optimal time path of consumption when young. The old generation finances consumption by the liquidation of accumulated wealth. It dies at the end of the second period without leaving any bequests. The macroeconomic framework models a stationary economy with uncertainty concerning the real rate of return of assets. We depart from the standard DSGE model by assuming that each young household maximizes the certainty equivalent. This enables us to give capital market risk as well as inflationary risk an explicit representation, even after we have linearized around the steady state.

Since we will compute an explicit formulation for the money demand function by using linearization techniques around the steady state, we start with a characterization of the long-run equilibrium and its consequences for optimal money holdings as well as optimal consumption and asset holdings.

2.2.1 Money Demand in the Long-Run

Household Sector In each period a young generation is born. For simplicity we normalize the size of the cohort to one. When young, the household maximizes its lifetime welfare, where we take the underlying utility functions to be of the CRRA type. Utility is derived from consumption when young and old as well as from holding money when young. In this respect we distinguish between interest-bearing time-deposits and non-interest-bearing cash holdings. Both types of money yield direct utility though at a different degree, depending on their different liquidity services.

Welfare maximization is subject to period budget constraints. We assume that the young household receives an exogenous labor income when young and has to pay a lump-sum tax. Net income is used to consume and to save for the old age when consumption has to be exclusively financed out of accumulated wealth. There are no bequests and hence the young household's initial wealth is zero. The household can use its savings for the accumulation of interest-bearing and interest-free cash holdings as well as for the purchase of an interest-bearing asset which serves to finance the given capital stock in the economy. By buying this asset the household acquires ownership rights in firms. When old the household liquidates its wealth in order to finance consumption. The old generation has to pay a lump-sum tax, too.

In accordance with the major bulk of macroeconomic DSGE models, we assume tat the steady state is characterized by the absence of uncertainty but not necessarily by the absence of inflation. In the absence of uncertainty the young household maximizes the following lifetime welfare function which is assumed to be strictly concave in all its components:

$$U = u\left(C_t^y\right) + \beta u\left(C_{t+1}^o\right) + v\left(\frac{M1_t^{yn}}{P_t}\right) + \gamma\left(\frac{T_t^{yn}}{P_t}\right) \rightarrow \max \qquad (1)$$

subject to the following period budget constraints:

$$\begin{aligned} C_t^y + \frac{M1_t^y}{P_t} + \frac{T_t^y}{P_t} + A_t^y &= Y_t^y - \Theta_t^y & (2) \\ C_{t+1}^o &= A_t^y\left(1 + r_{t+1}\right) + \frac{M1_t^{yn}}{P_t}\frac{P_t}{P_{t+1}} + \frac{T_t^{yn}}{P_t}\frac{P_t}{P_{t+1}}\left(1 + i_{t+1}\right) - \Theta_{t+1}^o & (3) \end{aligned}$$

where $M1_t^{yn}$ ($M1_t^y$)denotes nominal (real) interest-free money holdings and T_t^{yn} (T_t^y) nominal (real) time deposits, Y_t^y denotes real (labor) income accruing to the young household, C_t^y $\left(C_{t+1}^o\right)$ denotes real consumption by the young (old) household, Θ_t^y $\left(\Theta_{t+1}^o\right)$ real lump-sum taxes paid by the young and old generation, respectively, β denotes the subjective discount factor and A_t^y the real value of shares which is related to the capital stock K_t through the real share price (average Tobin's q):

$$A_t^y = K_t^y \frac{P_{kt}}{P_t} \equiv K_t^y q_t \qquad (4)$$

$q = \frac{P_k}{P}$ represents the relative price of the capital stock which is constant over time if shares offer a protection against inflation which we will assume throughout the paper. In the steady state q is always equal to one. However, since we abstract from investment, this will also be true outside the steady state:

$$q_t = 1 \qquad (5)$$

In order to facilitate computations outside the steady state, we use the following approximation:

$$\frac{P_t}{P_{t+1}} \approx 1 - \pi_{t+1} \qquad (6)$$

with π representing the rate of inflation. In doing so we assume that π^2 is close to zero.

Summarizing equations (2) and (3), we obtain the intertemporal budget constraint:

$$C_t^y + \frac{C_{t+1}^o}{1+r_{t+1}} = Y_t^y - M1_t^y\left(r_{t+1}+\pi_{t+1}\right) - T_t^y\left(r_{t+1}+\pi_{t+1}-i_{t+1}\right) - \Theta_t^y - \frac{\Theta_{t+1}^o}{1+r_{t+1}} \tag{7}$$

Note that $M1_t^y$ and T_t^y represent real values. In what follows we will treat real money holdings as the household's control variable. In order to obtain the optimality conditions, we maximize the Lagrangian:

$$\begin{aligned} \mathfrak{L} \;=\; & u\left(C_t^y\right) + \beta u\left(C_{t+1}^o\right) + v\left(M1_t^y\right) + \gamma\left(T_t^y\right) - \\ & \lambda\left[C_t^y + \frac{C_{t+1}^o}{1+r_{t+1}} - \left(Y_t^y - M1_t^y\frac{\left(r_{t+1}+\pi_{t+1}\right)}{1+r_{t+1}} - T_t^y\frac{\left(r_{t+1}+\pi_{t+1}-i_{t+1}\right)}{1+r_{t+1}} - \Theta_t^y - \frac{\Theta_{t+1}^o}{1+r_{t+1}}\right)\right] \end{aligned} \tag{8}$$

As first-order conditions we get:

$$u'\left(C_t^y\right) = \lambda \tag{9}$$

$$\beta u'\left(C_{t+1}^o\right) = \frac{\lambda}{1+r_{t+1}} \tag{10}$$

$$v'\left(M1_t^y\right) = \lambda\left(\frac{r_{t+1}+\pi_{t+1}}{1+r_{t+1}}\right) \tag{11}$$

$$\gamma'\left(T_t^y\right) = \lambda\left(\frac{r_{t+1}+\pi_{t+1}-i_{t+1}}{1+r_{t+1}}\right) \tag{12}$$

As is well known the optimal ratio of present and future consumption is determined by the ratio between the rate of the time preference and the real interest rate according to equation (13):

$$u'\left(C_t^y\right) = \beta\left(1+r_{t+1}\right)u'\left(C_{t+1}^o\right) \tag{13}$$

Note that in the overlapping generation case the steady state does not require the identity of the real interest rate and the rate of time preference of the young generation.

As the optimal ratio between interest-free money holdings and current consumption we obtain:

$$v'\left(M1_t^y\right) = u'\left(C_t^y\right)\left(\frac{r_{t+1}+\pi_{t+1}}{1+r_{t+1}}\right) \tag{14}$$

In order to interpret this optimality condition, assume that the household increases $M1_t^y$ by somewhat. This reduces the amount that alternatively can be channeled into capital, which also implies that the amount of capital interest income foregone goes down thus reducing the opportunity cost of current consumption and rendering higher money holdings less disadvantageous. This explains why an increase of the gross real interest rate on capital, $(1+r)$ increases the optimal ratio of interest-free money holdings and current consumption. On the other hand $M1$ does not yield interest, rather a positive rate of inflation reduces the purchasing power of a given nominal amount. In addition, a higher amount of interest-free cash holdings leads to real opportunity cost explained by a lower amount of capital which the household is able to purchase. The expression $\frac{r_{t+1}+\pi_{t+1}}{1+r_{t+1}}$ represents total opportunity cost of interest-free money holdings.

The optimal ratio between time deposits and current consumption

$$\gamma'\left(T_t^y\right) = u'\left(C_t^y\right)\left(\frac{r_{t+1}+\pi_{t+1}-i_{t+1}}{1+r_{t+1}}\right) \tag{15}$$

can be explained in a likewise manner with the difference that time deposits yield a nominal interest rate which reduces its opportunity cost correspondingly.

(14) and (15) taken together, informs us about the optimal ratio between interest-free and interest-bearing money holdings:

$$v'\left(M1_t^y\right) = \gamma'\left(T_t^y\right)\left(\frac{r_{t+1}+\pi_{t+1}}{r_{t+1}+\pi_{t+1}-i_{t+1}}\right) \tag{16}$$

Note that in the presence of a direct utility of time deposits, the real interest rate on capital and time deposits are allowed to deviate in the steady state. We observe the following: in the presence of a positive interest rate on time deposits, the household is only willing to hold interest-free money if its marginal utility exceeds that of time time deposits. Hence for the following we will always assume that $v'\left(M1_t^y\right) > \gamma'\left(T_t^y\right)$ for all amounts of $M1_t^y, T_t^y$.

A complete solution of the household's optimization problem requires the assumption of an explicit form of the utility function. For illustrative purposes we take household utility to be logarithmic:

$$U = \log C_t^y + \beta \log C_{t+1}^o + \zeta \log M_t^y + \kappa \log T_t^y \rightarrow \max \tag{17}$$

where $\zeta > 0$ ($\kappa > 0$) can be interpreted as a relative weight of interest-free (interest-bearing) money holdings hence $\zeta\left(\kappa\right) \in (0,1)$.

Solving the optimality problem by using (17) yields as the optimal ratio between current and future consumption:

$$C_{t+1}^o = \beta\left(1+r_{t+1}\right)C_t^y \tag{18}$$

and between money holdings and current consumption:

$$M1_t^y = \zeta\left(\frac{1+r_{t+1}}{r_{t+1}+\pi_{t+1}}\right)C_t^y \tag{19}$$

$$T_t^y = \kappa\left(\frac{1+r_{t+1}}{r_{t+1}+\pi_{t+1-}i_{t+1}}\right)C_t^y \tag{20}$$

Substituting (18), (19) and (20) into the intertemporal budget constraint, delivers the following optimal amounts of consumption (present and future), interest-free money holdings as well as interest-bearing money holdings:

$$C_t^y = \frac{Y_t^y - \Theta_t^y - \frac{\Theta_{t+1}^o}{1+r_{t+1}}}{1+\beta+\zeta+\kappa} \tag{21}$$

$$C_{t+1}^o = \beta\left(1+r_{t+1}\right)\left(\frac{Y_t^y - \Theta_t^y - \frac{\Theta_{t+1}^o}{1+r_{t+1}}}{1+\beta+\zeta+\kappa}\right) \tag{22}$$

$$M1_t^y = \zeta\left(\frac{1+r_{t+1}}{r_{t+1}+\pi_{t+1}}\right)\left(\frac{Y_t^y - \Theta_t^y - \frac{\Theta_{t+1}^o}{1+r_{t+1}}}{1+\beta+\zeta+\kappa}\right) \tag{23}$$

$$T_t^y = \kappa\left(\frac{1+r_{t+1}}{r_{t+1}+\pi_{t+1}-i_{t+1}}\right)\left(\frac{Y_t^y - \Theta_t^y - \frac{\Theta_{t+1}^o}{1+r_{t+1}}}{1+\beta+\zeta+\kappa}\right) \tag{24}$$

We observe that under the assumption of logarithmic utility neither present nor future consumption respond to variations of the interest rate on time deposits. The same applies to interest-free money holdings. In contrast time deposits are positively correlated with their "own" interest rate. Time deposits and interest-free money holdings are negatively correlated with the real interest rate on capital.

Production Sector We choose a rudimentary framework for production. In particular, we assume that in the long-run production is at its full-employment level and constant over time. The technology is characterized by a Cobb Douglas production function with constant returns to scale:

$$Y_t = K_t^{\zeta} N_t^{1-\zeta} \tag{25}$$

Assuming furthermore that the input of labor is exogenous, too, the capital stock has to adjust appropriately. In order to realize this, firms offer shares to young households amounting to

$$K_t = Y_t^{(1/\zeta)} N_t^{(\zeta/(1-\zeta))} \tag{26}$$

The Public Sector and the Banking Sector The banking sector in our model is rudimentary, too. First, we do not distinguish between commercial banks and the central bank. Second, the central bank and the government are consolidated into a homogeneous sector being responsible for price stability.

The government finances a deficit by increasing its supply of narrow money and by offering time deposits to young households. The government budget constraint hence is defined as:

$$G_t + T_{t-1}^y\left(1+i_t-\pi_t\right) + M1_{t-1}^y\left(1-\pi_t\right) - \Theta_t^y - \Theta_t^o = M1_t^s + T_t^s \tag{27}$$

2.2.2 Long-Run Macroeconomic Equilibrium

In a long-run macroeconomic equilibrium all components of real wealth as well as all rates of return on assets and the rate of inflation remain constant over time. This stationary economy is represented by a simultaneous equilibrium in the following four markets: aggregate commodity market, capital market, the market for time deposits and the market for cash. Capital market equilibrium requires that young households wish to hold the amount of capital which is necessary to realize the exogenous amount of production. This implies the assumption that old households sell their capital stock directly to firms which in their turn finance these transactions by selling capital to young households. Due to Walras' law one of the four market is redundant which we have chosen to be the aggregate commodity market. Equilibrium in the markets for capital, time deposits as well as for cash then serve to determine the real rate of return on capital, the real interest rate on time deposits as well as the rate of inflation.

Capital market equilibrium is characterized by the equality of capital desired by young households and by the amount of capital which is necessary to realize the full employment output level:

$$K_t = K_t^y \tag{28}$$

where the supply of capital is given by equation (26) and the demand for capital follows from the households' first period budget constraint taking the optimality conditions for time deposits and cash into account. The remaining markets concern the supply and demand for money. The old generation liquidates its time deposits and runs down cash balances in order to finance consumption. Note that we deviate from cash-in-advance-approaches by focusing on the store-of-value function which implies that transactions by the old generation do not show up on the supply side. Hence the supply side of both time deposits and cash is exclusively represented by decisions made in the consolidated government-banking sector. If the government wants to realize a specific desired (constant) rate of inflation, it can always do so by fixing the supply of time deposits and cash appropriately. In this case, either government expenditures and/or taxes will have to be adjusted in order to meet the budget constraint. We assume that this is the case. Equilibrium in the market for time deposits and cash then reads as:

$$T_t^s = T_t^y \tag{29}$$

$$M1_t^s = M_t^y \tag{30}$$

2.2.3 Household Optimization Outside the Steady State

Outside the steady state the young representative household plans under uncertainty about the future real rate of return on capital and the future rate of inflation Maximizing welfare now requires that the household builds expectations and evaluates possible expectation errors. In the standard intertemporal macro-model this is commonly modeled by a Bernoulli utility function according to which a risk-averse agent maximizes the expected utility of uncertain consumption instead of the utility of expected consumption. However, maximizing expected utility typically does not lead to explicit or linear optimal solutions. The usually applied linearization procedure rests on the application of a Taylor series of first order to the optimality conditions, which has the drawback that risk parameters drop out. One way to include risk parameters into the optimality conditions would be to use a second-order Taylor approximation. Assuming all random variables to be distributed normally, this would give a complete description of risk. A less challenging approach in this case, which we have decided to follow, consists of approximating expecting utility directly by a second-order Taylor series thus achieving the certainty equivalent (Groessl and Fritsche, 2007). Using CRRA utility functions then still does not provide us with explicit solutions for optimal consumption, asset and money holdings. However, now using a Tylor approximation of first order around their steady state values allows us to give risk parameters an explicit representation.

Optimization The young household then maximizes the following objective function:

$$U = u\left(C_t^y\right) + \beta u\left(CE_{t+1}\right) + v\left(\frac{M_t^{ny}}{P_t}\right) + \gamma\left(\frac{T_t^{ny}}{P_t}\right) \rightarrow \max \tag{31}$$

where CE_{t+1} denotes the certainty equivalent. As already mentioned, the certainty equivalent is based on the assumption that both the real rate of return on capital and the rate of inflation are normally distributed. It then combines expected consumption with its variance, where the link is established by the Arrow Pratt measure of absolute risk aversion. In the case of CRRA utility the absolute measure of risk aversion is not a constant but rather correlates negatively with expected consumption meaning that the household becomes less risk-averse if its expected consumption goes up. The certainty equivalent is thus given by

$$CE_{t+1} = E_t C_{t+1}^o - \frac{\alpha}{2E_t C_{t+1}^o} Var\left[C_{t+1}^o\right] \tag{32}$$

where α stands for the relative degree of risk aversion which is constant for CRRA utility functions, and $E_t C_{t+1}^o$ represents expected household consumption when old with $Var\left[C_{t+1}^o\right]$ as its variance.

The young household maximizes its lifetime welfare (31) subject to the period budget constraints. For the first period budget constraint we obtain:

$$A_t^y + \frac{T_t^{ny}}{P_t} + \frac{M_t^{ny}}{P_t} + C_t^y = Y_t^y - \Theta_t^y \tag{33}$$

$$A_t^y = K_t \frac{P_{kt}}{P_t} \equiv K_t q_t \tag{34}$$

Outside the steady state, too, we will not take investment activities into account implying

$$q_t = 1 \tag{35}$$

Uncertainty prevails both with respect to future inflation as well as with respect to the future level of the real rate of return on capital. This implies for expected consumption when old:

$$E_t C_{t+1}^o = M1_t^y \left(1 - E_t \pi_{t+1}\right) + T_t^y \left(1 + i_{t+1} - E_t \pi_{t+1}\right) + K_t^y \left(1 + E_t r_{t+1}\right) - \Theta_{t+1}^o \tag{36}$$

where $E_t r_{t+1}$ represents the expected real interest rate on capital and i_{t+1} the safe nominal interest rate on time deposits. In order to facilitate the algebra, we have again approximated the term $\frac{P_t}{E_t P_{t+1}} = \frac{1}{1+E_t \pi_{t+1}}$ by $1 - E_t \pi_{t+1}$ implying that $\left(E_t \pi_{t+1}\right)^2$ is assumed to be a negligible quantity. We continue to assume that the household takes *real* and not nominal money holdings as its decision variable, which implies that we can substitute $\frac{M1_t^{ny}}{P_t} \left(\frac{T_t^{ny}}{P_t}\right)$ by $M1_t^y \left(T_t^y\right)$ in the utility function. Substituting (32), (33) and (34) and (35) into (36), we obtain

$$\begin{aligned} E_t C_{t+1}^o = {} & \left(Y_t^y - C_t^y - \Theta_t^y\right)\left(1 + E_t r_{t+1}\right) - M1_t^y \left(E_t r_{t+1} + E_t \pi_{t+1}\right) \\ & - T_t^y \left(E_t r_{t+1} + E_t \pi_{t+1} - i_t\right) - \Theta_{t+1}^o \end{aligned} \tag{37}$$

The variance of old age consumption is then given by

$$\begin{aligned} Var\left[C_{t+1}^o\right] &= E\left[\left(C_{t+1}^o - E_t C_{t+1}^o\right)^2\right] \\ &= \left(Y_t^y - C_t^y - M1_t^y - T_t^y - \Theta_t^y\right)^2 \sigma_{r_t}^2 + \left(M1_t^y + T_t^y\right)^2 \sigma_{\pi_t}^2 \\ & \quad -2\left(Y_t^y - C_t^y - M1_t^y - T_t^y - \Theta_t^y\right)\left(M1_t^y + T_t^y\right)\sigma_{r\pi_t} \end{aligned} \tag{38}$$

where

$$\sigma_{r_t}^2 \equiv Var\left[r_{t+1}\right] = E\left[\left(r_{t+1} - E_t r_{t+1}\right)^2\right] \tag{39}$$

$$\sigma_{\pi_t}^2 \equiv Var\left[\pi_{t+1}\right] = E\left[\left(\pi_{t+1} - E_t \pi_{t+1}\right)^2\right] \tag{40}$$

$$\sigma_{r\pi_t} \equiv Cov\left[r_{t+1}, \pi_{t+1}\right] = E\left[\left(r_{t+1} - E_t r_{t+1}\right)\left(\pi_{t+1} - E_t \pi_{t+1}\right)\right] \tag{41}$$

We observe that an increase of consumption risk lowers household utility. The degree to which this happens depends on the size of the measure of absolute risk aversion. Given CRRA this is in turn negatively correlated with the expected level of old age consumption.

To simplify notations we define broad money

$$M2 \equiv M1 + T \tag{42}$$

In order to derive optimality conditions we form the Lagrangian:

$$\begin{aligned} \mathfrak{L} &= u\left(C_t^y\right) + \beta u\left(CE_{t+1}\right) + v\left(M_t^y\right) + \gamma\left(T_t^y\right) + \\ &\quad \lambda\left[E_t C_{t+1}^o - \left(Y_t^y - C_t^y - \Theta_t^y\right)\left(1 + E_t r_{t+1}\right) + M1_t^y\left(E_t r_{t+1} + E_t \pi_{t+1}\right) + T_t^y\left(E_t r_{t+1} + E_t \pi_{t+1} - i_{t+1}\right)\right] \end{aligned} \tag{43}$$

where CE_{t+1} is given by

$$CE_{t+1} = E_t C_{t+1}^o - \frac{\alpha}{2E_t C_{t+1}^o}\left[\begin{array}{c} \left(Y_t^y - C_t^y - M1_t^y - T_t^y - \Theta_t^y\right)^2 \sigma_{r_t}^2 + \left(M2_t^y\right)^2 \sigma_{\pi_t}^2 - \\ 2\left(Y_t^y - C_t^y - M1_t^y - T_t^y - \Theta_t^y\right)\left(M2_t^y\right)\sigma_{r_t \pi_t} \end{array}\right] \tag{44}$$

Computing the first derivative of the Lagrangian (43) with respect to current consumption we get:

$$\frac{\partial \mathfrak{L}}{\partial C_t^y} = U'\left(C_t^y\right) + \beta U'\left(CE_{t+1}\right)\frac{\alpha}{E_t C_{t+1}^0}\left[K_t^y \sigma_{r_t}^2 - M2_t^y \sigma_{\pi_t r_t}\right] - \lambda\left(1 + E_t r_{t+1}\right) = 0 \tag{45}$$

where

$$K_t^y = Y_t^y - \Theta_t^y - C_t^y - M2_t^y \tag{46}$$

We observe that the optimal size of present consumption does not only depend on its immediate utility and the opportunity cost measured by $\lambda\left(1 + E_t r_{t+1}\right)$ but is also determined by future consumption risk. If the household decides to spend more on current consumption out of a given income this implies lower purchases of capital and hence lower capital market risk. For the same reason it affects the impact of the covariance between cash holdings and equity. The covariance between r and π expresses how positive (negative) deviations of the real rate of return on capital and the rate of inflation from their averages are correlated. If both rates are positively correlated, this is equivalent to a negative correlation between the real interest rate on capital and the real rate of return on cash holdings. Such a negative correlation reduces consumption risk. In our writing this is the meaning of $\sigma_{\pi r} > 0$.. The opposite is true for a negative correlation between r and π.

