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A Microfounded Model of Money Demand Under Uncertainty, and some Empirical Evidence.*

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Abstract

In this article we derive a microfounded model of money demand under uncertainty built on intertemporally optimizing risk-averse households. Deriving a complete solution of the optimization problem taking the intertemporal budget constraint into account where linearization procedures in our paper take a risky steady state as benchmark. The solution leads to ambiguous effects w.r.t. to the impact of capital market risk as well as inflation risk, which is due to the interplay of substitution and opposing income effects. The econometric results reveal that U.S. households increase their demand for money in response to positive changes in inflation risk and capital market risk, respectively, with both effects lasting permanently.

JEL Classifications: C22, E41, E51, E58, G11

Key Words: Money Demand, Uncertainty, Inflation Risk, Capital Market Risk, Monetary Policy, Cointegration

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1 Introduction

The rise of monetarism and New Classical Economics in the 1970s and 1980s fueled an ongoing debate about the stability of money demand and its prominent role for the effectiveness of monetary policy (Barnett et al., 1992). In the course of the 1990s the attention shifted away from money to interest rates as a guide for monetary policy. However, in the aftermath of the financial crisis, interest in private actors' liquidity preference has regained academic interest. One line of argument points to quantitative easing policies exercised by central banks leading to growth rates of money which are considered as incompatible with real growth rates thus raising concerns about future inflation.[1] A second line of argument emphasizes the risk of protracted periods of secular stagnation for the world economy (Eggertsson and Mehrotra, 2014) with a high preference for liquidity as a major cause (Bossone, 2014). Both concerns suggest that theoretical as well as empirical research on the determinants of money demand should be resumed. However, different from the debate of the 1970s and 1980s which had a focus on the issue of a stable relationship between money demand and income, now fears of future inflation due to excessive monetary growth direct the attention to whether and how expected inflation as well as its volatility affect money holdings in the non-bank sector. A negative correlation between both variables and money demand implies that the non-bank sector wants to rid itself from high money holdings thus boosting purchases of goods and assets, and accompanied with that, prices. On the other hand, worries about secular stagnation also advocate an interest in how risks might affect actors' liquidity preference. The most prominent fear in this regard is that people do not believe in inflation but are instead afraid of lasting deflationary forces. In this case, a negative correlation between desired money holdings and expected inflation, too, would aggravate the situation whereas a positive correlation between inflation risk and money demand could act as a stabilizer. Finally, as a consequence of recent financial regulations, central banks will play a

[1] Of relevance here is the widespread consensus characterizing monetary policy prior to the crisis according to which inflation is always a monetary phenomenon, (Mishkin, 2011).

more active role in the process of financial supervision. This extension of authority has not gone uncriticized for reasons which point to a possible conflict of interest between financial and price stability. In this respect the demand for money, too, gains importance, where this time reactions to higher financial risk as compared to inflation risk are of interest. All these arguments suggest that forecasts of monetary demand will play a pivotal role for both the assessment of the future macroeconomic development as well as for the effectiveness of monetary policy.

There is indeed an increasing number of publications examining the impact of diverse risks on money demand. Overwhelmingly, these studies are empirical basing their estimations on either plausibility or on Euler equations. With our paper we aim at contributing to this line of research. In doing so, we offer a microeconomic foundation assuming intertemporally optimizing households. Contrary to the literature, however, we do not content ourselves with the Euler equations but rather propose a complete solution to the optimization problem taking the intertemporal budget constraint into account where linearization procedures in our paper take a risky steady state as benchmark (Coeurdacier et al., 2011). As one main difference between models using the Euler equation as a monetary demand function and our approach, we do not only consider substitution effects but in addition possibly countervailing income effects. As a second difference, in our approach expectations about future income and not only about current consumption determine households' money holdings. Finally, by taking interest-bearing bank deposits into account, we consider narrow as well as broad money. Unlike most empirical applications, we estimate both the long run money demand relationship as well as its short run dynamics. Our most important finding is the dominance of income effects as compared to substitution effects with respect to variations of inflation risks. This result casts serious doubts on the usefulness of approaches which exclusively take the Euler equations into account. In line with the evidence found by Bloom (2009, 2014) that uncertainty matters for business cycle fluctuations, our results indicate another channel through which uncertainty may affect macroeconomic developments.