The first derivative of the Lagrangian with respect to expected future old age consumption delivers:

$$\frac{\partial \mathfrak{L}}{\partial E_t C_{t+1}^o} = \beta U'\left(CE_{t+1}\right) + \beta U'\left(CE_{t+1}\right)\frac{\alpha}{2\left(E_t C_{t+1}^0\right)^2} Var\left[C_{t+1}^o\right] = \lambda \tag{47}$$

An increase in future expected consumption increases the certainty equivalent both by increasing the utility of consumption and by lowering the Arrow Pratt measure of absolute risk aversion.

The first derivative of the Lagrangian with respect to $M1_t^y$ leads to:

$$\begin{aligned} \frac{\partial \mathfrak{L}}{\partial M1_t^y} &= \gamma'\left(M1_t^y\right) + \beta U'\left(CE_{t+1}\right) \frac{\alpha}{E_t C_{t+1}^0} \left[K_t^y \sigma_{r_t}^2 - M2_t^y \sigma_{\pi_t}^2 + \left(K_t^y - M2_t^y\right) \sigma_{\pi_t r_t}\right] \\ &- \lambda\left(E_t r_{t+1} + E_t \pi_{t+1}\right) \\ &= 0 \end{aligned} \tag{48}$$

where $M2_t^y = M1_t^y + T_t^y$.

In the optimum, an increase in welfare due to higher cash holdings equals its opportunity cost. An increase in cash holdings leads to higher welfare due to the assumption that cash yields direct utility. Higher cash holdings, however, also have ambiguous effects on the certainty equivalent. On the one hand, a higher level of cash holdings increases inflationary risk. On the other hand, capital risk declines since higher cash holdings lower the accumulation of capital. The impact of the covariance between the real rate of return on capital and inflation does now not only depend on the covariance between the two variables. In addition it plays a role whether the capital stock exceeds money holdings, whether they are equal in size or whether the capital stock is smaller than money holdings. Note that if both have the same size, then the covariance has no impact at all.

The first derivative of the Lagrangian with respect to T_t^y leads to:

$$\begin{aligned} \frac{\partial \mathfrak{L}}{\partial T_t^y} &= v'\left(T_t^y\right) \\ &+ \beta U'\left(CE_{t+1}\right) \frac{\alpha}{E_t C_{t+1}^0} \left[K_t^y \sigma_{rt}^2 - M2_t^y \sigma_{\pi t}^2 + \left(K_t^y - M2_t^y\right) \sigma_{\pi r t}\right] \\ &- \lambda\left(E_t r_{t+1} + E_t \pi_{t+1} - i_{t+1}\right) \\ &= 0 \end{aligned} \tag{49}$$

Higher time deposits have the same effect on future consumption risk as higher cash holdings. However, the opportunity cost of holding time deposits are lower compared to cash holdings.

Combining (45) and (47), we get an expression for the optimal ratio between current and future consumption:

$$\begin{aligned} &\left[\begin{array}{c} U'\left(C_t^y\right) + \\ \beta U'\left(CE_{t+1}\right) \frac{\alpha}{E_t C_{t+1}^0} \left(K_t^y \sigma_{rt}^2 - M2_t^y \sigma_{\pi r t}^2\right) \end{array}\right] \\ &= \beta U'\left(CE_{t+1}\right)\left(1 + E_t r_{t+1}\right) + \beta U'\left(CE_{t+1}\right) \frac{\alpha\left(1 + E_t r_{t+1}\right)}{\left(2 E_t C_{t+1}^0\right)^2} Var\left[C_{t+1}^o\right] \end{aligned} \tag{50}$$

Combining (45) and (49) delivers the optimal ratio between cash holdings and present consumption:

$$
\begin{aligned}
& \gamma'\left(M1_t^y\right) + \beta U'\left(CE_{t+1}\right) \frac{\alpha}{E_t C_{t+1}^0} \left[K_t^y \sigma_{rt}^2 - M2_t^y \sigma_{\pi t}^2 + \left(K_t^y - M2_t^y\right) \sigma_{\pi rt}\right] \qquad (51) \\
= & \left(\frac{1 + E_t\left(r_{t+1}\right) + E_t \pi_{t+1}}{1 + E_t\left(r_{t+1}\right)}\right) \left(U'\left(C_t^y\right) + \beta U'\left(CE_{t+1}\right) \frac{\alpha}{E_t C_{t+1}^0} \left[K_t^y \sigma_{rt}^2 - M2_t^y \sigma_{\pi rt}\right]\right)
\end{aligned}
$$

Combining (45) and (50) delivers the optimal ratio between time deposits and present consumption:

$$
\begin{aligned}
& v'\left(T_t\right) + \beta U'\left(CE_{t+1}\right) \frac{\alpha}{E_t C_{t+1}^0} \left[\overline{K}^y \sigma_{rt}^2 - \overline{M2}^y \sigma_{\pi t}^2 + \left(\overline{K}^y - \overline{M2}^y\right) \sigma_{\pi rt}\right] \qquad (52) \\
= & \frac{E_t r_{t+1} + E_t \pi_{t+1} - i_{t+1}}{1 + E_t\left(r_{t+1}\right)} \left(U'\left(C_t^y\right) + \beta U'\left(CE_{t+1}\right) \frac{\alpha}{E_t C_{t+1}^0} \left[K_t^y \sigma_{rt}^2 - M2_t^y \sigma_{\pi rt}\right]\right)
\end{aligned}
$$

Note that these optimal ratios do not only depend on a comparison between rates of return and marginal utilities but also on a comparison between reactions of consumption risk.

Linearization We symbolize percentage deviations of a variable x from its steady state value by $\widehat{x}_t$. Linearizing (50) around its steady state value yields:

$$
\begin{aligned}
E_t \widehat{c}_{t+1}^o = & \ \widehat{c}_t^y + \frac{E_t \widehat{r}_{t+1}}{\alpha\left(1 + \overline{r}\right)} + \left(\frac{\overline{K}^y}{\overline{C}^o}\right) \left[\frac{\left(\frac{\overline{K}^y}{\overline{C}^o}\right)(1+\alpha)(1+\overline{r}) - 2}{1 + \overline{r}}\right] \sigma_{rt}^2 \qquad (53) \\
+ & \ \frac{1+\alpha}{1+\overline{r}} \left(\frac{\overline{M}2^y}{\overline{C}^o}\right)^2 \sigma_{\pi t}^2 - \left(\frac{\overline{M}2^y}{\overline{C}^o}\right) \left[\frac{\left(\frac{\overline{K}^y}{\overline{C}^o}\right)(1+\alpha)\left(1+\overline{r}\right) - 1}{\left(1 + \overline{r}_{t+1}\right)}\right] \sigma_{\pi rt}
\end{aligned}
$$

Equation (50) describes how deviations of expected future consumption from its steady state value are related to deviations of current consumption from the steady state. An excess of deviations of future expected consumption over current consumption from their steady state values is positively correlated with the real rate of return on capital, but also with inflation risk. Underlying the last effect is the result that due to a higher expected future consumption, the household becomes less risk-averse implying that the marginal disutility of the variance of future consumption goes down. By contrast the impact of capital market risk is ambiguous: On the one hand we observe the same effect as in the case of inflation risk. On the other hand a higher capital market risk, too, increases the marginal utility of present consumption. Which effect dominates, depends on whether $\left(\overline{K}^y\right)(1+\alpha)\left(1+\overline{r}\right) - 2\overline{C}^o \gtreqless 0$. The impact of $\sigma_{\pi rt} > 0$ remains ambiguous no. This ambiguity holds irrespective of how the rate of inflation and the real rate of return on capital are correlated.

Linearizing (51) around its steady state value leads to:

$$\begin{aligned}
\widehat{m1}_t^y \quad = \quad & \frac{\alpha}{\eta}\widehat{c}_t^y - \frac{1-\overline{\pi}}{\eta\left(1+\overline{r}\right)\left(\overline{r}+\overline{\pi}\right)}E_t\widehat{r}_{t+1} - \frac{E_t\widehat{\pi}_{t+1}}{\eta\left(\overline{r}+\overline{\pi}\right)} + \\
& \frac{\alpha}{\eta}\left(\frac{\overline{K}^y}{\overline{C}^o}\right)\left(\frac{1-\overline{\pi}}{\left(1+\overline{r}\right)\left(\overline{r}+\overline{\pi}\right)}\right)\sigma_{rt}^2 - \\
& \frac{\alpha}{\eta}\left(\frac{\overline{M}2^y}{\overline{C}^o}\right)\frac{\sigma_{\pi t}^2}{\left(\overline{r}+\overline{\pi}\right)} + \\
& \frac{\alpha}{\eta}\left[\frac{\left(1+\overline{r}\right)\left(\frac{\overline{K}^y}{\overline{C}^o}\right)+\frac{\overline{M}2^y}{\overline{C}^o}\left(1+\overline{\pi}\right)}{\left(1+\overline{r}\right)\left(\overline{r}+\overline{\pi}\right)}\right]\sigma_{r\pi}
\end{aligned} \tag{54}$$

where

$$\eta \equiv -\frac{\gamma''\left(M1^y\right)}{\gamma'\left(M1^y\right)}M1^y \tag{55}$$

An excess of deviations of the demand for narrow money from the steady state over deviations of current consumption from the steady state is negatively correlated with the real interest rate on capital, the expected rate of inflation and inflation risk. A positive correlation results for capital market risk and a positive covariance $\sigma_{r\pi}$.

Linearizing (52) around its steady state value delivers:

$$\begin{aligned}
\widehat{t}_t^y \quad = \quad & \frac{\alpha}{\mu}\widehat{c}_t^y - \frac{1-\overline{\pi}}{\eta\left(1+\overline{r}\right)\left(\overline{r}+\overline{\pi}-\overline{i}\right)}E_t\widehat{r}_{t+1} - \frac{E_t\widehat{\pi}_{t+1}}{\mu\left(\overline{r}+\overline{\pi}-\overline{i}\right)} \\
& + \frac{\widehat{i}_{t+1}}{\mu\left(\overline{r}+\overline{\pi}-\overline{i}\right)} + \\
& \frac{\alpha}{\mu}\left(\frac{\overline{K}^y}{\overline{C}^o}\right)\left(\frac{1-\overline{\pi}}{\left(1+\overline{r}\right)\left(\overline{r}+\overline{\pi}+\overline{i}\right)}\right)\sigma_{rt}^2 - \\
& \frac{\alpha}{\mu}\left(\frac{\overline{M}2^y}{\overline{C}^o}\right)\frac{\sigma_{\pi t}^2}{\left(\overline{r}+\overline{\pi}-\overline{i}\right)} + \\
& \frac{\alpha}{\mu}\left[\frac{\left(1+\overline{r}\right)\left(\frac{\overline{K}^y}{\overline{C}^o}\right)+\frac{\overline{M}2^y}{\overline{C}^o}\left(1+\overline{\pi}\right)}{\left(1+\overline{r}\right)\left(\overline{r}+\overline{\pi}-\overline{i}\right)}\right]\sigma_{\pi rt}
\end{aligned} \tag{56}$$

and

$$\mu \equiv -\frac{\gamma''\left(T^y\right)}{\gamma'\left(T^y\right)}T^y \tag{57}$$

$\widehat{t}_t^y$ is positively correlated with its own rate. For the remaining variables we obtain the same results as for cash, at least qualitatively.

In order to obtain a complete solution of the optimization problem, we have to linearize the intertemporal budget constraint around its steady state value. In doing so we assume that lump-sum taxes always retain their steady state value. After linearizing around its steady state value the intertemporal budget constraint reads as follows:

$$\begin{aligned}\overline{C}^{o}E_t\widehat{c}^{o}_{t+1}+(1+\overline{r})\,\overline{C}^{y}\widehat{c}^{y}_{t} \quad = \quad & \overline{Y}^{y}\,(1+\overline{r})\,\widehat{y}^{y}_{t}-(\overline{r}+\overline{\pi})\,\overline{M}1^{y}\widehat{m}1^{y}_{t}- \\ & \left(\overline{r}+\overline{\pi}-\overline{i}\right)\overline{T}^{y}\widehat{t}^{y}_{t}+\Big(\underbrace{\overline{Y}^{y}-\overline{C}^{y}-\overline{M}1-\overline{T}^{y}}_{\overline{K}^{y}}\Big)E_t\widehat{r}_{t+1}+ \\ & \overline{T}^{y}\widehat{i}_{t+1}-\left(\underbrace{\overline{T}^{y}+\overline{M}1}_{\overline{M}2^{y}}\right)E_t\widehat{\pi}_{t+1}\end{aligned} \tag{58}$$

The right-hand side reveals percentage deviations of lifetime resources from their steady state values. Lifetime resources are higher if labor income as well as the expected real interest rate and the nominal interest rate on deposits exceed their long-run equilibrium value. Life time resources are lower if the expected rate of inflation is higher than its steady state value. If for example deviations of the expected real interest rate from the long-run equilibrium increase, this allows to consume more both in the present and in the future.[1] We also recognize that the expected rate of inflation has a stronger effect on present and future consumption than the nominal interest rate on deposits because expected inflation does not only determine the real interest rate on deposits but also the real rate of return on interest-free cash.

We now use the linearized intertemporal budget constraint in order to obtain complete solutions to the household optimization problem. In doing so we start with young age consumption. Given old age consumption and furthermore given the levels of deposits and cash, young age consumption is entirely determined by the behavior of lifetime resources which also implies that changes in the (expected) rates of return on assets and expected inflation affect current consumption exclusively through income effects. This explains why given the assumptions we have just made, current consumption correlates positively with the rates of return on assets and negatively with the expected rate of inflation. However, neither old age consumption nor the size of deposits and cash are given quantities but are endogenously determined by the optimality conditions. For the sake of clarity we proceed in steps and start with a discussion how the optimal ratio of current

[1] For the following we suppress the fact that the linearized model represents deviations from the steady state. Note that we do so in order to simplify the argument only. Implicitly our interpretation of results refer to deviations from the steady state.

and future consumption affects $\widehat{c}_t^y$. Inserting equation (53) into (58) yields:

$$\begin{aligned}
\left(\overline{C}^o + (1+\overline{r})\,\overline{C}^y\right)\widehat{c}_t^y \;=\;& \overline{Y}^y (1+\overline{r})\,\widehat{y}_t^y - \\
& (\overline{r}+\overline{\pi})\,\overline{M}1^y \widehat{m}1_t^y - \left(\overline{r}+\overline{\pi}-\overline{i}\right)\overline{T}^y \widehat{t}_t^y + \\
& \left(\overline{K}^y - \frac{\overline{C}^o}{\alpha(1+\overline{r})}\right) E_t \widehat{r}_{t+1} + \\
& \overline{T}^y \widehat{i}_{t+1} - \left(\underbrace{\overline{T}^y + \overline{M}1^y}_{\overline{M}2^y}\right) E_t \widehat{\pi}_{t+1} - \\
& \overline{K}^y \left[\frac{\frac{\overline{K}^y}{\overline{C}^o}(1+\alpha)(1+\overline{r}) - 2}{(1+\overline{r})}\right]\sigma_{rt}^2 - \\
& \overline{C}^o \left(\frac{\overline{M}2^y}{\overline{C}^o}\right)^2 \left(\frac{1+\alpha}{1+\overline{r}}\right)\sigma_{\pi t}^2 + \\
& \overline{M}2^y \left[\frac{\frac{\overline{K}^y}{\overline{C}^o}(1+\alpha)(1+\overline{r}) - 1}{(1+\overline{r})}\right]\sigma_{\pi r t}
\end{aligned} \tag{59}$$

Obviously now the impact of the expected real interest rate depends on the relative strength of the substitution effect compared to the income effect. Note also that since the optimal ratio of current and future consumption remains unaffected if expected inflation changes, current consumption continues to be unambiguously negatively correlated with $E_t\widehat{\pi}_{t+1}$. Furthermore now risk parameters, too, have to be taken into account. We have seen from equation (47) that a higher level of expected future consumption increases the certainty equivalent by lowering the absolute Arrow Pratt measure of risk aversion. The magnitude of this effect depends positively on the size of consumption risk. This in turn explains why expected future consumption will be expanded at the cost of current consumption if inflationary risk becomes more severe. Since therefore inflationary risk and old age consumption are positively correlated due to the Arrow Pratt measure of absolute risk aversion, a negative correlation with current consumption will follow. Ambiguity prevails with respect to capital market risk. A higher current consumption increases the certainty equivalent through a lower variance of future consumption due to lower savings in the form of capital. On the other hand, a higher level of future consumption increases the certainty equivalent as a consequence of a lower Arrow Pratt measure of risk aversion. Ambiguity, too, holds with respect to $\sigma_{\pi rt}$.

We now extend our analysis by inserting equation (54) into (58) thus taking the optimal ratio of current consumption and cash holdings into account as explained by equation (54). This changes equation (59) as follows:

$$
\begin{aligned}
\left(\overline{C}^{o}+(1+\overline{r})\,\overline{C}^{y}+(\overline{r}+\overline{\pi})\,\overline{M}1^{y}\frac{\alpha}{\eta}\right)\widehat{c}_{t}^{y} &= \\
\overline{Y}^{y}(1+\overline{r})\,\widehat{y}_{t}^{y}-\left(\overline{r}+\overline{\pi}-\overline{i}\right)\overline{T}^{y}\widehat{t}_{t}^{y} &+ \\
\left(\overline{K}^{y}-\frac{\overline{C}^{o}}{\alpha(1+\overline{r})}+\frac{(1-\overline{\pi})\,\overline{M}1^{y}}{\eta(1+\overline{r})}\right)E_{t}\widehat{r}_{t+1}+\overline{T}^{y}\widehat{i}_{t+1} &- \\
\left(\underbrace{\overline{T}^{y}+\overline{M}1^{y}}_{\overline{M}2^{y}}-\frac{\overline{M}1^{y}}{\eta}\right)E_{t}\widehat{\pi}_{t+1} &- \\
\left[\overline{K}^{y}\left(\frac{\frac{\overline{K}^{y}}{\overline{C}^{o}}(1+\alpha)(1+\overline{r})-2}{(1+\overline{r})}\right)+\frac{\alpha}{\eta}\left(\frac{\overline{K}^{y}}{\overline{C}^{o}}\right)\frac{\overline{M}1^{y}(1-\overline{\pi})}{(1+\overline{r})}\right]\sigma_{rt}^{2} &- \\
\left[\overline{C}^{o}\left(\frac{\overline{M}2^{y}}{\overline{C}^{o}}\right)^{2}\left(\frac{1+\alpha}{1+\overline{r}}\right)-\frac{\alpha}{\eta}\left(\frac{\overline{M}2^{y}}{\overline{C}^{o}}\right)\overline{M}1^{y}\right]\sigma_{\pi t}^{2} &+ \\
\left[\overline{M}2^{y}\left(\frac{\frac{\overline{K}^{y}}{\overline{C}^{o}}(1+\alpha)(1+\overline{r})-1}{(1+\overline{r})}\right)-\frac{\alpha}{\eta}\overline{M}1^{y}\frac{\left(\frac{\overline{K}^{y}}{\overline{C}^{o}}\right)(1+\overline{r})+\left(\frac{\overline{M}2^{y}}{\overline{C}^{o}}\right)(1+\overline{\pi})}{(1+\overline{r})}\right]\sigma_{\pi rt} & \qquad (60)
\end{aligned}
$$

Obviously, the optimal ratio of cash holdings and current consumption strengthens the income effect of changing expected real interest rates on capital because higher real interest rates on capital lower optimal cash holdings compared to current consumption. On the other hand, a higher rate of expected inflation lowers optimal cash holdings in relation to current consumption, and this substitution effect introduces ambiguity into the correlation between current consumption and expected inflation. By contrast, higher capital market risk implies a higher level of cash holdings compared to current consumption. This strengthens a negative correlation between current consumption and capital market risk. A higher inflationary risk lowers optimal cash holdings in relation to current consumption thus introducing ambiguity with respect to the direction of correlation between inflationary risk and current consumption. Ambiguity, too, can still be observed with respect to the covariance of the real interest rate on capital and inflation, where a negative correlation gains strength.

The final step consists of introducing the optimal ratio of deposits and current consumption into (60). A complete solution of current consumption can then be represented by

$$\widehat{c}_t^y = \frac{1}{\Sigma}\left(\Psi_1\widehat{y}_t^y + \Psi_2 E_t\widehat{r}_{t+1} + \Psi_3\widehat{i}_{t+1} + \Psi_4 E_t\widehat{\pi}_{t+1} + \Psi_5\sigma_{rt}^2 + \Psi_6\sigma_{\pi t}^2 + \Psi_7\sigma_{\pi rt}\right) \tag{61}$$

$$\Sigma = \left(\overline{C}^o + (1+\overline{r})\,\overline{C}^y + (\overline{r}+\overline{\pi})\,\overline{M}1^y\frac{\alpha}{\eta} + \left(\overline{r}+\overline{\pi}-\overline{i}\right)\overline{T}^y\frac{\alpha}{\mu}\right) \tag{62}$$

$$\Psi_1 = \overline{Y}^y(1+\overline{r}) > 0 \tag{63}$$

$$\Psi_2 = \left(\overline{K}^y - \frac{\overline{C}^o}{\alpha(1+\overline{r})} + \frac{(1-\overline{\pi})\,\overline{M}1^y}{\eta(1+\overline{r})} + \frac{(1-\overline{\pi})\,\overline{T}^y}{\mu(1+\overline{r})}\right) \gtreqless 0 \tag{64}$$

$$\Psi_3 = \overline{T}^y - \frac{\overline{T}^y}{\mu} < 0 \text{ if } \mu < 1 \tag{65}$$

$$\Psi_4 = \left(-\underbrace{\left(\overline{T}^y + \overline{M}1^y\right)}_{\overline{M}2^y} + \frac{\overline{M}1^y}{\eta} + \frac{\overline{T}^y}{\mu}\right) > 0 \text{ if } \mu < 1, \eta < 1 \tag{66}$$

$$\Psi_5 = -\overline{K}^y\left(\frac{\frac{\overline{K}^y}{\overline{C}^o}(1+\alpha)(1+\overline{r}) - 2}{(1+\overline{r})}\right) - \frac{\alpha}{\eta}\left(\frac{\overline{K}^y}{\overline{C}^o}\right)\frac{\overline{M}1^y(1-\overline{\pi})}{(1+\overline{r})} - \frac{\alpha}{\mu}\left(\frac{\overline{K}^y}{\overline{C}^o}\right)\frac{\overline{T}^y(1-\overline{\pi})}{(1+\overline{r})} \lesseqgtr 0 \tag{67}$$

$$\Psi_6 = -\overline{C}^o\left(\frac{\overline{M}2^y}{\overline{C}^o}\right)^2\left(\frac{1+\alpha}{1+\overline{r}}\right) + \frac{\alpha}{\eta}\left(\frac{\overline{M}2^y}{\overline{C}^o}\right)\overline{M}1^y + \frac{\alpha}{\mu}\left(\frac{\overline{M}2^y}{\overline{C}^o}\right)\overline{T}^y \gtreqless 0 \tag{68}$$

$$\Psi_7 = \overline{M}2^y\left(\frac{\frac{\overline{K}^y}{\overline{C}^o}(1+\alpha)(1+\overline{r}) - 1}{(1+\overline{r})}\right) - \frac{\alpha}{\eta}\overline{M}1^y\frac{\left(\frac{\overline{K}^y}{\overline{C}^o}\right)(1+\overline{r}) + \left(\frac{\overline{M}2^y}{\overline{C}^o}\right)(1+\overline{\pi})}{(1+\overline{r})} - \frac{\alpha}{\mu}\overline{T}^y\frac{\left(\frac{\overline{K}^y}{\overline{C}^o}\right)(1+\overline{r}) + \left(\frac{\overline{M}2^y}{\overline{C}^o}\right)(1+\overline{\pi})}{(1+\overline{r})} \tag{69}$$

We observe that by integrating the optimal relationship between deposits and current consumption into (60), the positive income effect of changing real interest rates on capital for current consumption will be strengthened further. We also note that now a higher rate of expected inflation will lead to a higher level of current consumption, given that μ and η are smaller than one. A negative correlation between current consumption and capital market risk attains a higher probability. Furthermore, a higher level of current consumption due to higher inflationary risk now has an even greater chance. With caution we may furthermore conclude that if the rate of inflation and the real interest rate on capital are positively correlated, then a higher covariance between the two variables will lead to a lower level of current consumption. Finally, we observe that now current consumption and the nominal interest rate on deposits are negatively correlated for $\mu < 1$ and hence the substitution effect dominates.

2.3 Implications for Money Demand

For the following analysis we take cash and deposits together to obtain broad money according to

$$M2_t^y = M1_t^y + T_t^y \tag{70}$$

or in percentage deviations from the steady state:

$$\hat{m}2_t^y = \frac{\overline{M}1^y}{\overline{M}2^y}\hat{m}1^y + \frac{\overline{T}^y}{\overline{M}2^y}\hat{t}_t^y \tag{71}$$

where $m1_t^y$ and t_t^y are given by equations (54) and (56). Of importance for the following analysis between direct effects (substitution effects) of changing rates of return and risk parameters and indirect effects due to the impact of these same variables on current consumption.

As a straightforward result we obtain that due to its dependence on current consumption, money demand correlates positively with (labor) income according to:

$$\begin{aligned}
\frac{\partial \hat{m}2_t^y}{\partial \hat{y}_t} &= \left(\frac{\alpha}{\eta}\left(\frac{\overline{M}1^y}{\overline{M}2^y}\right) + \left(\frac{\overline{M}1^y}{\overline{M}2^y}\right)\frac{\alpha}{\mu}\right)\frac{\partial \hat{c}_t^y}{\partial \hat{y}_t} \\
&= \left(\frac{\alpha}{\eta}\left(\frac{\overline{M}1^y}{\overline{M}2^y}\right) + \left(\frac{\overline{T}^y}{\overline{M}2^y}\right)\frac{\alpha}{\mu}\right)\frac{\Psi_1}{\Sigma}
\end{aligned}$$

$$\left(\frac{\alpha}{\eta}\left(\frac{\overline{M}1^y}{\overline{M}2^y}\right) + \left(\frac{\overline{T}^y}{\overline{M}2^y}\right)\frac{\alpha}{\mu}\right) \equiv \Delta$$

Taking equations (54) and (55) together, the reaction of broad money to changes of the expected real interest rate on capital is as follows:

$$\begin{aligned}
\frac{\partial \hat{m}2_t^y}{\partial E_t \hat{r}_{t+1}} &= \Delta\frac{\Psi_2}{\Sigma} - \frac{1-\pi}{\eta(1+\overline{r})(\overline{r}+\overline{\pi})}\left(\frac{\overline{M}1^y}{\overline{M}2^y}\right) - \\
&\quad \frac{1-\pi}{\eta(1+\overline{r})(\overline{r}+\overline{\pi}-\overline{i})}\left(\frac{\overline{T}^y}{\overline{M}2^y}\right)
\end{aligned} \tag{72}$$

Ambiguity exists because do not know whether current consumption will increase or fall due to a higher expected real interest rate on capital. The interesting point is that the direction of this correlation itself is affected by money demand where in this case only the unambiguous substitution effect is relevant.

An increase in the nominal interest rate on deposits leads to a higher demand for money provided that the impact of consumption on money is smaller than the impact of deposits:

$$\frac{\partial \hat{m}2_t^y}{\partial \hat{i}_{t+1}} = \Delta\frac{\Psi_3}{\Sigma} + \frac{\frac{\overline{T}^y}{\overline{M}2^y}}{\mu(\overline{r}+\overline{\pi}-\overline{i})}$$

A change in expected inflation changes money demand as follows:

$$\frac{\partial \hat{m}2_t^y}{\partial E_t \hat{\pi}_{t+1}} = \Delta \frac{\Psi_4}{\Sigma} - \frac{\frac{\overline{M}1^y}{\overline{M}2^y}}{\eta(\overline{r}+\overline{\pi})} - \frac{\frac{\overline{T}^y}{\overline{M}2^y}}{\mu(\overline{r}+\overline{\pi}-\overline{i})}$$

Since current consumption and expected inflation are positively correlated for $\eta < 1$ and $\mu < 1$, the net effect is again ambiguous.

Turning to capital market risk, we observe that given current consumption, a higher capital market risk leads to a lower level of money demand. Taking into account, however, that we cannot rule out a negative correlation between current consumption and capital market risk, we are again unable to indicate clear effects.

$$\begin{aligned} \frac{\partial \hat{m}2_t^y}{\partial \sigma_{rt}^2} &= \Delta \frac{\Psi_5}{\Sigma} + \frac{\alpha}{\eta}\left(\frac{\overline{M}1^y}{\overline{M}2^y}\right)\left(\frac{\overline{K}^y}{\overline{C}^o}\right)\left(\frac{1-\overline{\pi}}{\eta(1+\overline{r})(\overline{r}+\overline{\pi})}\right) + \\ &\quad \left(\frac{\overline{T}^y}{\overline{M}2^y}\right)\left(\frac{\overline{K}^y}{\overline{C}^o}\right)\left(\frac{1-\overline{\pi}}{\mu(1+\overline{r})(\overline{r}+\overline{\pi}-\overline{i})}\right) \end{aligned}$$

Again we face the situation that the direct effects of higher capital market risk promote a fall in current consumption which in its turn feeds back to a lower level of money demand. Qualitatively, the same applies to effects of inflationary risk:

$$\frac{\partial \hat{m}2_t^y}{\partial \sigma_{\pi t}^2} = \Delta \frac{\Psi_6}{\Sigma} - \frac{\frac{\alpha}{\eta}\frac{\overline{M}1^y}{\overline{M}2^y}\frac{\overline{M}2^y}{\overline{C}^o}}{(\overline{r}+\overline{\pi})} - \frac{\frac{\alpha}{\mu}\frac{\overline{T}^y}{\overline{M}2^y}\frac{\overline{M}2^y}{\overline{C}^o}}{(\overline{r}+\overline{\pi}-\overline{i})}$$

According to the direct effects of higher inflationary risks, money demand goes down. However, these direct effects lead to a higher level of current consumption which again leads to a higher level of money demand. How a correlation between the real interest rate on capital and the rate of inflation affects money demand is explained by the following equation:

$$\begin{aligned} \frac{\partial \hat{m}2_t^y}{\partial \sigma_{r\pi t}^2} &= \Delta \frac{\Psi_7}{\Sigma} + \frac{\alpha}{\eta}\left(\frac{\overline{M}1^y}{\overline{M}2^y}\right)\left[\frac{(1+\overline{r})\left(\frac{\overline{K}^y}{\overline{C}^o}\right) + \left(\frac{\overline{M}2^y}{\overline{C}^o}\right)(1+\overline{\pi})}{(1+\overline{r})(\overline{r}+\overline{\pi})}\right] + \\ &\quad \frac{\alpha}{\mu}\left(\frac{\overline{T}^y}{\overline{M}2^y}\right)\left[\frac{(1+\overline{r})\left(\frac{\overline{K}^y}{\overline{C}^o}\right) + \left(\frac{\overline{M}2^y}{\overline{C}^o}\right)(1+\overline{\pi})}{(1+\overline{r})(\overline{r}+\overline{\pi}-\overline{i})}\right] \end{aligned} \tag{73}$$

Given current consumption, money demand is positively correlated with a positive covariance of inflation and the real interest rate on capital. These reaction contributes to a negative correlation of current consumption with a positive covariance $\sigma_{r\pi}$, which again acts as a countervailing effect on money demand. In summary we may say that the dependency of money demand on consumption explains ambiguity in the behavior of money demand. If money demand reacts strongly to changes of consumption, then it becomes possible that for example higher inflationary risks will even increase households' willingness to increase their money holdings.

3 The Empirical Section

3.1 Literature Review On Money Demand Under Capital Market and Inflation Risks

The existing empirical literature on money demand is rich but almost all of these studies formulate *ad hoc* models based on story-telling or plausibility. Given the large number of research articles, we will review only empirical papers explicitly considering economic risks/uncertainty on money demand. Furthermore, solely empirical studies referring to the North American or EMU area will be reviewed.[2]

As uncertainty has a latent nature it can be measured only indirectly. The concrete measure depends on the aspects one wants to evaluate. The focus can be either on the microeconomic or macroeconomic level. In this paper we put the accent on macroeconomic aspects with regard to price inflation risk and capital market risk. Recently, the IMF has emphasized the relevance of uncertainty measures as major macroeconomic stress factors (IMF, 2012, 49). Economic theory suggests that macroeconomic as well as policy uncertainty may affect the economy's demand side through its impact on household consumption or firm investment. Additionally, there are various supply side channels through which economic uncertainty may have repercussions on the economy (Bloom, 2009; Bloom et al., 2013).[3] In this subsection, we will highlight the findings of the existing empirical literature on the relationship between risk factors and money demand behavior on the aggregate.

Carpenter and Lange (2003) estimate a risk-augmented money demand relationship for the U.S. economy. They add a volatility index of the equity market into a standard equilibrium money demand relationship. According to their results a positive change in equity risk leads to higher demand for M2 in the long-run. It is argued that risky assets are substituted for safe alternatives such as cash.

Choi and Oh (2003) stress the importance of uncertainty about output and monetary policy for money demand decisions. The authors derive a general equilibrium model showing that output uncertainty and monetary uncertainty, among other explanatory variables, affect U.S. money demand significantly. They apply a bi-variate rolling window VAR model including the growth rates of real GNP and M1 money measure, respectively, to extract the time-varying innovations of both series. As a result, Choi and Oh find that output uncertainty has a negative effect while monetary uncertainty (interpreted as an unexpected shift in monetary policy) positively affects

[2] For recent and more detailed literature surveys on empirical money demand studies see Belke and Czudaj (2010) as well as Setzer and Wolff (2009). For an overview using Panel data see also Dobnik (2011) and Kumar et al. (2013).

[3] For empirical evaluations see for instance IMF (2012, 49 pp.).

money demand in their sample.

Based on data for the Canadian economy, Atta-Mensah (2004) constructs an economic uncertainty index. The author fits GARCH models to a vector of variables, namely the stock market index, the long-term yield of the bond market, the 90-day commercial paper rate, the US-CAN exchange rate and real GDP in order to compile a single index capturing economic uncertainty which enters the short-term dynamics of an error-correction model. The results indicate that a positive change in economic uncertainty is accompanied by an increase in the demand for M1 but a reduction in M2. Atta-Mensah concludes that increasing economic uncertainty "*...reduces agent's appetites for risky assets (guaranteed investment certificates and money market mutual funds). In addition, uncertainty surrounding the production and supply of goods and services in periods of increased economic uncertainty induces agents to increase their level of money holding for precautionary reasons. Furthermore, in periods of economic uncertainty, real assets, such as houses and precious metals, are more attractive than nominal assets.*" (Atta-Mensah, 2004, 10).

In their study on the Euro area, Bruggeman et al. (2003) examine the effects of stock market volatility on M3 money demand. In a first step, a leverage GARCH model is specified in order to construct a risk series. The estimated (conditional) volatility measure is added to a VECM as a weakly exogenous variable in the second step. However, the authors do not find a significant effect of stock market volatility on money demand. Nevertheless, they admit that this might be due to the selected sample which does not cover pronounced periods of stock market volatility (Bruggeman et al., 2003, 35).

Greiber and Lemke (2005) conduct some research on both the Euro area as well the U.S. economy. Among the standard set of regressors two economic uncertainty measures are estimated in a first step using an unobserved component model consisting of six variables. These variables comprise the correlation between stock and bond returns, a stock market loss measure, a stock market volatility measure, a measure on stock market returns as well as a consumer and industry-sector confidence measure, respectively. For the Euro area, the authors find that a standard money demand relationship augmented by the estimated $I(1)$ uncertainty factor, which is interpreted as a liquidity preference indicator, helps to re-establish a stable cointegrating relationship. Furthermore, the second but stationary factor, mainly reflecting idiosyncratic consumer and industry sentiments, improves the short-run fit of the model substantially. The application of this estimation strategy to the U.S. economy reveals for both monetary aggregates M2M and MZM that the inclusion of uncertainty factors improves the statistical fit of the model.

In an application to the Euro area Carstensen (2006) argues that the observed overshoot of M3 at the end of 2001 can partly be explained by a decline in equity returns as well as increased stock market volatility. The inclusion of these two additional factors re-establishes the standard long-run money demand relationship. The respective stock market volatility measure is estimated by a leverage GARCH model based on daily returns of the nominal stock price index.[4]

The role of inflation uncertainty on money demand was examined by Higgins and Majin (2009) for both M1 and M2 U.S. money measures. In order to quantify latent inflation uncertainty, the authors fit a conventional backward-looking Phillips curve model with GARCH errors to derive the conditional variance of inflation. The authors find that increased inflation uncertainty has negative impacts on the demand for M1 as concerns about higher expected inflation put low-interest bearing assets under stress. This triggers a substitution away from M1 to higher-interest bearing components of M2. Furthermore, M1 includes long-term assets which agents may want to substitute for money market instruments in order to reduce the risk associated with long-term assets. This is confirmed by the results as higher inflation uncertainty is positively correlated with M2 holdings.

de Bondt (2009) studies the effects of equity risk and macroeconomic uncertainty on M3 money demand for the Euro area. The results suggest that equity markets play a significant role for money demand dynamics. The demand for M3 is found to be negatively related to the expected risk-adjusted real rate of equity return. This is in line with previous findings that there exists a substitution effect away from equity markets during turbulent times on these markets. Additionally, the author finds that precautionary motives, stemming from the labor market, also have a significant effect on money demand holding.

The work by Seitz and von Landesberger (2010) is a recent synthesis of previous work done by de Bondt (2009) on the relevance of precautionary motives as well asl the studies conducted by Greiber and Lemke (2005) and Carstensen (2006) on the effect of stock and bond market risks on money demand. Seitz and Landesberger find for the Euro area that financial market uncertainty is positively correlated with the demand for M3 through the substitution channel which is in line with former studies.

Lastly, Cronin et al. (2011) apply a slightly different econometric framework using U.S. data. Instead of estimating long-run relationships, a multivariate GARCH framework is applied. This allows one to analyze the causality between money demand growth and macroeconomic as well as monetary uncertainty. In contrast to Choi and Oh (2003), who employ M1 as their money measure, it is found that a positive change in macroeconomic uncertainty leads to an increase in the demand for M2. Furthermore, Cronin et al.'s measure of monetary uncertainty does not cause changes in money demand. Rather the causality runs the other way around: monetary uncertainty may be caused by (excessive) money growth.

Overall, there is strong evidence that capital market risk as well as inflation risk are economically meaningful in explaining money demand behavior. Our own empirical application and the estimation results are provided in the next section. Following most studies, we also apply

[4]Further recent studies examining the relevance of stock prices for money demand in the Euro area are written by Dreger and Wolters (2009, 2010) and Nautz and Rondorf (2011).

the cointegrating method. However, we also study the short-run dynamics as well as potential time-variation of the money demand relationship.

3.2 The Modeling Strategy

The starting point for the empirical analysis is given by the linearized money demand function stated in deviations from an empirically latent steady-state, as stated in eq. (71). The linearized money demand (for M2) function can be stated implicitly as follows[5]

$$\widehat{m_t} = f(\widehat{y}_t, E_t(\widehat{r}_{t+1}), \widehat{i}_{t+1}, E_t(\widehat{\pi}_{t+1}), \sigma^2_{rt}, \sigma^2_{\pi t}, \sigma_{\pi rt}) \tag{74}$$

where *hat* denotes deviations from steady state, E is the expectations operator and t refers to the time subscript. Thus, money demand m_t is a function of current income (y_t), the one-period ahead own rate of M2 (i_{t+1}), the expected real rate of return on stocks ($E_t(r_{t+1})$), expected inflation ($E_t(\pi_{t+1})$), the current variance of the real rate of return (σ^2_{rt}), the current variance of inflation ($\sigma^2_{\pi t}$) as well as the current covariance between the real rate of return and inflation ($\sigma^2_{\pi rt}$).

Combining theory and evidence poses a major issue. Under specific conditions, dynamic models (e.g. DSGE) translate into highly restricted VAR models which do not fit empirical data well (see e.g. Juselius and Franchi (2007)). Different methods to deal with this issue were suggested in the literature (see for an overview about modeling techniques Garratt et al. (2012)). However, Kapetanios et al. (2007) and Hoover et al. (2008) have stressed that regardless of the method used to combine theory and evidence, the empirical model selected needs to take into account cointegrating relationships once some shocks have permanent effects. Thus, it may not be appropriate to pre-filter any of the variables with the intention to remove its permanent component, as the model will not be able to track the levels of the data, which is important for forecasting. Furthermore, measuring deviations from some *ad hoc* trend[6], e.g. by means of any univariate filter, introduces some severe estimation bias and makes it complicated to conduct inference from the long-run properties of the model (Garratt et al., 2012, 29 p.).

We take this critical perspective into consideration and specify an econometric model which differs from the theoretical approach derived in eq. (71), in order to reconcile theory and evidence. To be specific, we exploit the information of the levels of the variables by estimating a long-run relationship between the series. This is in contrast to the frequently applied approach of directly

[5]Note that we have re-stated the expression in eq. (71) in terms of current income, as this expression is used in the following empirical application.

[6]The *ad hoc* feature refers to the fact that such trend-extracting methods do not allow for series-dependent characteristics that guarantee consistency with the data. For instance, for the well-known Hodrick-Prescott filter, a (smoothing) λ-value of 100 is recommended for annual data. This value, however, has no sound theoretical justification, and the optimal smoothing parameter may depend on additional time-series characteristics. Model-based frameworks provide more precise estimates of cyclical and trend components. See e.g. Garratt et al. (2012, ch. 10) for more arguments in this.

using deviations from some pre-determined steady-state. Since we are not interested in the size of the structural parameter values but rather in their algebraic signs, this does not pose further theoretical issues. We follow the argumentation of Pesaran and Smith (2011) stating that it is preferable to include long-run relations and to leave the short-run dynamics less restricted in order to estimate the steady-state. Furthermore, the majority of empirical approaches to money demand apply the cointegration and error-correction modeling technique. This allows us to compare our results directly with former studies. Lastly, within this modeling framework, we can distinguish between long-run and short-run dynamics of various variables in a consistent way in order to study numerous aspects of the model. However, it should be noted that the majority of empirical studies focuses solely on the long-run properties as the main concern is about the long-run stability of the money demand relationship. Nevertheless, for the conduct of monetary policy it is particularly important to analyze the short- to medium-run response of money demand to specific shocks as well (Ball, 2012).

In this study we stick to the single-equation cointegrating framework for various reasons. First, the theoretical model outlined is a partial model analyzing solely the determinants and dynamics of the money demand behavior. We do not analyze a completely closed macroeconomic system at this stage. Secondly, as shown by Pesaran and Shin (1998) and Pesaran et al. (2001) the ARDL model framework provides super-consistent estimates of the long-run parameters even in the presence of endogeneity. Hence, for estimating the money demand long-run relationship consistently, a system framework, which is much more sensitive to (mis-)specification issues, is not necessarily required. Of course, this does not rule out the use of the VECM framework *per se*. Thirdly, the ARDL single-equation framework allows for the mixture of both $I(1)$ as well as $I(0)$ variables in the long-run relationship. The VECM framework is less flexible with respect to this problem. Overall, there are good reasons to test our theoretical model using the frequently-used single-equation approach.

3.3 Construction of Variables

The detailed definition of variables and its data sources are provided in the Data Appendix. Most of the variables in our dataset cover observations from the 1960s to the end of 2014. However, the empirical analysis is restricted to the period 1978q1 to 2013q4 as for some variables no observations are available earlier.[7]

As we are interested in the money demand behavior of U.S. households, we decided to use the M2 definition as the starting point. The focus on households requires the appropriate adjustment of the original M2 time series by subtracting the sum of firm sector money demand (consisting of time deposits, savings deposits and mutual fund shares of corporates). The resulting series

[7] We have also used an extended sample ending in 2014q4. However, the results indicate substantial parameter changes in the long-run relationship which leads to implausible parameter estimations.

is expected to reflect $M2$ money holdings of the household sector. We again refer to the Data Appendix for more details. The nominal variable is deflated by the GDP deflator, P, $m_t = log(M_t/P_t)$.

The log of real households' disposable income is denoted by $y_t = \log(Y_t)$, and taken as the income measure. Expected inflation is approximated by the median of expected relative price changes over the next 12 months of households based on survey data, π_t^e. The own rate of M2 is denoted by i_t. Expected real stock market returns are measured by current real stock market returns, r_t, which is a convention in the literature. As the original real stock market return series is characterized by excess volatility reflecting short-term sentiments and speculation driving the underlying process, we decided to get rid of excess volatility by calculating the moving-average over three months. The co-variance between current price inflation (again based on the GDP deflator) and real stock market returns, $\sigma_{\pi r_t}$, is estimated by means of a rolling-window bi-variate VAR.[8]

In order to control for other sources of macroeconomic risk, we add two measures of capturing different types of risk. First, the time series measure of macroeconomic uncertainty proposed by Jurado et al. (2015) is considered.[9] They define macroeconomic uncertainty as "...uncertainty that may be observed in many economic indicators at the same time, across firms, sectors, markets, and geographic regions. And we are interested in the extent to which this macroeconomic uncertainty is associated with fluctuations in aggregate real activity and financial markets." (2015, 1212). The common component across many indicators is derived from a dynamic factor model. In the following the series is denoted $econunc_t$. The second index is the so called economic policy uncertainty measure, and accounts for economic policy risks. Here, we consider the widely-used *Economic Policy Uncertainty index* compiled by Baker et al. (2013) [10] The series is denoted by $polunc_t$.[11]

Our inflation risk measure takes the literature on constructing uncertainty measures seriously. The method applied for the construction of inflation risk starts from the general observation that inflation follows a unit root process. Stock and Watson (2007) formulate an univariate unobserved component model with stochastic volatility (UCSV henceforth), and show that it provides reasonable forecasting properties for inflation. The variances of the permanent and transitory disturbances evolve randomly over time. An alternative approach to quantify inflation risk was proposed by Andrade et al. (2012) and is coined the inflation-at-risk measure using survey-based density forecasts. Their inflation risk index illustrates a more general framework

[8]The window size is 20 quarters. For each iteration the estimated co-variance is stacked into a vector in order to construct the co-variances time-series. We also tried a window-size of 32 quarters but the results remain qualitatively unchanged.

[9]The data series is obtained from `http://www.econ.nyu.edu/user/ludvigsons`.

[10]The historical time series is available from `http://www.policyuncertainty.com/us_monthly.html`.

[11]It should be noted that the two series indeed account for different aspects of economic risk, as the contemporaneous correlation between the series is only 0.07 for the sample 1978q1 to 2013q4.

since it allows for potential asymmetry of inflation risks evolving over time.[12]

The setup of the UCSV model is as follows: It is assumed that the series of interest, x_t, can be decomposed into a permanent and transitory component with time-varying volatility. Allowing for time-variations is based on the empirical fact that parameter shifts in the estimated variances of the components have occurred over time for the U.S. economy (Stock and Watson, 2007). The dynamics of inflation closely follow an integrated moving-average process which can be re-written as an unobserved component model. It is assumed that x_t is driven by a stochastic trend, τ_t, with serially uncorrelated innovations η_t. The stochastic trend is driven by another white noise innovation ϵ_t:

$$x_t = \tau_t + \eta_t \tag{75}$$

$$\tau_t = \tau_{t-1} + \epsilon_t \ . \tag{76}$$

Both innovations η_t and ϵ_t are *i.i.d* normally distributed. Furthermore, the logarithms of the variances of both the transitory part, $\sigma^2_{\eta,t}$ ($\eta_t \sim N(0, \sigma^2_{\eta,t})$), as well as permanent part, $\sigma^2_{\epsilon,t}$ ($\epsilon_t \sim N(0, \sigma^2_{\epsilon,t})$), evolve as separate random-walks according to:

$$log\ \sigma^2_{\eta,t} = log\ \sigma^2_{\eta,t-1} + \nu_{\eta,t} \tag{77}$$

$$log\ \sigma^2_{\epsilon,t} = log\ \sigma^2_{\epsilon,t-1} + \nu_{\epsilon,t} \ . \tag{78}$$

The innovations to the variances, $\nu_t = (\nu_{\eta,t}, \nu_{\epsilon,t})'$, are *i.i.d.* $N(0, \gamma I_2)$ and orthogonal to each other. The parameter γ controls the smoothness of the stochastic volatilities $\sigma^2_{*,t}$. Thus, this approach models heteroskedasticity in inflation explicitly and might be preferred e.g. to standard (S)VAR models based on the (eventually) restrictive assumption of homoskedasticity, as argued by Chua et al. (2011). The model is estimated using the Gibbs sampling approach.[13] The studies by Wright (2011) and Dovern et al. (2009) have applied this model to inflation series before. Grimme et al. (2011, 7) interpret the permanent component as a measure of inflation uncertainty, whereas the transitory part may reflect some type of short-run risk measure. We fit the UCSV(0.2) model to our quarterly inflation expectation time series, π^e_t, for the sample from 1978q1 to 2013q4 using a prior for the initial condition of $\gamma = 0.2$.[14] The estimated time-varying standard deviation of the permanent component is plotted in Figure 2(a), and discussed in more detail below.

[12] Another alternative but maybe less sophisticated approach was employed by Higgins and Majin (2009) who estimate Phillips-curves with (G)ARCH errors. The conditional heteroskedasticity series is used as a proxy for inflation risk. However, the constructed risk series is transitory by construction, and no trend-risk component is estimated.

[13] We thank Peter Summers for providing his `gretl` code to us.

[14] This prior was also used by Stock and Watson (2007) for GDP inflation. We found that the results were robust against different prior values. We also applied the model to monthly data, but the results do not differ substantially, which is in line with the findings by Stock and Watson (2007).

As a measure of stock market risk the stock market premium, as e.g. suggested by Fama and French (1988), is used. It should be noted that in our theoretical model, risk is based on actor's aversion against the volatility of rates of return, which is measured by the variance of stock market returns, while the stock market premium additionally takes the strength of risk aversion into account. Hence, changes in the stock market premium may be the result of capital market risk and/or changes in the risk attitudes. This should be considered when interpreting the empirical results. The stock market premium, $\sigma^2_{r_t}$, is given by the ratio of the dividend yield on the S&P 500 stock price index, $divy_t = 100\frac{Dividends_t}{SP500_{t-4}}$, over the yield on 10-year U.S. Treasury notes, $GS10$:

$$\sigma^2_{r_t} = log(\frac{1 + divy_t}{1 + GS10_t}) \ . \tag{79}$$

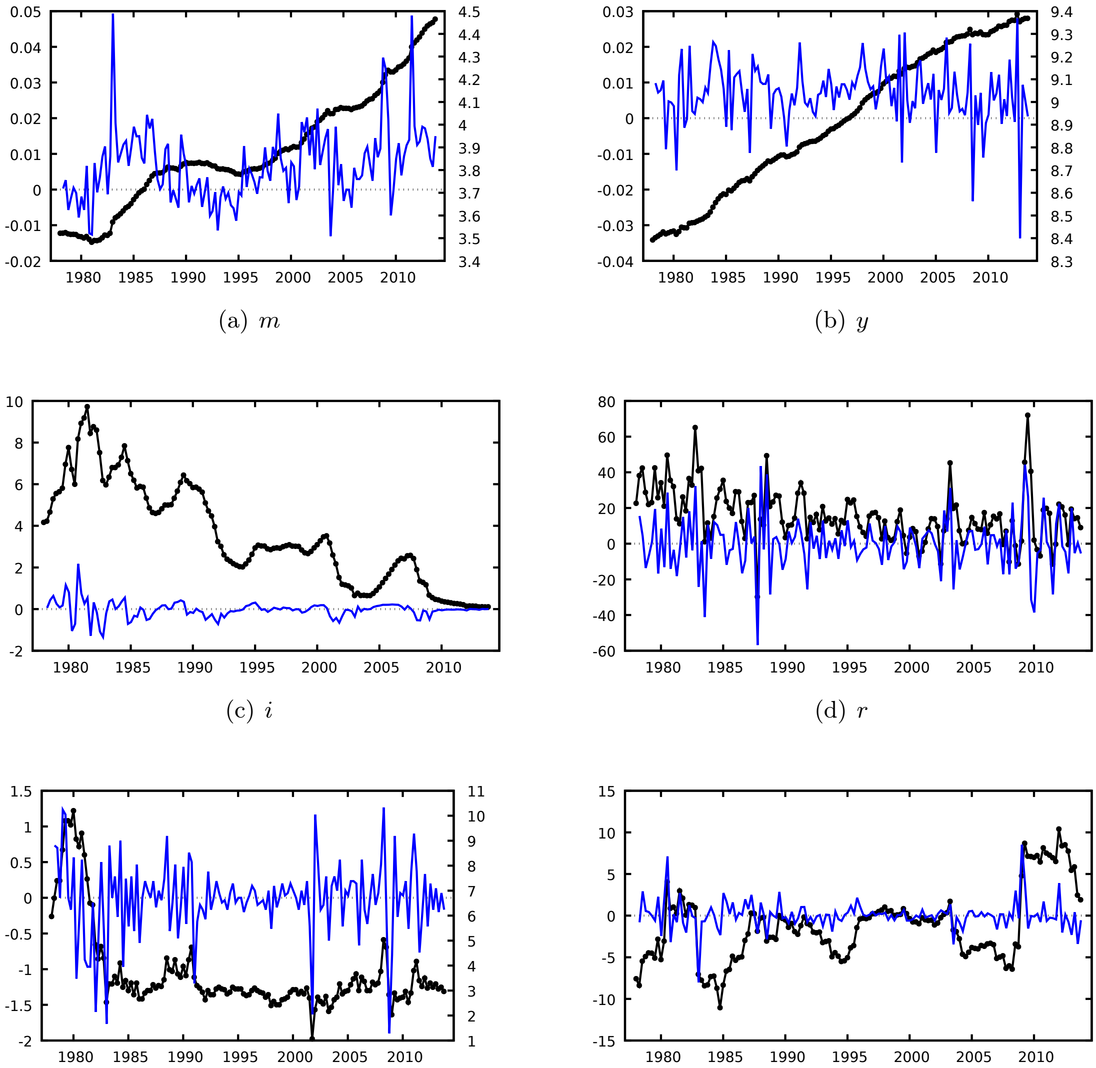
Recall that the expected partial effect of stock market risk on money demand is ambiguous: Most likely an increase in stock market risk reduces current consumption and hence money demand. However, the countervailing portfolio shift effect is positive such that the total impact is not definite.

3.4 Visual Inspection of the Time-Series and Initial Correlation Analysis

All time series are depicted in Figures 1 and 2. As expected, both the monetary aggregate m and the income measure y are upward trending over time (see Figures 1(a) and 1(b)). Additionally, the own rate of M2, i, (see Figure 1(c)) has shown a declining trend since the 1980s and reached the zero line as a result of unconventional monetary policy since 2012. Real stock market returns r are characterized by high variance and high-frequency fluctuations, as depicted in Figure 1(d). Real returns were temporarily negative during the bust of the New Economy bubble and also for about five quarters between 2008 and 2009 as a result of the recent great financial crisis (GFC, henceforth). Expected inflation shows a remarkable stability over time, with a few exceptional changes in the early 1980s, during the Iraq-war, the beginning of 2002/3 and during 2008/9 (see Figure 1(e)). However, overall there is no tendency of a fundamentally changed trend in inflation perceptions. Inflation risk, σ^2_π, as depicted in Figure 2(a), had been stable on a rather high level between 1978 and 1987 before its level has shifted downwards during of the Great Moderation period. Since then, the risk level has remained stable accompanied by modest cyclical fluctuations. The Clinton era boom years, the New Economy bubble and the surge in oil prices since the early 2000s were accompanied by a mild increase in inflation risk. The recent temporary increase in expected inflation is accompanied by a temporary but mild increase in the permanent component of inflation risk. However, the recent level of inflation risk is still low in historical comparison in the U.S. The risk associated with capital markets, σ^2_r, has been rather stable between 1978 and 2004 with a temporary decline between 1996 and 2000 (see Figure 2(b)). However, the bust of the New Economy bubble led to an increase in capital market risk.

In historical comparison, the GFC has led to a sharp positive level-shift in capital market risk since 2008, reflecting the high risk associated with capital market investments.

Interestingly, the macroeconomic uncertainty index (*econunc*) shows two spikes: first during the second oil crisis in the early 1980s and another one between 2008 and 2010 (see Figure 2(c)). The impact of the U.S. financial market crisis in the late 1980s as well as the bust of the New Economy in 2001 have had mild impacts on macroeconomic uncertainty. Different to the *econunc* measure, the economic policy uncertainty measure, *polunc*, has successively risen as a result of the GFC (see Figure 2(d)) and remains on a historically high plateau. Lastly, the covariance between inflation and real stock market returns, $\sigma_{\pi r}$, is slightly negative for most of the sample. The time series shows sharp negative downturns in 1978 and 1985. However, since 2009 the covariance has turned strongly positive (see Figure 1(f)).

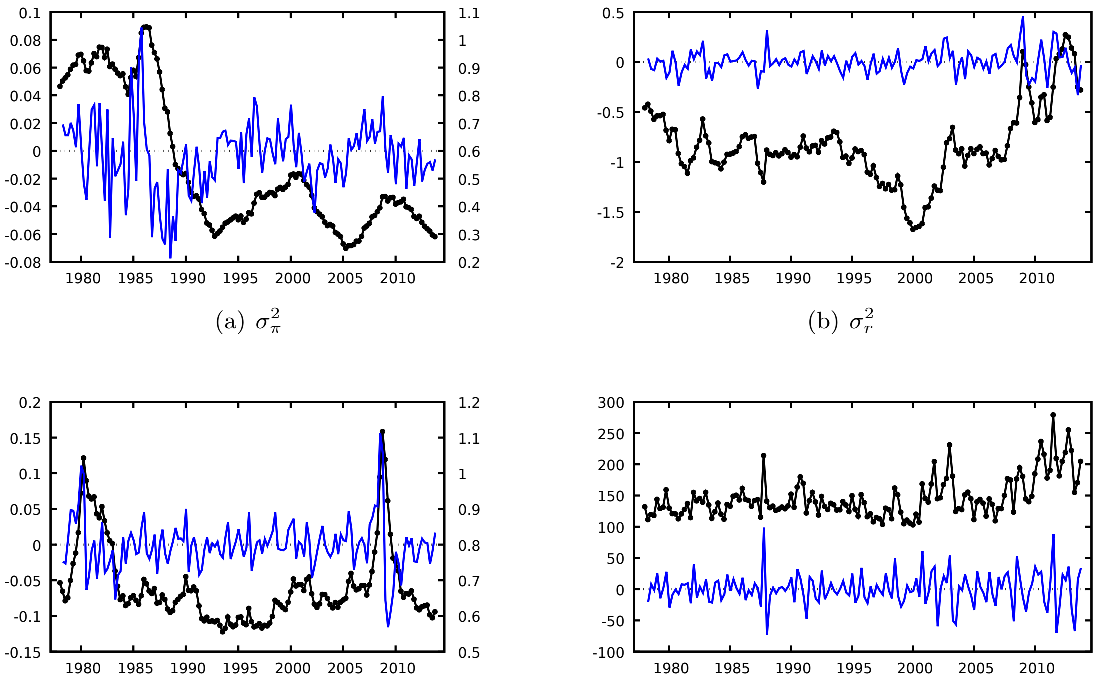


(a) m (b) y

(c) i (d) r

(e) π^e (f) $Cov(\pi, r)$

Figure 1: Time series plots of the level variables (point-lines) and its corresponding first differences. If a second y-axis is given, it refers to the level variable. Sample: 1978q1 – 2013q4.



(a) σ_π^2 (b) σ_r^2

(c) *econunc* (d) *polunc*

Figure 2: Time series plots of the level variables (point-lines) and its corresponding first differences. If a second y-axis is given, it refers to the level variable. Sample: 1978q1 – 2013q4.

In Figure 6 we depict the contemporaneous correlation between the change in money stock, Δm_t, and the first difference of the variables of interest. The unconditional correlation analysis reveals a positive link between money demand and income changes (see Figure 6(a)) as expected.

In contrast to the OLS estimator, which reveals a slightly negative correlation between Δm and changes in the real rate of returns, the least-absolute deviations estimator (LAD)[15] suggests a positive correlation 6(b). Theoretically, it was shown that the total effect is ambiguous and depends on the response of current consumption on changes in the expected real return on capital as well as a substitution effect, as described in Section 2.3. If the substitution effect is sufficiently strong, however, the total effect is most likely negative.

Furthermore, we find a negative unconditional correlation between changes in the own rate of M2 and changes in money holdings (see Figure 6(c)). This rather counter-intuitive result is also contained as a possibility in the theoretical model where the direct effect points to a positive correlation between the demand for M2 and the own rate whereas the indirect effect, which is represented by the reaction of per capita consumption, indicates the opposite, as described in Section 2.3.

The link between changes in expected inflation and the growth of money demand is negative, as displayed in Figure 6(d). According to the theoretical model, the total effect is again ambiguous but most likely current consumption responds positively to an increase in expected

[15]The LAD estimator is resistant to outliers as it gives equal emphasizes to all observations.

inflation.

Changes in money demand and inflation risk are not unconditionally correlated at all, as depicted in Figure 6(f). It seems that the indirect positive effect on current consumption just compensates the direct negative impact on money demand. Lastly, we find a positive unconditional correlation between the change in money holdings and changes in stock market risk (see Figure 6(e)).

3.5 Unit Root Properties

In this sub-section the univariate time-series properties of the variables of interest are briefly analyzed. Instead of following the classical cointegration approach by initially testing each time series for (non-)stationarity before estimating the long-run relationship, we follow the error-correction modeling (ECM henceforth) procedure. Putting the focus on the direct estimation of the ARDL or ECM has several advantages. First, it should be recalled that unit root tests can suffer from inflated Type I error rates when data are cointegrated (Reed, 2014). Secondly, the residual-based Engel-Granger (Engle and Granger, 1987) two-step estimation strategy involves additional uncertainty as all variables have to be tested for unit roots before the long-run equilibrium is also tested for stationarity. The single-step ECM-based or ARDL bounds test on cointegration involves less uncertainty and the power as well as size of the associated cointegration tests is higher as it uses available information more efficiently (Kremers et al., 1992). Additionally, the bounds test approach on cointegration also allows for a mixture of $I(1)$ and $I(0)$ series in the long-run relationship. Lastly, standard unit root tests also suffer from non-normality and structural breaks (Perron, 1989). However, instead of applying unit-root tests allowing for parameter changes, we prefer to estimate the ARDL model of interest and apply a test on parameter stability afterwards.

In order to check for the statistical properties of the separate time series, we run the ADF-GLS (Elliott et al., 1996) as well as the KPSS (Kwiatkowski et al., 1992) unit-root tests for our sample ranging from 1978q1 to 2013q4.[16] The results for both the ADF-GLS and KPSS test are provided in Tables 2 and 3 in the Appendix.

The null of a unit-root cannot be rejected for m, y, i, σ_r^2 and σ_π^2 at standard significance levels and lag lengths tested. This finding is confirmed by the KPSS test according to which the null of stationarity can be fairly rejected at least at the 5% level for these series.

The ADF-GLS test and KPSS tests suggest some conflicting results for the real stock market return series (r), expected inflation (π^e), the covariance measure ($\sigma_{\pi r}$) as well as for the macroeconomic uncertainty (*econunc*) and economic policy uncertainty (*polunc*) series. Thus, the tests do not present clear-cut results. However, we proceed by assuming that inflation follows

[16] All computation in this paper is done by the open-source econometric package `gretl` (Cottrell and Lucchetti, 2013).

a random-walk which is a generally acknowledged finding (Stock and Watson, 2005). The visual inspection of the covariance series as well as both uncertainty measures rather suggests stationary processes accompanied either by level-shifts or temporary outliers resulting in non-normality.[17] Both properties affect the power and size of standard unit-root tests, as shown by Perron (1989) and others. Similar ambiguities remain w.r.t. the stock market return series. However, the good news is that the cointegration bounds test proposed by Pesaran et al. allows one to remain open with regard to the stationarity assumptions as will be explained below.

3.6 Econometric Long-Run Specification, Testing, and Dynamic Multipliers

We proceed with the determination and estimation of possible long-run relationships. The following five long-run model specifications are tested, where Z denotes a 1 by k time series vector:

1. $Z_{1t} = [m_t \; y_t \; i_t \; r_t]'$
2. $Z_{2t} = [m_t \; y_t \; i_t \; r_t \; \pi_t^e]'$
3. $Z_{3t} = [m_t \; y_t \; i_t \; r_t \; \pi_t^e \; \sigma_{r_t}^2]'$
4. $Z_{4t} = [m_t \; y_t \; i_t \; r_t \; \pi_t^e \; \sigma_{\pi_t}^2]'$
5. $Z_{5t} = [m_t \; y_t \; i_t \; r_t \; \pi_t^e \; \sigma_{r_t}^2 \; \sigma_{\pi_t}^2]'$.

The benchmark Model 1 includes among the dependent money series the standard set of explanatory variables namely an income measure (y) and an opportunity cost measure comprising the own rate of M2 (i) and the stock market real rate of return (r).[18] Step-by-step, inflation expectations (π^e) and the two risk variables σ_π^2 and σ_r^2 are added to the remaining four specifications. We estimate these different specifications in order to check whether the baseline long-run money demand relationship fits the data or not. If this is not the case other explanatory variables are required to eventually restore a plausible and stable long-run relationship which explains the data sufficiently well.

It may be surprising that both risk variables enter the long-run relationship, even though they are not included in the deterministic steady-state of the theoretical model obtained after a first-order Taylor expansion. In our empirical analysis we follow the argumentation of Pesaran and Smith, and allow for the "use of long-run cointegrating relations where they exist" (Pesaran and Smith, 2011, 13). As already shown in the literature review, there is overwhelming evidence that financial as well as risk variables help to re-establish a long-run money demand relationship. Thus, the inclusion of both inflation risk and capital market risk allows us to test empirically the hypothesis that both risk factors affect the households' money demand behavior.

[17] The stationarity assumption of the covariance is a sound assumption as the correlation between two series is bounded between -1 and 1.

[18] We also ran specifications using the cumulative sum of r instead of the rate itself, as the stationarity properties of r are ambiguous. Qualitatively, the results remain unchanged, irrespective of the chosen sample.

The co-variance between inflation and stock market returns ($\sigma_{\pi r}$) is taken to be I(0), and enters the model as an unrestricted exogenous, as described below. The same assumption is made for the macroeconomic uncertainty (*econunc*) and the economic policy uncertainty (*polunc*) measures.

Bounds Testing Approach to the Analysis of Long-Run Relations Classical cointegration methods require all the underlying variables to follow integrated stochastic processes of the same order. The unit-root pre-testing introduces additional uncertainty into the estimation process. Recently, Pesaran, Shin and Smith (2001) (PSS henceforth) have suggested a bounds testing methodology which allows the long-run modeling of mixed $I(1)$ and $I(0)$ processes. A brief introduction into the model and estimation strategy follows.

For illustrative purposes, an unrestricted error correction model with a single regressor, x_t, and an intercept term is assumed. The conditional error-correction model (ECM)

$$\Delta y_t = \delta + \rho y_{t-1} + \theta x_{t-1} + \sum_{j=1}^{p-1} \gamma_j \Delta y_{t-j} + \sum_{j=0}^{q-1} \phi_j \Delta x_{t-j} + u_t \qquad t = 1, ..., T \tag{80}$$

can be derived from an underlying ARDL(p,q) model which can be estimated consistently by OLS.

The parameters δ, ρ, θ, γ and ϕ denote the intercept, speed of adjustment towards the long-run attractor, the effect of the lagged level of the exogenous $I(1)$ variable and the short-run effects of the endogenous as well exogenous series, respectively. Additional $I(0)$ series and deterministic variables can be added without causing further issues for estimation and inference. The lag-order of the ARDL(p,q) model can be determined by means of information criteria and specification tests such that the residuals fulfill the standard assumptions. We follow the argumentation by Hassler and Wolters (2006) and also consider the contemporaneous Δx_t as a regressor, as it was found that the conditional ECM outperforms the unconditional ECM as long as Δx_t does not respond to past equilibrium deviations.[19]

The null hypothesis of no long-run relationship (with a restricted intercept) is stated as $H_0^{PSS} : \rho = \theta = \delta = 0$ and can be tested by using a Wald test for which the asymptotic distribution of the test statistics is non-standard under the null hypothesis irrespective of whether the regressors are $I(0)$, $I(1)$ or mutually cointegrated. Instead of exact critical values for an arbitrary mix of $I(0)$ and $I(1)$ variables, Pesaran et al. (2001) provide two sets of critical values: one which assumes that all regressors are $I(1)$, and the other one assuming that all series are $I(0)$. If the computed test-statistics falls below the $I(0)$ bound, one can conclude that the variables are $I(0)$, and hence no long-run relationship is possible. If the statistics exceeds the $I(1)$ bound a long-run relationship between the variables exists. The test is inconclusive if the statistics falls

[19] This assumption is frequently made in empirical applications such as the one by Shin et al. (2014).

inside the bounds, and some knowledge about the order of integration of the underlying variables will be needed. To improve the power and size of the PSS test under potential heteroskedasticity, we apply a bootstrap version of the PSS test. Furthermore, it was just recently shown by Cavaliere et al. (2014) in a multivariate framework that in the presence of heteroskedasticity in the innovations process, the wild bootstrap approach significantly outperforms the i.i.d. bootstrap analogue. We expect that this also holds in the univariate context. In the Appendix in Section C.2 the corresponding bootstrap algorithm is described. Additionally, we report the results of the standard residual-based Engle-Granger test of cointegration using asymptotic critical values, instead.[20]

In case the null hypothesis of no long-run relationship can be rejected, the long-run coefficient is given by the non-linear estimate of $\widehat{\beta} = -\frac{\widehat{\theta}}{\widehat{\rho}}$ where 'hat' refers to the OLS estimate. Inference on $\widehat{\beta}$ can be conducted by means of the Delta method, as described in Pesaran and Shin (1998), or, as conducted in this study, by means of bootstrap methods (Efron and Tibshirani, 1993, ch. 5).

Recall that the conditional specification of the ARDL model provides super-consistent estimates of the long-run parameters even in the presence of endogeneity issues. However, this is not the case for the short-run parameters which are contaminated by the contemporaneous correlations (Pesaran and Shin, 1998).

Dynamic Multipliers The cumulative dynamic multiplier effects of x_t on y_t can be evaluated as follows:

$$m_h = \sum_{j=0}^{h} \frac{\partial y_{t+j}}{\partial x_t} \text{ , } h = 0, 1, 2, ... \tag{81}$$

Notice that, by construction, and $h \to \infty$, $m_h \to \beta$, where β is the long-run coefficient.

Additionally to the $I(1)$ variable we add both contemporaneous and lagged values of the $I(0)$ regressors $Cov_{(\pi_t, r_t)}$, $econunc_t$ and $polunc_t$ up to order $q-1$. The co-variance measure enters the model due to theoretical reasons whereas the uncertainty measures account for other sources of risk different from inflation risk and capital market risk. In order to determine the optimal lag length of the ARDL model, we apply a type of general-to-specific modeling approach as well as automatic outlier detection, as described in detail in the Appendix in Section C.1. Given the small sample size, we provide the bootstrap estimation results of the error-correction adjustment term $\widehat{\rho}$, the long-run coefficients $\widehat{\beta(.)}$ jointly with bootstrap standard errors and the R^2. A battery of standard specification tests on serial correlation, heteroskedasticity, functional form and parameter stability are performed on the final specification estimated.

[20] As a cross-check one could apply the test suggested by Banerjee et al. (1998) testing the null $\rho = 0$ of no cointegration against the alternative $\rho < 0$, for which Pesaran et al. (2001) also provide critical values. However, it is expected that the bootstrap PSS test clearly outperforms this test using asymptotic critical values.

3.7 Estimation Results

Table 1 provides the estimation and test results of all five models. The bootstrap PSS cointegration test indicates only for Models 3 and 5 (significant at the 10% level) an existing long-run relationship between the variables. These results are in contrast to the residual-based Engle-Granger test (using asymptotic 5% critical values) according to which for none of the specifications a cointegration relationship does exist.

None of the models suffers from remaining autocorrelation. Furthermore, we do not find any evidence of remaining issues with heteroskedasticity or residual non-normality problems, and Ramsey's RESET test does not indicate any issues with the functional form. The QLR test on parameter stability is performed to three categories of variables: A) join test on all regressors, B) joint test on $I(1)$ regressors, and C) jointly on all $I(0)$ regressors.[21] The null of joint stability of all parameters can be rejected at least at the 5% level for specifications 2, 4 and 5. Performing the test on the $I(1)$ level regressors results in clear rejection of parameter stability (at least at the 5% level) for all specifications. Interestingly, with respect to the $I(0)$ variables the null of parameter stability can only be rejected (at the 5% level) for Models 2, 4 and 5. Overall, the results indicate some significant parameter changes over time.

A visual inspection of the long-run equilibrium errors reveals a mixed picture (see Figure 3). For Models 1 to 3 one can observe a permanent downward level-shift in the long-run equilibrium errors at the beginning of the 1980s. This level shift disappears after the inclusion of inflation risk into Models 4 and 5 (see Figures 3(d) and 3(e)). Overall, the long-run errors of the preferred specification Model 5 show a low persistence. However, one can observe a temporary decline in the errors during the 1980s as well as a slight negative trend in the time series between the mid 1990s and 2000. Furthermore, the impact of the GFC is visible as a negative spike in 2009. Overall, the long-run equilibrium error of Model 5 looks much more stationary compared to the baseline specification.

Given that structural breaks may result in biased parameter estimates, we decided to re-estimate the specifications using a smaller sample ending in 2008q3; just before the GFC started. The estimation results are provided in Table 4 in the Appendix. We find evidence for some differences in the estimated parameters. For instance, the error-correction coefficient (ρ) is smaller for the full sample in comparison to the restricted sample: For Model 5 the corresponding ρ-coefficient is about -0.06 for the sample ending in 2013q4 but -0.12 for the sample ending in 2008q3. Furthermore, a comparison of the long-run income elasticity of money demand reveals some differences for specifications 1, 2 and 4. Fort specification 5 we find an income elasticity of about 1.2 for the full sample in comparison to 0.997 for the restricted sample which indicates only minor differences.

[21] The likelihood ratio test for a break, maximized over all possible break dates in the inner 70% of the full sample 1978q1–2013q4.

As the QLR-test searches recursively for potential breaks only in the inner 70% of the sample, parameter changes at the sample beginning and end (including the recent GFC period), are not detected. To circumvent this problem, we will compute the rolling-window multipliers for the period between 1998 and 2013.[22] The results support the view that parameter changes have occurred during this period for some of the variables as will be shown in more detail below.

Lastly, it should be mentioned that we also worked with a restricted sample starting in 1985q1 in order to avoid the inclusion of the very turbulent periods in the early 1980s. In total, the results reported do not change. However, restricting the sample to start in the mid 1980s does not seem to be a reasonable choice as we are actually interested in explicitly considering periods of high inflation risk and capital market risk.

[22] An alternative approach allowing for end-of-sample stability testing was recently proposed by Andrews (2003).

(A) ESTIMATION RESULTS

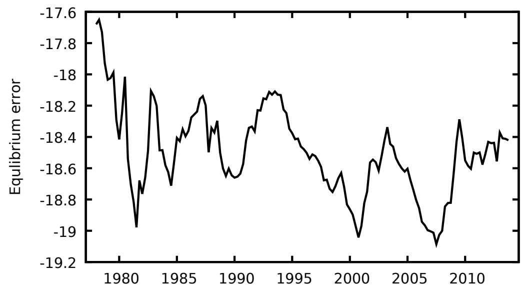
	Model 1	Model 2	Model 3	Model 4	Model 5
ρ	-0.015^{*}	-0.019^{**}	-0.070^{***}	-0.015^{*}	-0.060^{***}
	(0.008)	(0.009)	(0.018)	(0.008)	(0.019)
$\beta(y)$	2.445	1.491	0.865^{***}	3.260	1.208^{***}
	(4.246)	(2.166)	(0.105)	(6.983)	(0.223)
$\beta(r)$	-0.004	0.004	-0.002	-0.002	-0.003^{*}
	(0.023)	(0.021)	(0.002)	(0.036)	(0.002)
$\beta(i)$	0.187	0.121	0.041^{***}	0.079	0.025^{**}
	(0.432)	(0.240)	(0.012)	(0.292)	(0.012)
$\beta(\pi^e)$		-0.165	-0.041^{**}	-0.168	-0.030
		(0.432)	(0.016)	(0.665)	(0.018)
$\beta(\sigma_r^2)$			0.187^{***}		0.268^{***}
			(0.042)		(0.055)
$\beta(\sigma_\pi^2)$				2.888	0.578^{**}
				(11.041)	(0.257)
R^2	0.735	0.787	0.822	0.825	0.819
	(0.656/0.796)	(0.730/0.837)	(0.769/0.869)	(0.767/0.874)	(0.768/0.870)

(B) DIAGNOSTIC STATISTICS

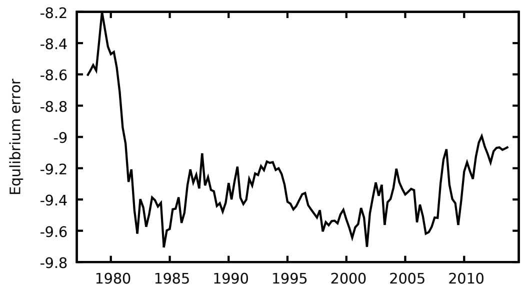
$F_{SC(1)}$	0.802	0.497	0.811	0.708	0.852
	(0.063)	(0.465)	(0.058)	(0.141)	(0.035)
$F_{SC(4)}$	0.832	0.157	0.139	0.096	0.536
	(0.366)	(1.694)	(1.780)	(2.031)	(0.788)
χ^2_H	0.098	0.333	0.484	0.537	0.284
	(65.527)	(64.160)	(68.818)	(66.263)	(60.503)
χ^2_N	0.197	0.175	0.235	0.142	0.372
	(3.250)	(3.490)	(2.896)	(3.902)	(1.980)
F_{FF}	0.648	0.322	0.271	0.503	0.453
	(0.436)	(1.144)	(1.324)	(0.692)	(0.798)
QLR	0.066	0.019	0.129	0.014	0.028
	(44.056)	(48.970)	(39.661)	(50.130)	(47.494)
$QLR_{I(1)}$	0.005	0.009	0.035	0.036	0.011
	(24.292)	(24.836)	(22.907)	(22.798)	(27.983)
$QLR_{I(0)}$	0.083	0.048	0.311	0.014	0.031
	(41.675)	(45.395)	(35.059)	(50.130)	(44.488)
F^b_{PSS}	0.441	0.470	0.070	0.619	0.059
EG	0.414	0.347	-2.000	0.026	-1.955
EG_{5pct}	-3.741	-4.096	-4.415	-4.415	-4.707

NOTE: ρ and β denote the bootstrapped mean value of the error-correction coefficient and the long-run coefficients, respectively. The bootstrap standard error are reported in rounded parentheses. $***$, $**$ and $*$ denote the 1pct., 5 pct. and 10 pct. rejection probabilities. For R^2 the bootstrapped 95pct. intervals are provided. All results are based on 999 stable bootstrap iterations. The optimal lag length of the ARDL(p,q) model as well as potential impulse dummmies are determined by an automatic algorithm as described in Section C.1 in the Appendix. $F_{SC(1)}$, $F_{SC(4)}$, χ^2_H, χ^2_N and F_{FF} denote the p-values for the tests of no serial correlation of order 1 or 4 (respectively), White's test of homoskedasticity, the Doornik-Hansen test of residual normality and Ramsey's RESET test of the correct functional form. The Quandt likelihood ratio test, QLR, tests for a structural break at an unknown point in time, with 15pct. trimming. QLR, $QLR_{I(1)}$ and $QLR_{I(0)}$ are tests on joint parameter stability of all regressors, only of the I(1) and I(0) regressors, respectively. For these tests the p-values are provided and the test statistics are reported in rounded parentheses below. F^b_{PSS} refers to the bootstrap version of the Pesaran et al. (2001) F-test on cointegration (bootstrapped p-values are reported) while EG denotes the test statistics of the Engle-Granger residual based cointegration test. EG_{5pct} is the corresponding 5 pct. critical value. The restricted intercept with no trend case is considered.

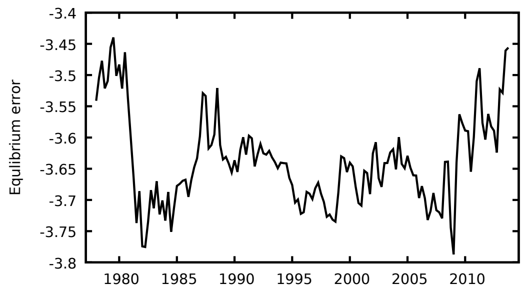
Table 1: Estimation results of the money demand relationship. Sample: 1978q1 to 2013q4.



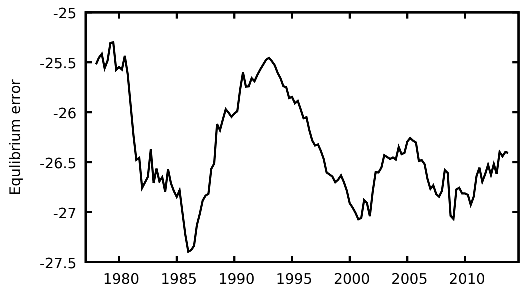
(a) Model 1



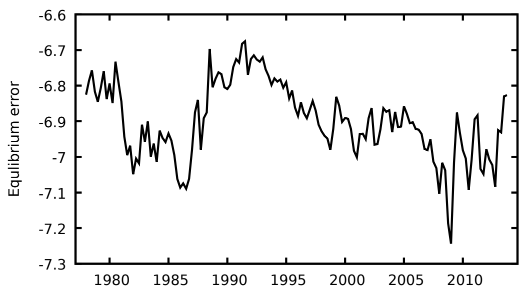
(b) Model 2



(c) Model 3



(d) Model 4



(e) Model 5

Note: The optimal lag length of the ARDL(p,q) model as well as potentially required impulse dummies are determined by an automatic algorithm, as described in Section C.1 in the Appendix. The long-run equilibrium error is computed using bootstrap mean values of the long-run coefficients.

Figure 3: Long-run equilibrium error of money demand. Sample: 1978q1 – 2013q4.

Long-run Effects The long-run estimation results are reported in Table 1. The bootstrap mean value of the long-run income elasticity of money demand is severely upward biased for Models 1, 2 and 4. We find a long-run elasticity of $\beta(y) = 2.445$ for Model 1, $\beta(y) = 1.491$ for Model 2, and $\beta(y) = 3.260$ for Model 4. Most importantly, the coefficients are not significantly different from zero using bootstrap standard errors. However, the consideration of capital market risk in Model 3 and additionally inflation risk in Model 5 results in a significant (at the 1% level) and close to unity long-run income elasticity of money demand (for Model 4 $\beta(y) = 0.865$ and Model 5 $\beta(y) = 1.208$). This suggests that the separate consideration of capital market risk or the joint account of both risk variables in the cointegrating space helps to restore a plausible and widely-acknowledged assumption that there is a (probably one-to-one) long-run relationship between money demand and income. An income elasticity of money demand above unity is often interpreted as proxying omitted wealth effects (Coenen and Vega, 2001), and hence not implausible.

Overall, Model 5 is the favorable specification on which we focus in the following, as the specification includes both risk factors.

The point estimate of the long-run impact of real stock market return is significant at the 10% level and negative for Model 5. A 10 percentage-point increase in stock market returns is associated with a 3% reduction in money demand in the long-run as households shift their portfolio away from low-interest bearing money holdings to stocks. This means that the unambiguous substitution effect dominates. Based on the restricted sample ending in 2008q3, we find for Model 5 no significant long-run effect, as reported in Table 4 in the Appendix.

The results reveal evidence for a significant (at the 5% level) and positive long-run effect of a change in the own rate. Long-run Money demand holdings increase by about 2.5% as a result of a one percentage point increase in the own rate. Almost the same long-run effect is obtained for the restricted sample ending in 2008q3 for which we find a semi-elasticity of 2.2%. Theoretically, the total effect is ambiguous. However, the empirical finding indicates the dominance of the direct positive substitution effect of deposits on money holdings.

For the full sample, we do not find a significant long-run effect of expected inflation money demand. This implies that the positive effect of higher expected inflation on current consumption (and hence money demand) just equals the direct negative impact on the demand for money. However, based on Model 3 the impact of expected inflation on money demand is negative and significant (at the 5% level). Hence, the joint consideration of stock market as well as inflation risk in the long-run relationship in Model 5, cancels out the long-run effect of expected inflation. The picture is slightly different for the restricted sample ending in 2008q3 where the point estimate is negative and significant at the 5% level. Here we find for Models 2 to 5 a significant (at least at the 5% level) negative effect of expected inflation. As will be shown below in the rolling-window dynamic multiplier exercise, the effect of expected inflation crucially depends on the sample period considered.

According to the estimation results, households shift their portfolio towards safer assets away from risky stocks in response to higher perceived or actual stock market risks. The long-run effect of a change in the stock market premium on money demand is positive and significant at the 1% level. A 0.1 percentage point increase in σ_r^2 is accompanied by a long-run increase in money demand of about 2.7%. Interestingly, this long-run effect disappears if the sample ends in 2008q3. Furthermore, the point estimate is much lower for the sample not covering the recent GFC period. This suggest that the long-run responsiveness of households to stock market risk has recently increased due to the crisis episode in the U.S. economy. Overall, the full sample findings confirm the results of Cook and Choi who find a positive long-run relationship between stock market risk and the demand for M2, and who argue that the "...relative risk effect dominates the relative return effect" (Cook and Choi, 2007, 15).

Even though our findings (based on the full sample) rather indicate that U.S. households do not react to changes in inflation expectations in the long-run, there is stark evidence that they respond to inflation risk. For the full sample, the long-run effect is significant at the 5% level, and a 0.1 unit increase in inflation risk results in a 5.8% increase in money demand in the long-run. This effect remains positive and significant for the restricted sample ending in 2008q3, even though the long-run effect is found being slightly smaller being 3.3%. The positive effect can be explained by a relatively strong positive response of current consumption to an increase in inflationary risk which outperforms the direct negative substitution effect. It should be noted that the result is in line with previous findings by Higgins and Majin (2009).[23]

Overall, we find strong support for the inclusion of both risk factors into the long-run relationship. First, their inclusion helps to restore a plausible economic long-run money demand relationship, indicating that inflation risk as well as capital market risk variables are crucial factors in explaining the economic behavior of U.S. households over the period considered. Secondly, the respective long-run coefficients of both risk factors are statistically significant using the sample covering the recent financial crisis episode. This finding is in stark contrast to the standard and frequent assumption that the steady-state is characterized by full certainty *per definition* which rules out that higher moments of shocks may have a permanent effect.

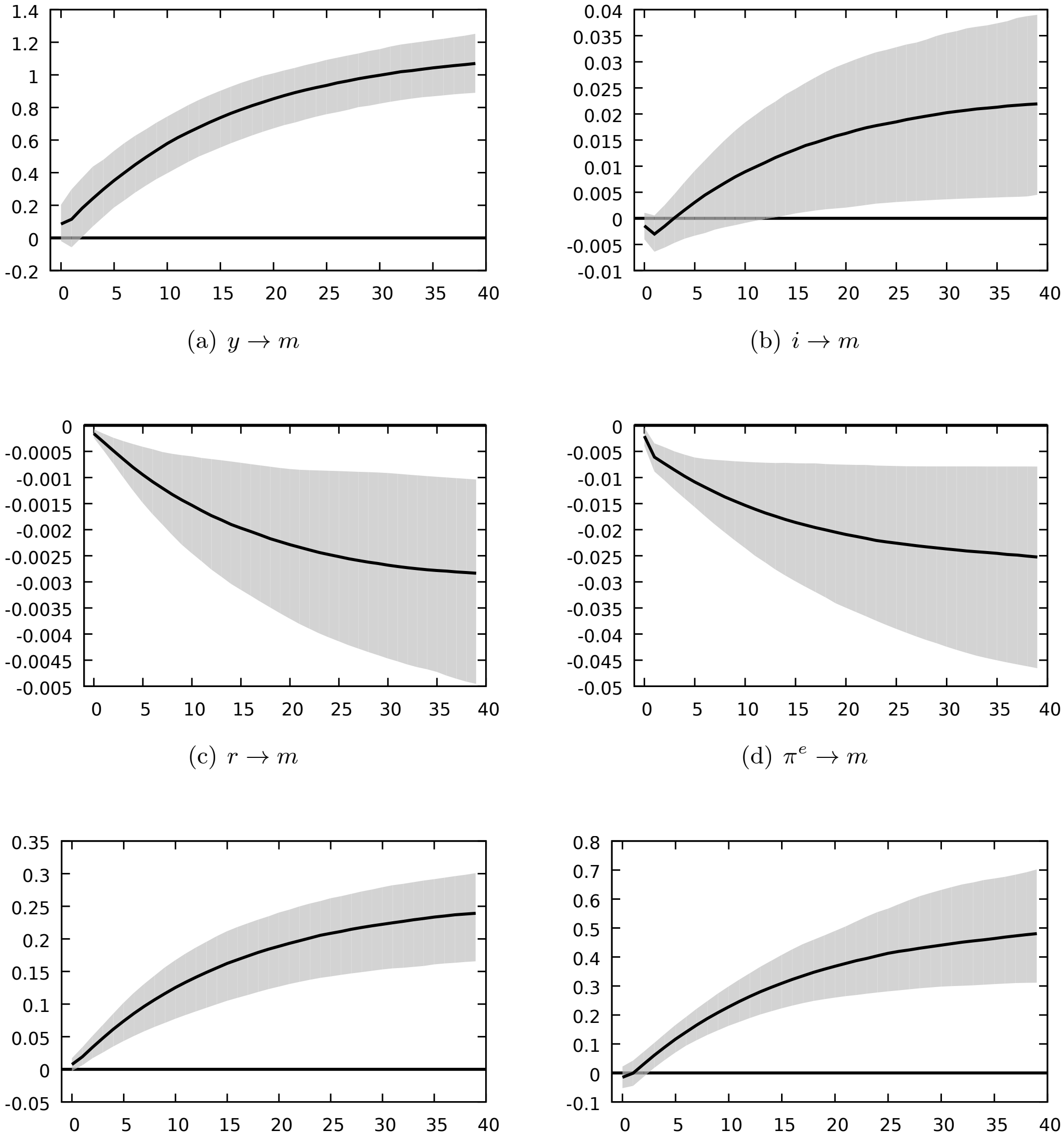
Dynamic Multipliers In Figure 4 the dynamic multipliers of money demand are depicted. Still, the estimation results are based on Model 5, even though almost identical results are obtained using Model 3. The dynamics reveal find that a positive unit change in income leads to a significant increase in the demand for money after a mild two quarter lag and lasts permanently. The effect is significant over the entire horizon (see Figure 4(a)), and remains valid even if the sample ends in 2008q3, as shown in Figure 7(a) in the Appendix.

U.S. households shift their portfolio immediately towards higher-interest-bearing assets away from money after a positive change in real stock market returns (see Figure 4(c)). This effect is significantly negative and lasts permanently. For the pre-GFC period we find a totally changed picture (see Figure 7(c)): The point estimate is positive and significant. However, as shown in Table 4 the long-run effect is not significantly different from zero.

A positive change in the own rate has a significant positive effect after about two to three years, as displayed in Figure 4(b). The effect lasts permanently. Again, the dynamics do not change qualitatively for the pre-GFC period with the only difference that the effect is found being significant already after five quarters, as depicted in Figure 7(b) in the Appendix.

[23]It should be mentioned that we also used different inflation series such as CPI and core inflation for estimating the inflation risk series using the UC-SV model. However, the results stay robust against alternative inflation rates applied.

Irrespective of the selected sample end, we do find evidence for an impact of expected inflation on money demand holdings in the short and medium term. The point estimate is negative over the entire horizon of forty quarters, as displayed in Figure 4(d). Similar holds for the restricted sample (see Figure 7(d)). However, it should be recalled that the long-run effect is only significant for the restricted sample ending in 2008q3 but not for the full sample.



(a) $y \rightarrow m$ (b) $i \rightarrow m$

(c) $r \rightarrow m$ (d) $\pi^e \rightarrow m$

(e) $\sigma_r^2 \rightarrow m$ (f) $\sigma_\pi^2 \rightarrow m$

NOTE: The optimal lag length of the ARDL(p,q) model as well as potentially required impulse dummies are determined by an automatic algorithm, as described in Section C.1 in the Appendix. The 90% Efron percentiles are based on a wild bootstrap method using 999 iterations.

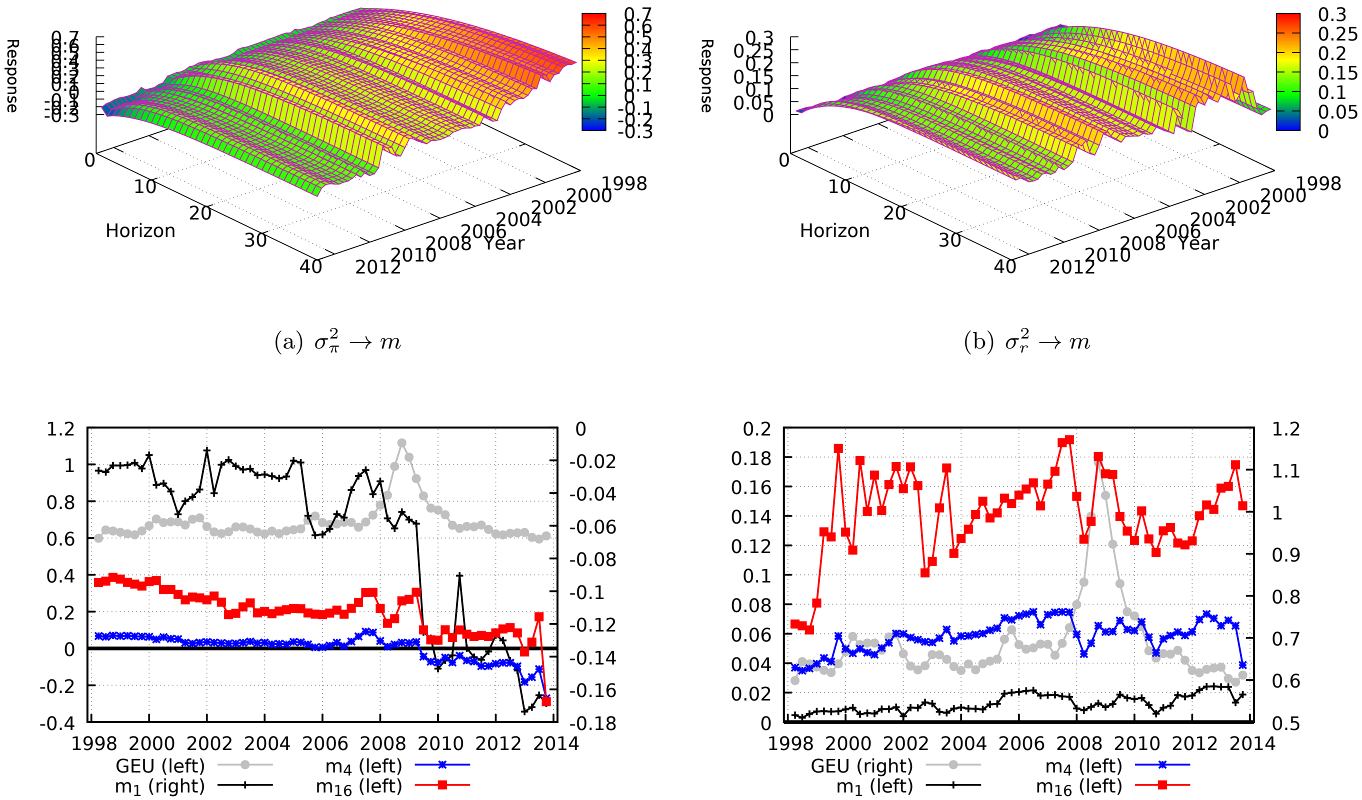
Figure 4: Dynamic multipliers of money demand with 90% non-parametrically bootstrapped confidence intervals (Efron percentiles) based on Model 5 after general-to-specific model reduction. Sample: 1978q1 – 2013q4.

U.S. households do not only respond to changes in stock market risk in the long-run but also in the short- and medium-term, as depicted in Figure 4(e). We find an immediate increase in money demand in response to a positive change in this type of risk. However, the dynamics change fundamentally using the restricted sample ending in 2008q3: the dynamic multiplier is only temporarily significant in the third quarter after a change in stock market risk (see Figure 7(e)). This strengthens the argument that the capital market risk has become a crucial determinant of money demand during the GFC episode which was accompanied by an increase in capital market risk.

The adjustment dynamics for a positive change in inflation risk are displayed in Figure 4(f). U.S. households respond to an increase in inflation risk by increasing their safe money holdings after a mild lag of two to four quarters. The effect stays positive over the entire horizon which can be explained by an increase in households' current consumption expenditures. Hence, the positive effect is stronger compared to the countervailing negative substitution effect. The dynamics for the pre-crisis period are very similar, as depicted in Figure 7(f) in the Appendix.

Rolling-Window Dynamics In Figure 5 the rolling-window dynamic multipliers of money demand are depicted. The purpose of this exercise is to study eventual parameter-variations over time and to control for time-varying conditional heteroskedasticity. Given that our sample covers turbulent times such as the second oil price crisis, the late banking crisis in the 1980s, the New Economy Boom and Bust as well as the current recent financial crisis, structural shifts are likely to have occurred. The lag length of the ARDL(p,q) model is set to the full sample equivalent, as determined in the previous step.[24] Potential outliers are again automatically detected at each iteration. The window-size is fixed to eighty quarters to ensure sufficient degrees of freedom. Again the reported results are based on Model 5.

[24]We also allowed for the determination of the optimal lag length at each iteration, but the results remain unchanged.



(a) $\sigma_\pi^2 \to m$

(b) $\sigma_r^2 \to m$

(c) $\sigma_\pi^2 \to m$

(d) $\sigma_r^2 \to m$

Note: The impact multiplier (m_1), the effect after four (m_4) and sixteen periods (m_{16}) are reported, respectively. *GEU* refers to general economic uncertainty (*econunc*). The window size is 80 quarters. The optimal lag length of the ARDL(p,q) model as well as potentially required impulse dummies are determined by an automatic algorithm, as described in Section C.1 in the Appendix.

Figure 5: Rolling-window dynamic multipliers based on model 5. Sample: 1978q1 – 2013q4.

Figures 5(a) and 5(c) depict the dynamic multipliers of inflation risk over time.[25] The impact multiplier ($m_1^{\sigma_\pi^2}$) is found to be fairly stable between 1998 and 2008 with a mean level of about -0.04. The 4th-quarter multiplier also behaves stable between 1998 and 2013 but is close to zero. The medium-term 16th-quarter multiplier is about 0.3 until 2009 before it decreases to 0.1 in the following.[26] The downward shift in the impact as well as the 16th-quarter multipliers just coincide with a spike in the macroeconomic uncertainty measure in 2009. The associated increase (in absolute terms) in the impact multiplier from about -0.04 to -0.14 indicates that U.S. households' money demand holdings have become more sensitive to inflation risk during this period which is also accompanied by the reduction of the Federal Funds rate close to zero. Furthermore, in mid 2008 the FED initiated its program of unconventional monetary policy accompanied by quantitative easing and forward-guidance which led to some temporary increase in expected inflation (see again Figure 1(e)) and some further increase in inflation risk.

It is interesting to see that the qualitative properties of the stock market risk effect on money demand have remained stable throughout the time period considered. It can be observed that the magnitude of the impact multiplier is about $m_1^{\sigma_r^2} = 0.01$ between 1998 and 2013 (see 5(b) and 5(d)). Also the 4th-quarter multiplier effect stays constantly around $m_4^{\sigma_r^2} = 0.06$ and no tendencies of breaks are visible. However, the medium-term multiplier after sixteen quarters shows some strong cyclical dependency over time. Additionally, its point estimate has increased at the end of 1999 from about 0.06 to about 0.16 has started to fluctuate around this level. Lastly, one can observe that the GFC led to a temporary decline in the mean value of the $m_{16}^{\sigma_r^2}$ multiplier between 2009 and 2012 before it has bounced back to its pre-crisis level. Nevertheless, the observed shifts in the point estimates are rather modest, indicating parameter constancy.

In the Appendix we also depict the rolling-window dynamic multipliers for the remaining variables. In Figure 8(a) and 8(c) the time-varying multiplier effects of an income change are depicted. The impact multiplier has increased between 1998 and 2004 from about zero to 0.2. Since then, the effect is found being stable. A very similar development can be observed for the fourth-quarter multiplier effect. No substantial disruptions are visible for the medium-term multiplier after sixteen quarters which is about 0.85 over the entire period considered. It is interesting to see that the GFC, the monetary policy programs initiated and the increase in macroeconomic uncertainty did not have any impact on the income elasticity of money demand.

Based on the full sample estimations, we found a positive but delayed dynamic multiplier effect of an increase in the own rate on money demand. The rolling-window exercise indicates changes in the dynamic relationship between the own rate of M2 and money demand: The multiplier effects are fairly stable between 1998 and 2002 prior to a lasting reduction in the point estimate of the absolute value of the impact multiplier from about zero to -0.01 and for the

[25] The date reported on the x-axis refers to the sample end of the specific window.

[26] However, since no formal tests are applied at this stage, no decisive conclusion can be made whether the parameter changes over time are statistically significant or not.

16th-quarter effects from about 0.03 to 0.01 until 2008 (see Figures 8(b) and 8(d)). Since 2009 the impact multiplier has declined (in absolute terms) to around zero. A similar tendency can be observed for the medium-term effect after sixteen quarters. Thus, both periods the New Economy bust as well as the period after 2008 were accompanied by a reduction in the responsiveness of money demand to changes in the own rate in the U.S. economy. This may not be that surprising given that the nominal own rate of M2 declined to almost zero as a result of the conducted zero-lower bound policy strategy.

The responsiveness of households to changes in real stock market returns, r, has experienced some changes between 1998 and 2013, as depicted in Figures 9(a) and 9(c)). While the impact multiplier stays stable just below zero during this episode, one can observe some declining tendency in the 4th-quarter multiplier since 2008 from about -0.0005 to -0.001. For the medium-term multiplier after sixteen quarters one can see a first decline between 2000 and 2002 before the effect stabilizes at a rather low level (in absolute terms) between 2003 and 2008. However, since the end of 2008–again just coinciding with the spike in macroeconomic uncertainty–the multiplier effect has more than doubled (in absolute terms) from -0.0015 to about -0.0035 at the end of 2013. It remains hard to say what exactly has triggered those changed responsiveness of money demand to real stock market returns. A potential cause may have been the increase in the relative yields of stocks over deposits as a result of the low-interest environment accompanied by a strong stock market development.

The time-varying effects of expected inflation on money demand are depicted in Figures 9(b) and 9(d). As shown before, the short- and long-run multipliers are negative and statistically different from zero using the full sample. The rolling-window exercise indicates severe instability in the multiplier effects between the entire period considered. During both episodes between 2000 and 2002 as well as 2008 and 2009 the 4th- and 16th-quarter dynamic multipliers turned negative. The dynamics of the multiplier effects indicate the the responsiveness of money demand on expected inflation is counter-cyclical: During upswings the correlation is positive but turn negative during recession periods.

4 Concluding Remarks

We investigated the demand for narrow as well as broad money both within a theoretical as well as empirical framework. In doing so our primary focus was directed to the impact of inflationary and stock market risks. In our theoretical analysis we distinguished between a deterministic stationary state implying the absence of uncertainty and hence risks and deviations from this long-run equilibrium marked by information deficiencies with respect to inflation and the real rate of return on capital. Two differences compared to standard DSGE models stand out: First, risk parameters enter the household's objective function directly which is a due consequence of using the certainty equivalent instead of expected utility. This procedure enabled us to give risk parameters an explicit representation in the Euler equations even after linearization around the steady state. Second, demand for money in our model is the result of a complete solution to the household optimization problem taking the intertemporal budget constraint into account. This implies that the impact of rates of return as well as risk parameters on money demand do not only depend on substitution effects but also on income effects. Most notably both effects proved to be countervailing leading to ambiguous results concerning the role of higher inflationary as well as stock market risks. In particular we were not able to rule out a higher demand for cash and deposits due to higher inflationary risks, which has to be expected whenever money demand reacts strongly to changes in consumption.

We used a single-equation error-correction model to test the underlying theoretical model of money demand under uncertainty. Some of the estimation results are in contrast to theoretical assumptions made in our model. For instance, both inflation risk and stock market risk significantly enter the long-run money demand relationship (using quarterly data between 1978q1 and 2013q4) implying that the empirical steady-state is not characterized by a fixed-point with full certainty as higher moments of shocks play a role. This questions the frequent theoretical assumption that the (deterministic) steady-state incorporates no information about the stochastic nature of the economic environment. There is a growing literature introducing the concept of a *risky steady state* which is associated with our findings (see e.g. Coeurdacier et al. (2011); de Groot (2013)). Future theoretical research should consider this perspective if it wants to build more realistic models which are closer in line with empirical evidence.

The dynamics show that U.S. households increase their demand for safe assets when confronted by an increase in either inflation risk or stock market risk. The recursive empirical analysis reveals evidence for non-constancy of structural parameters which also questions a frequent assumption of fixed preferences. The rolling-window dynamic multiplier analysis allows us to compare similarities and differences in both the short- and long-run relations. Particularly, we find a general decline in the impact multiplier effect of inflation risk on money demand since the late 1998s accompanied by an acceleration in this trend since 2008. Similar holds, correspondingly, for the long-run effect. The sensitivity of money demand to stock market risk has been rather stable since 2000 even though it the long-run multiplier is associated with some cyclical variation over time. Furthermore, the dynamic effects of both the own rate of M2 and expected inflation are found being time-varying. The changes in these effects coincide with the start of unconventional monetary policy in the U.S. in 2008. Lastly, we find that the negative long-run responsiveness of money holdings to real stock market returns has become stronger in absolute terms since mid 2009. Interestingly, most of the parameter shifts coincide with the height of economic policy uncertainty as well as macroeconomic uncertainty in the U.S., as approximated by the now widely-used measures of Baker et al. (2013) and Jurado et al. (2015), respectively.

In line with the evidence found by Bloom (2009, 2013) that uncertainty matters for business cycle fluctuations, our results indicate another channel through which uncertainty may affect macroeconomic developments.

With respect to the analysis of the long-run relationship, this econometric approach provides a valid framework as the long-run coefficients are still super-consistent even in the presence of endogeneity issues. However, in a general-equilibrium context feedback-relationships between the variables may arise. Thus, for future work a system modeling framework should be applied.

As the cost of investing in stocks and bonds has declined and households hold broader sets of monetary assets, it can be argued that money holdings may have become more sensitive to financial as well as inflation risk (Cook and Choi, 2007). Assuming that the money-growth-to-inflation nexus remains relevant, an inflation-targeting central bank needs to monitor financial and inflation risk to future inflation. Future research also needs to take into account the interaction between financial and inflation risk developments as well as money demand. Our results provide another argument for the inclusion of financial stability measures into a central bank's objective function, as the stabilization of financial markets can be seen an additional pillar for ensuring price stability (Cronin et al., 2011).

Data Appendix

All, except two series, were collected from the Federal Reserve Economic Data Service. The variables are defined as follows:

Real money demand, m_t, is the difference between M2 money stock (FRB: M2, SA) and the sum of demand for money by the firm sector which consists of the sum of time and saving deposits held by nonfinancial corporate business (FRB: NCBTSDQ027S, SA) and nonfinancial noncorporate business (FRB: NNBTTDQ027S, SA) as well as money market mutual fund shares of both the nonfinancial corporate business (FRB: NCBMASQ027S, SA) and nonfinancial noncorporate business (FRB: NNBMFTQ027S, SA). The resulting nominal series is deflated by the GDP price deflator (FRB: GDPDEF, SA) and logged.

Real disposable income, y_t, is the log of real disposable income (FRB: DPIC96, SA).

The own rate, i_t, refers to the own rate of M2 (FRB: M2OWN, NSA) converted from monthly to quarterly frequency.

The real stock market rate of return, r_t, is the 3-period moving average of the real rate of return of the S&P 500 Stock Price Index plus dividends on S&P 500 (both data are available at: `http://www.econ.yale.edu/~shiller/data/ie_data.xls`). Inflation rate is based on the GDP price deflator (FRB: GDPDEF, SA). The series is expressed at an annual rate and converted from monthly to quarterly frequency.

Expected price level inflation, π_t^e, is the University of Michigan Inflation Expectation (FRED: MICH, NSA).

For the construction of the covariance series, $Cov(\pi_t, r)$, we use the log-difference of the GDP price deflator (FRB: GDPDEF, SA) to approximate inflation and the real stock market rate of return, r_t.

The Economic Policy Uncertainty measure, $polunc_t$, is constructed by Baker et al. (2013), and can be downloaded from `http://www.policyuncertainty.com/media/US_Policy_Uncertainty_Data.xlsx`. The series is converted from monthly to quarterly frequency.

The macroeconomic uncertainty measure, $econunc_t$, is constructed by Jurado et al. (2015), and available from `http://www.econ.nyu.edu/user/ludvigsons/MacroUncertainty_update.zip`. The series is converted from monthly to quarterly frequency.

References

Andrade, P., Ghysels, E., and Idier, J. (2012). Tails of Inflation Forecasts and Tales of Monetary Policy. Working papers 407, Banque de France.

Andrews, D. (2003). End-of-sample instability tests. *Econometrica*, 71(6):1661–1694.

Atta-Mensah, J. (2004). Money demand and economic uncertainty. Working Papers 04-25, Bank of Canada.

Baker, S. R., Bloom, N., and Davis, S. J. (2013). Measuring economic policy uncertainty. Technical report.

Ball, L. (2012). Short-run money demand. *Journal of Monetary Economics*, 59(7):622–633.

Banerjee, A., Dolado, J., and Mestre, R. (1998). Error-correction mechanism tests for cointegration in a single-equation framework. *Journal of Time Series Analysis*, 19:267–283.

Barnett, W. A., Fisher, D., and Serletis, A. (1992). Consumer theory and the demand for money. *Journal of Economic Literature*, 30:2086–2119.

Belke, A. and Czudaj, R. (2010). Is euro area money demand (still) stable?: Cointegrated var versus single equation techniques. Discussion Papers of DIW Berlin 982, DIW Berlin, German Institute for Economic Research.

Bloom, N. (2009). The impact of uncertainty shocks. *Econometrica*, 3:623–685.

Bloom, N. (2013). Fluctuations in uncertainty. NBER Working Papers 19714, National Bureau of Economic Research, Inc.

Bloom, N., Floetotto, M., Jaimovich, N., Saporta-Eksten, I., and Terry, S. (2013). Really uncertain business cycles. LSE Research Online Documents on Economics 51526, London School of Economics and Political Science, LSE Library.

Bossone, B. (2014). Secular stagnation. Economics Discussion Papers 2014-47, Kiel Institute for the World Economy.

Bruggeman, A., Donati, P., and Warne, A. (2003). Is the demand for euro area m3 stable? Working Paper Series 255, European Central Bank.

Carpenter, S. B. and Lange, J. (2003). Money demand and equity markets. FEDS Working Paper 2003-03, Board of Governors of the Federal Reserve System.

Carstensen, K. (2006). Stock market downswing and the stability of european monetary union money demand. *Journal of Business and Economic Statistics*, 24(4):395–402.

Cavaliere, G., Rahbek, A., and Taylor, A. M. R. (2014). Bootstrap determination of the cointegration rank in heteroskedastic var models. *Econometric Reviews*, 33(5-6):606–650.

Choi, W. G. and Oh, S. (2003). A money demand function with output uncertainty, monetary uncertainty, and financial innovations. *Journal of Money, Credit, and Banking*, 35(5):685–709.

Chua, C. L., Kim, D., and Suardi, S. (2011). Are empirical measures of macroeconomic uncertainty alike? *Journal of Economic Surveys*, 4:801–827.

Coenen, G. and Vega, J.-L. (2001). The demand for m3 in the euro area. *Journal of Applied Econometrics*, 16(6):727–748.

Coeurdacier, N., Rey, H., and Winant, P. (2011). The Risky Steady State. *American Economic Review*, 101(3):398–401.

Cook, D. and Choi, W. G. (2007). Financial market risk and u.s. money demand. IMF Working Papers 07/89, International Monetary Fund.

Cottrell, A. and Lucchetti, R. (2013). *Gretl User's Guide.*

Cronin, D., Kelly, R., and Kennedy, B. (2011). Money growth, uncertainty and macroeconomic activity: a multivariate garch analysis. *Empirica*, 38(2):155–167.

de Bondt, G. J. (2009). Euro area money demand - empirical evidence on the role of equity and labour markets. Working Paper Series 1086, European Central Bank.

de Groot, O. (2013). Computing the risky steady state of DSGE models. *Economics Letters*, 120(3):566–569.

Dobnik, F. (2011). Long-run money demand in oecd countries – cross-member cointegration. Ruhr Economic Papers 0237, Rheinisch-Westfälisches Institut für Wirtschaftsforschung, Ruhr-Universität Bochum, Universität Dortmund, Universität Duisburg-Essen.

Dovern, J., Fritsche, U., and Slacalek, J. J. (2009). Disagreement among forecasters in g7 countries. Working Paper Series 1082, European Central Bank.

Dreger, C. and Wolters, J. (2009). Money velocity and asset prices in the euro area. *Empirica*, 36(1):51–63.

Dreger, C. and Wolters, J. (2010). Investigating m3 money demand in the euro area. *Journal of International Money and Finance*, 29(1):111–122.

Efron, B. and Tibshirani, R. J. (1993). *An Introduction to the Bootstrap.* Chapman and Hall.

Eggertsson, G. B. and Mehrotra, N. R. (2014). A Model of Secular Stagnation. NBER Working Papers 20574, National Bureau of Economic Research, Inc.

Elliott, G., Rothenberg, T., and Stock, J. (1996). Efficient tests for an autoregressive unit root. *Econometrica*, 64(4):813–836.

Engle, R. F. and Granger, C. W. J. (1987). Co-integration and error-correction: Representation, estimation, and testing. *Econometrica*, 55(2):251–276.

Fama, E. F. and French, K. R. (1988). Dividend yields and expected stock returns. *Journal of Financial Economics*, 22(1):3–25.

Feenstra, R. C. (1986). Functional equivalence between liquidity costs and the utility of money. *Journal of Monetary Economics*, 17(2):271–291.

Garratt, A., Lee, K., Pesaran, M. H., and Shin, Y. (2012). *Global and National Macroeconometric Modelling. A Long-Run Structural Approach.* Oxford Univ. Press, reprinted (2006) edition.

Greiber, C. and Lemke, W. (2005). Money demand and macroeconomic uncertainty. Discussion Paper: Series 1 26/2005, Deutsche Bundesbank.

Grimme, C., Henzel, S., and Wieland, E. (2011). Inflation uncertainty revisited: Do different measures disagree? Ifo Working Paper Series Ifo Working Paper No. 111, Ifo Institute for Economic Research at the University of Munich.

Groessl, I. and Fritsche, U. (2007). Default option, risk-aversion and household borrowing behaviour. Macroeconomics and Finance Series 200705, Hamburg University, Department Wirtschaft und Politik.

Hassler, U. and Wolters, J. (2006). Autoregressive distributed lag models and cointegration. *AStA Advances in Statistical Analysis*, 90(1):59–74.

Higgins, M. and Majin, S. (2009). Inflation uncertainty and money demand. *Applied Economics Letters*, 16(13):1323–1328.

Hoover, K. D., Johansen, S., and Juselius, K. (2008). Allowing the Data to Speak Freely: The Macroeconometrics of the Cointegrated Vector Autoregression. *American Economic Review*, 98(2):251–55.

IMF (2012). World economic outlook. coping with high debt and sluggish growth. Technical report, IMF.

Ingersoll, J. E. (1987). *Theory of Financial Decision Making*. Rowman and Littlefield, Totowa, NJ.

Jurado, K., Ludvigson, S. C., and Ng, S. (2015). Measuring Uncertainty. *American Economic Review*, 105(3):1177–1216.

Juselius, K. and Franchi, M. (2007). Taking a dsge model to the data meaningfully. *Economics: The Open-Access, Open-Assessment E-Journal*, 1(2007-4).

Kapetanios, G., Pagan, A., and Scott, A. (2007). Making a match: Combining theory and evidence in policy-oriented macroeconomic modeling. *Journal of Econometrics*, 136(2):565–594.

Kremers, J. J. M., Ericsson, N. R., and Dolado, J. J. (1992). The Power of Cointegration Tests. *Oxford Bulletin of Economics and Statistics*, 54(3):325–48.

Kumar, S., Chowdhury, M. B., and Rao, B. B. (2013). Demand for money in the selected oecd countries: a time series panel data approach and structural breaks. *Applied Economics*, 45(14):1767–1776.

Kwiatkowski, D., Phillips, P. C. B., Schmidt, P., and Shin, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of Econometrics*, 54(1–3):159–178.

MacKinnon, J. G. (1996). Numerical distribution functions for unit root and cointegration tests. *Journal of Applied Econometrics*, 11(6):601–618.

McCallum, B. T. and Goodfriend, M. S. (1987). *The New Palgrave: A Dictionary of Economics*, chapter Demand for Money: Theoretical Studies. Macmillan, London.

Nautz, D. and Rondorf, U. (2011). The (in)stability of money demand in the euro area: lessons from a cross-country analysis. *Empirica*, 38(4):539–553.

Patinkin, D. (1965). *Money, Interest, and Prices*. Harper, New York, 2nd edition.

Perron, P. (1989). Testing for a random walk: A simulation experiment when the sampling interval is varied. In Raj, B., editor, *Advances in econometrics and modelling*. Kluwer, Dordrecht.

Pesaran, M. H. and Shin, Y. (1998). An autoregressive distributed lag modelling approach to cointegration analysis. In Strom, S., editor, *Econometrics and Economic Theory: The Ragnar Frisch Centennial Symposium. Econometric Society Monographs.*, pages 371–413. Cambrdige: Cambridge University Press.

Pesaran, M. H., Shin, Y., and Smith, R. J. (2001). Bounds testing approaches to the analysis of level relationships. *Journal of Applied Econometrics*, 16(3):289–326.

Pesaran, M. H. and Smith, R. P. (2011). Beyond the dsge straitjacket. *Manchester School*, 79(s2):5–16.

Reed, W. R. (2014). Unit Root Tests, Size Distortions, and Cointegrated Data. Working Papers in Economics 14/28, University of Canterbury, Department of Economics and Finance.

Saving, T. R. (1971). Transactions costs and the demand for money. *American Economic Review*, 57:534–544.

Seitz, F. and von Landesberger, J. (2010). Household money holdings in the euro area: An explorative investigation. Working Paper Series 1238, European Central Bank.

Sephton, P. S. (1995). Response surface estimates of the kpss stationarity test. *Economics Letters*, 47(3-4):255–261.

Setzer, R. and Wolff, G. B. (2009). Money demand in the euro area: new insights from disaggregated data. European Economy - Economic Papers 373, Directorate General Economic and Monetary Affairs, European Commission.

Shin, Y., Yu, B., and Greenwood-Nimmo, M. (2014). Modelling asymmetric cointegration and dynamic multipliers in a nonlinear ardl framework. In Horrace, W. and Sickles, R., editors, *Festschrift in Honor of Peter Schmidt: Econometric Methods and Applications*. New York: Springer Science and Business Media.

Stock, J. and Watson, M. (2005). Has inflation become harder to forecast? mimeo, Harvard University.

Stock, J. H. and Watson, M. W. (2007). Why has u.s. inflation become harder to forecast? *Journal of Money, Credit, and Banking*, 39:3–33.

Tobin, J. (1956). Liquidity Preference as Behavior Towards Risk. Cowles Foundation Discussion Papers 14, Cowles Foundation for Research in Economics, Yale University.

Walsh, C. E. (2003). *Monetary Theory and Policy.* MIT Press, Cambridge/London, UK, 2nd edition.

Wright, J. H. (2011). Term premia and inflation uncertainty: Empirical evidence from an international panel dataset. *American Economic Review*, 101(4):1514–34.

Appendix

A Tables

Variable	Lag=1	Lag=2	Lag=3	Lag=4
(i) For the levels (test statistics)				
m	-0.590	-0.723	-1.072	-1.373
y	-0.671	-0.893	-1.164	-1.190
i	-2.156	-1.786	-2.220	-2.216
r	-7.246	-9.172	-5.168	-5.120
π^e	-2.132	-1.927	-2.275	-3.015
σ_r^2	-2.045	-1.614	-1.669	-1.532
σ_π^2	-1.539	-1.781	-2.415	-2.573
$Cov(\pi, r)$	-2.749	-2.790	-3.062	-3.047
$polunc$	-5.379	-3.730	-3.039	-2.505
$econunc$	-3.393	-2.718	-3.050	-3.149
(ii) For the first differences (*p*-values)				
Δm	0.000	0.000	0.003	0.013
Δy	0.000	0.000	0.000	0.000
Δi	0.000	0.000	0.000	0.000
Δr	0.000	0.000	0.002	0.045
$\Delta \pi^e$	0.000	0.008	0.081	0.075
$\Delta \sigma_r^2$	0.000	0.000	0.000	0.000
$\Delta \sigma_\pi^2$	0.000	0.005	0.014	0.010
$\Delta Cov(\pi, r)$	0.000	0.000	0.000	0.000
$\Delta polunc$	0.000	0.000	0.000	0.007
$\Delta econunc$	0.000	0.000	0.000	0.000

NOTE: When applied to the first differences, augmented Dickey-Fuller using the GLS procedure suggested by Elliott et al. (1996) (ADF-GLS) test statistics with an intercept and p lagged first differences of dependent variable, while when applied to levels, ADF-GLS statistics are computed using regression with an intercept, a linear time trend and p lagged first differences of dependent variable. The relevant 1%, 5% and 10% critical values for the ADF-test on the levels are -3.46, -2.93 and -2.64, respectively and are taken from Elliott et al. (1996, Table 1). For the first differences the p-values are provided. The calculation is based on MacKinnon (1996).

Table 2: ADF-GLS unit root test results. Sample: 1978q1 – 2013q4.

Variable	Lag=1	Lag=2	Lag=3	Lag=4
(i) For the levels				
m	0.871	0.589	0.449	0.365
y	0.757	0.517	0.397	0.325
i	0.245	0.172	0.136	0.114
r	0.227	0.203	0.214	0.225
π^e	0.878	0.602	0.462	0.379
σ_r^2	1.061	0.731	0.566	0.467
σ_π^2	0.978	0.658	0.499	0.404
$Cov(\pi, r)$	0.282	0.197	0.154	0.129
$polunc$	0.534	0.427	0.359	0.309
$econunc$	0.724	0.500	0.389	0.324
(ii) For the first differences				
Δm	0.821	0.667	0.571	0.501
Δy	0.342	0.330	0.312	0.302
Δi	0.135	0.127	0.119	0.112
Δr	0.008	0.008	0.014	0.018
$\Delta \pi^e$	0.070	0.077	0.075	0.069
$\Delta \sigma_r^2$	0.145	0.145	0.148	0.153
$\Delta \sigma_\pi^2$	0.183	0.149	0.125	0.108
$\Delta Cov(\pi, r)$	0.040	0.040	0.039	0.039
$\Delta polunc$	0.017	0.027	0.037	0.046
$\Delta econunc$	0.047	0.042	0.040	0.039

NOTE: KPSS represents the test suggested by Kwiatkowski et al. (1992) (KPSS). In first difference equations, KPSS test statistics are obtained including only an intercept and p lagged first differences of dependent variable, while when applied to levels, KPSS statistics are computed using regression with an intercept, a linear time trend and p lagged first differences of dependent variable. The relevant 1%, 5% and 10% critical values for the KPSS test on the levels are 0.216, 0.148 and 0.120, respectively. The relevant 1%, 5% and 10% critical values for the KPSS test on the first differences are 0.735, 0.465 and 0.349, respectively. All critical values are provided by Sephton (1995).

Table 3: KPSS unit root test results. Sample: 1978q1 – 2013q4.

(A) Estimation Results

	Model 1	Model 2	Model 3	Model 4	Model 5
ρ	-0.072^{***}	-0.081^{***}	-0.075^{***}	-0.111^{***}	-0.120^{***}
	(0.017)	(0.018)	(0.023)	(0.024)	(0.028)
$\beta(y)$	0.881^{***}	0.766^{***}	0.693^{***}	1.006^{***}	0.997^{***}
	(0.097)	(0.097)	(0.149)	(0.114)	(0.110)
$\beta(r)$	-0.000	0.003	0.002	0.004^{***}	0.002
	(0.001)	(0.002)	(0.003)	(0.001)	(0.002)
$\beta(i)$	0.035^{***}	0.027^{**}	0.021	0.019^{**}	0.022^{***}
	(0.011)	(0.010)	(0.023)	(0.008)	(0.008)
$\beta(\pi^e)$		-0.047^{***}	-0.051^{*}	-0.027^{**}	-0.025^{**}
		(0.015)	(0.028)	(0.013)	(0.012)
$\beta(\sigma_r^2)$			-0.065		0.033
			(0.157)		(0.060)
$\beta(\sigma_\pi^2)$				0.307^{***}	0.328^{***}
				(0.091)	(0.086)
R^2	0.741	0.826	0.839	0.867	0.895
	(0.665/0.812)	(0.765/0.878)	(0.786/0.887)	(0.823/0.905)	(0.858/0.927)

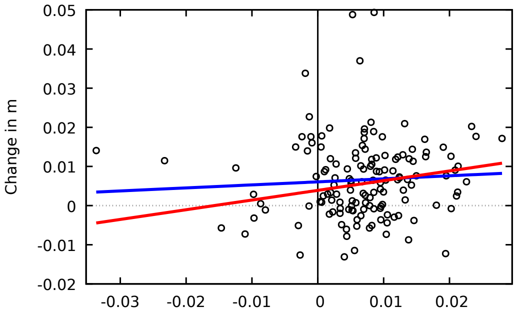
(B) Diagnostic Statistics

	Model 1	Model 2	Model 3	Model 4	Model 5
$F_{SC(1)}$	0.041	0.449	0.295	0.438	0.437
	(4.286)	(0.578)	(1.111)	(0.608)	(0.611)
$F_{SC(4)}$	0.087	0.656	0.567	0.127	0.013
	(2.099)	(0.611)	(0.741)	(1.860)	(3.442)
χ^2_H	0.235	0.602	0.736	0.435	0.303
	(45.019)	(55.583)	(59.341)	(85.465)	(100.555)
χ^2_N	0.529	0.308	0.363	0.132	0.589
	(1.274)	(2.355)	(2.028)	(4.045)	(1.059)
F_{FF}	0.865	0.737	0.913	0.024	0.046
	(0.146)	(0.306)	(0.091)	(3.909)	(3.234)
QLR	0.000	0.594	0.155	0.050	0.007
	(62.118)	(26.734)	(36.146)	(41.196)	(48.559)
$QLR_{I(1)}$	0.000	0.033	0.011	0.009	0.010
	(35.154)	(21.184)	(26.216)	(26.817)	(28.464)
$QLR_{I(0)}$	0.001	0.669	0.102	0.067	0.050
	(45.664)	(25.660)	(38.083)	(40.009)	(41.207)
F^b_{PSS}	0.455	0.577	0.082	0.965	0.098
EG	-1.544	-1.234	-2.029	-1.230	-2.032
EG_{5pct}	-3.741	-4.096	-4.415	-4.415	-4.707

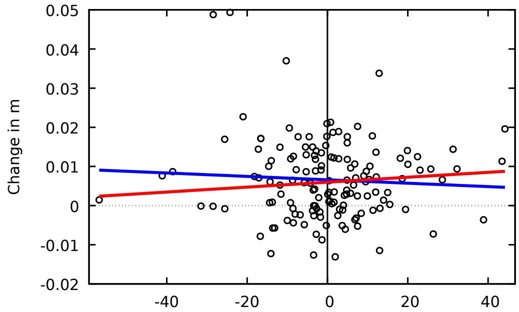
Note: ρ and β denote the bootstrapped mean value of the error-correction coefficient and the long-run coefficients, respectively. The bootstrap standard error are reported in rounded parentheses. $***$, $**$ and $*$ denote the 1pct., 5 pct. and 10 pct. rejection probabilities. For R^2 the bootstrapped 95pct. intervals are provided. All results are based on 999 stable bootstrap iterations. The optimal lag length of the ARDL(p,q) model as well as potential impulse dummmies are determined by an automatic algorithm as described in Section C.1 in the Appendix. $F_{SC(1)}$, $F_{SC(4)}$, χ^2_H, χ^2_N and F_{FF} denote the p-values for the tests of no serial correlation of order 1 or 4 (respectively), White's test of homoskedasticity, the Doornik-Hansen test of residual normality and Ramsey's RESET test of the correct functional form. The Quandt likelihood ratio test, QLR, tests for a structural break at an unknown point in time, with 15pct. trimming. QLR, $QLR_{I(1)}$ and $QLR_{I(0)}$ are tests on joint parameter stability of all regressors, only of the I(1) and I(0) regressors, respectively. For these tests the p-values are provided and the test statistics are reported in rounded parentheses below. F^b_{PSS} refers to the bootstrap version of the Pesaran et al. (2001) F-test on cointegration (bootstrapped p-values are reported) while EG denotes the test statistics of the Engle-Granger residual based cointegration test. EG_{5pct} is the corresponding 5 pct. critical value. The restricted intercept with no trend case is considered.

Table 4: Estimation results of the money demand relationship. Sample: 1978q1 to 2008q3.

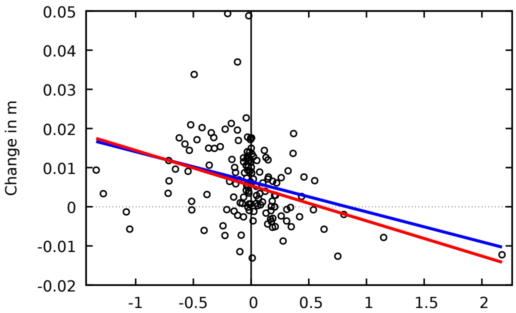
B Figures



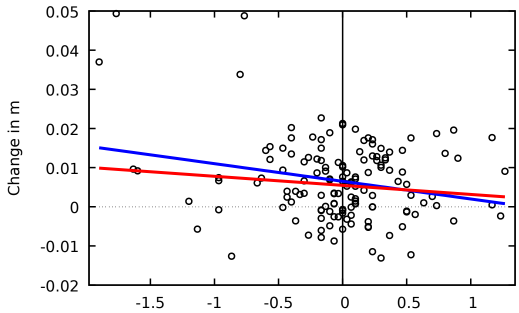
(a) Δy



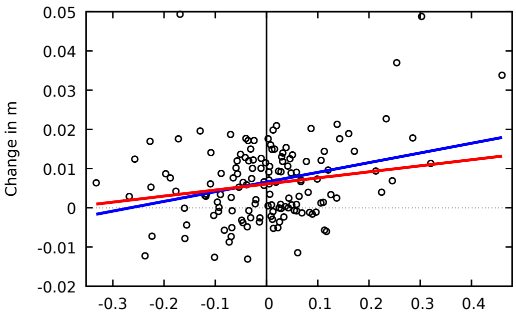
(b) Δr



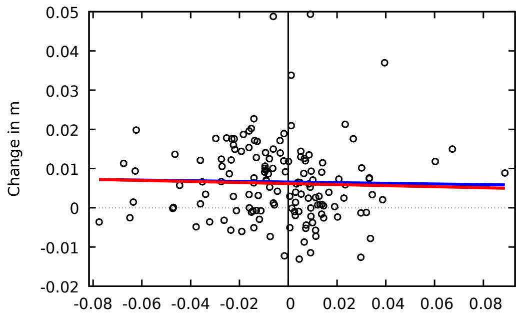
(c) Δi



(d) $\Delta \pi^e$

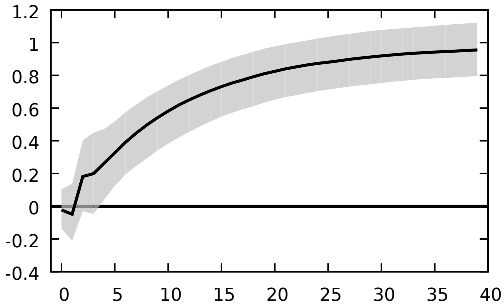


(e) $\Delta \sigma_r^2$

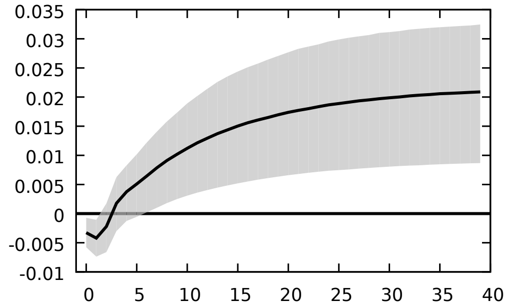


(f) $\Delta \sigma_\pi^2$

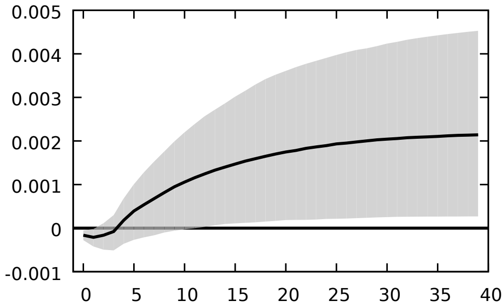
Figure 6: Scatter plot between the log-change in money demand (m) and the first difference of the respective variable. The blue (red) line depicts the OLS (LAD) fitted line. Sample: 1978q1 – 2013q4.



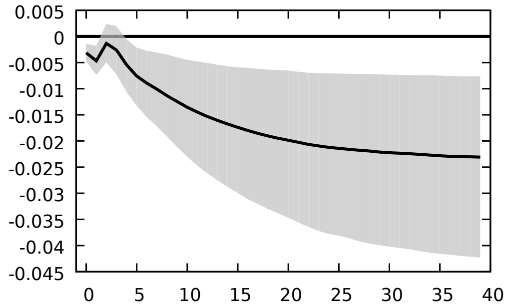
(a) $y \rightarrow m$



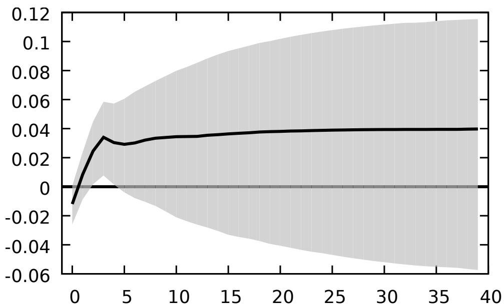
(b) $i \rightarrow m$



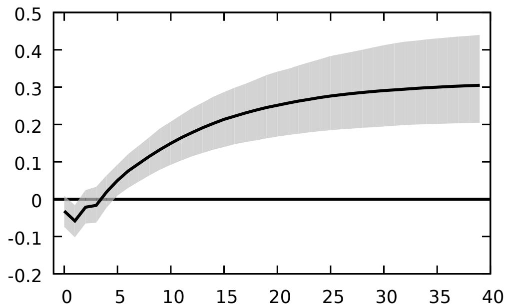
(c) $r \rightarrow m$



(d) $\pi^e \rightarrow m$



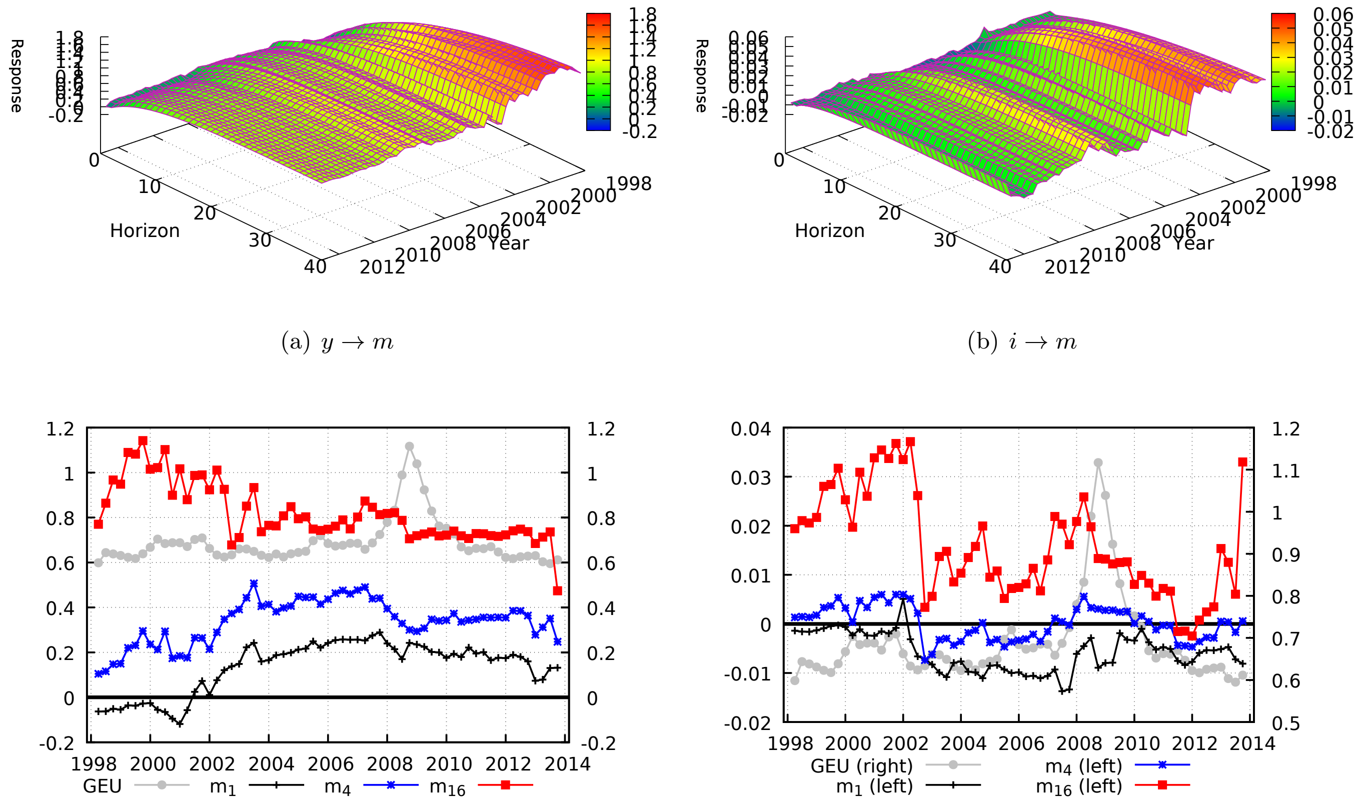
(e) $\sigma_r^2 \rightarrow m$



(f) $\sigma_\pi^2 \rightarrow m$

NOTE: The optimal lag length of the ARDL(p,q) model as well as potentially required impulse dummies are determined by an automatic algorithm, as described in Section C.1 in the Appendix. The 90% Efron percentiles are based on a wild bootstrap method using 999 iterations.

Figure 7: Dynamic multipliers of money demand with 90% non-parametrically bootstrapped confidence intervals (Efron percentiles) based on Model 5 after general-to-specific model reduction. Sample: 1978q1 – 2008q3.

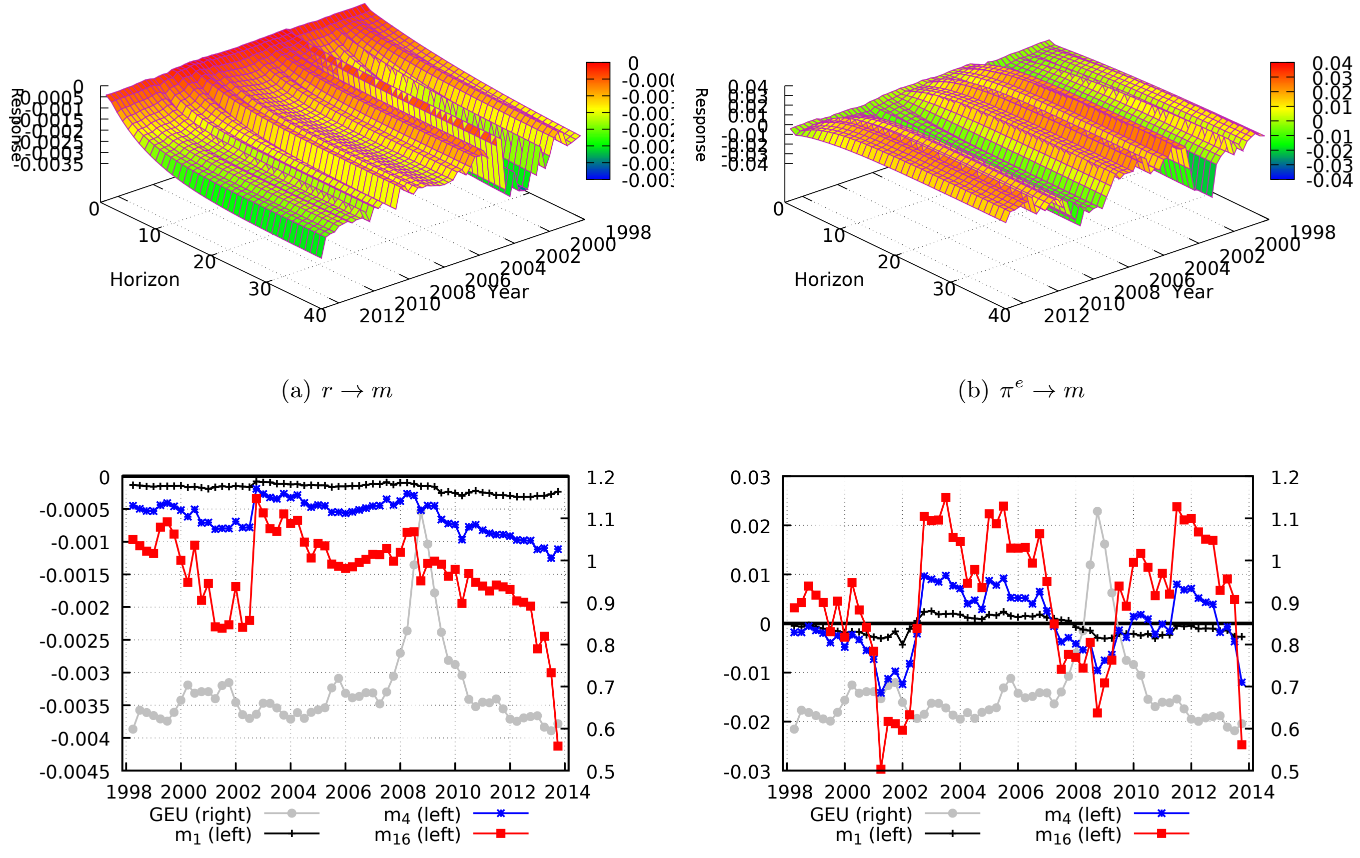


(a) $y \rightarrow m$ (b) $i \rightarrow m$

(c) $y \rightarrow m$ (d) $i \rightarrow m$

NOTE: The impact multiplier (m_1), the effect after four (m_4) and sixteen periods (m_{16}) are reported, respectively. *GEU* refers to general economic uncertainty (*econunc*). The window size is 80 quarters. The optimal lag length of the ARDL(p,q) model as well as potentially required impulse dummies are determined by an automatic algorithm, as described in Section C.1 in the Appendix.

Figure 8: Rolling-window dynamic multipliers based on model 5. Sample: 1978q1 – 2013q4.



(a) $r \rightarrow m$

(b) $\pi^e \rightarrow m$

(c) $r \rightarrow m$

(d) $\pi^e \rightarrow m$

NOTE: The impact multiplier (m_1), the effect after four (m_4) and sixteen periods (m_{16}) are reported, respectively. *GEU* refers to general economic uncertainty (*econunc*). The window size is 80 quarters. The optimal lag length of the ARDL(p,q) model as well as potentially required impulse dummies are determined by an automatic algorithm, as described in Section C.1 in the Appendix.

Figure 9: Rolling-window dynamic multipliers based on model 5. Sample: 1978q1 – 2013q4.

C `gretl` Code Description

All `gretl` code[27] used in the paper is available upon request. The results presented in the paper are based on `gretl` version `1.10.0 cvs`. See the file `readme.txt` included in the zip archive for further details.

To recreate the results in the paper run the main file `MAIN.inp`. This program performs the following steps:

- Load the gretl-type data file `DATASET_1978q1_2013q4.gdt`.
- Call `gretl_urtest.inp` to conduct the ADF-GLS and KPSS unit root test, and compile the Latex-tables.
- Call `ARDL.inp`. This file comprises the whole setup for the following ARDL model estimations, cointegration test analysis, dynamic multiplier computation, and rolling-window dynamic multiplier computation. The required sub-procedures are included in the file named `PROCEDURES.inp` and automatically called. All Latex-tables and figures are compiled automatically.

C.1 Notes on the General-To-Specific Algorithm and Outlier Detection Procedure on the ARDL Model

The following algorithm is applied to determine the lag order of the ARDL(p,q) model as well as the need for impulse dummy variables:

1. Estimate the ARDL(p,q) and set the lag length to $p = q = k$ where k is an integer value and $k = 1..4$. The BIC information criteria is used to select the lag length which minimizes the BIC criteria. The maximum lag order tested is $k = 4$. The optimal lag order is denoted by ARDL(p^*,q^*).
2. Store the residuals $\widehat{u}$ of the estimated ARDL(p^*,q^*) model. Create impulse dummies taking unit for observations for which $\widehat{u}_t >= 2\sigma(\widehat{u})$, otherwise zero, where $\sigma(\widehat{u})$ refers to the estimated standard deviation.
3. Re-estimate the ARDL(p^*,q^*) model including all dummy variables determined in the step before. Sequentially eliminate the dummy variables with a p-value greater 0.1, until all remaining dummy variables have a p-value not greater than 0.1.

C.2 Notes on PSS Wild Bootstrap Test on Cointegration

The bootstrap estimator of the cointegration relationship, denoted $\widehat{PSS}_b$ in what follows, iterates over the following steps:

1. Estimate model 82 under null hypothesis $H_0 : \rho = \theta = 0$ using OLS yielding the estimates $\widehat{\gamma}^r_1, ..., \widehat{\gamma}^r_{p-1}$ and $\widehat{\phi}^r_1, ..., \widehat{\phi}^r_{p-1}$ together with the corresponding residuals $\widehat{u}_t$:

$$\Delta y_t = \rho y_{t-1} + \theta x_{t-1} + \sum_{j=1}^{p-1} \gamma_j \Delta y_{t-j} + \sum_{j=1}^{q-1} \phi_j \Delta x_{t-j} + u_t \qquad t = 1, ..., T \tag{82}$$

   where the initial values, $y_{1-p}, ..., y_0$ and $x_{1-q}, ..., x_0$, are taken to be fixed in the statistical analysis.

[27] See Cottrell and Lucchetti (2013) for the more information on the open-source econometric software package `gretl`.

2. Construct the bootstrap sample, $\{y_t^*\}$, recursively from the first step with the T bootstrap errors u_t^*, generated using the re-centered residuals, $\widehat{u}_t^c := \widehat{u}_t - T^{-1}\sum_{i=1}^{T}\widehat{u}_t$, for the wild bootstrap, where for each $t = 1, ..., T$, $u_t^* := \widehat{u}_t^c w_t$, where w_t, $t = 1, ..., T$, is an i.i.d. $N(0, 1)$ sequence.
3. Using the bootstrap sample, $\{y_t^*\}$, estimate model 82 under the alternative $H_1 : \rho \neq \theta \neq 0$ using OLS. Check that the error-correction term $\rho <= 0.0001$ and that stability is ensured. If the condition is fulfilled, proceed with the next step, otherwise go back to step 2 and draw from another set of residuals.
4. Using the bootstrap sample, $\{y_t^*\}$, compute the bootstrap PSS test statistics, $\widehat{PSS}_b$.
5. Repeat steps 1 to 4 B times.
6. The bootstrap p-value is computed as $F_{PSS}^b = \#\{\widehat{PSS}_b \geq \widehat{PSS}\}/B$ where $\widehat{PSS}$ is the observed value of the statistics.