The remainder of the paper starts with a literature review on both theoretical and empirical studies on money demand. In the subsequent theoretical part we develop a macroeconomic model of money demand using an OLG framework distinguishing between a long run and short run perspective. Our analysis is partial in the sense that we do not set up a complete macroeconomic model but concentrate on the demand for alternative assets. The empirical part comprises the solid testing on cointegration and the estimation of error-correction models. The model dynamics are studied by means of the dynamic multiplier analysis.

2 Literature review

The examination of risk variables as components of the money demand function directs the attention to money as a store of value. Depending on the type of risk we focus on different motives of storing money. In particular, in the aftermath of the recent great financial crisis (GFC, henceforth) liquidity risks have gained importance. For example Telyukova and Visschers (2013) assume an uninsurable idiosyncratic liquidity risk which gives rise to precautionary savings in the form of money holdings. The theoretical discussion whether money holdings are useful within the framework of portfolio decisions has a long history. That non-interest bearing cash holdings serve to protect investors from capital market risk was emphasized by J.M. Keynes and formally elaborated by James Tobin within a static portfolio framework (Tobin, 1956). On the other hand, a reduction of portfolio risk can also be achieved by holding interest-bearing assets provided that they are considered as riskless (Ingersoll, 1987). And indeed, due to numerous financial innovations the supply of interest-bearing assets promising safety to their holders, has increased over the years. Hence, a further argument is needed to legitimate cash as a store-of-value. In this respect cash as immediate liquidity gained importance, which came to be incorporated into microeconomic models of optimizing behavior either by assigning direct utility to money (based on Patinkin, 1965) or by assuming transaction costs of transforming assets into immediate liquidity (Saving, 1971; McCallum and Goodfriend, 1987). That both approaches are equivalent in terms of their results for optimal cash holdings, was shown for example by Feenstra

(1986). Overwhelmingly, in these approaches money is defined as cash thus legitimating its status as immediate liquidity. However, taking into account that due to improved payment technologies, costs of liquidating a broad range of assets have been reduced to a rather negligible quantity, central banks nowadays resort to broad aggregates of money as indicators of the effectiveness of their policies as well as of macroeconomic liquidity preferences. Arguably, this, too, has not gone criticized for reasons which doubt that the components of either monetary aggregate should be considered as perfect substitutes (see Barnett et al., 1992, for a review). On the other hand, already the existence of just a few distinct monetary aggregates acknowledges that private actors hold different types of riskless assets reaching from cash to interest-bearing deposits simultaneously, which requires explanation. Macroeconomic theory so far has not taken up this issue (with an exception of Bossone, 2014).

In DSGE models, which have come to serve as the workhorse model for monetary policy, cash yields direct utility thus legitimating positive cash holdings even in the presence of a riskless but interest-bearing security of indeterminate maturity. Since this class of models generally exclude the derivation of explicit solutions, log-linearization around the steady state is chosen, which leads to percentage deviations of optimal cash holdings as a function of both deviations of current consumption from steady state values and the riskless nominal rate of interest (Walsh, 2003, as one example). Moreover, due to the application of a Taylor expansion of first order, risk variables are excluded from the analysis.

It is finally worth noting at this point that typically intertemporal macroeconomic models do not offer complete solutions for household optimization problems taking the intertemporal budget constraint into account, but derive all types of behavioral functions directly from the Euler equations. This implies that the relationship between money demand and its explanatory variables reflects substitution effects only thus telling merely half of the story. This procedure is also followed in Choi and Oh (2003) who derive a money demand function from a general equilibrium model focusing on the impact of output as well as monetary uncertainty which has its origins in information deficiencies concerning the money supply process. By assuming that both output and the supply of money are log-normally distributed, they are able to consider risk by including variances and covariances as components of optimal cash holdings. Furthermore they do not need to resort to log-linearization procedures around some equilibrium in order to derive explicit optimality conditions. Money demand here, too, depends on current consumption but furthermore both output shock variances and monetary shock variances play a role though the direction of impact is ambiguous. The authors explain this ambiguity by the coincidence of a substitution and precautionary effect. For example higher monetary uncertainty motivates households to reduce money balances (substitution effect). On the other hand, the authors argue that higher uncertainty as such also motivates higher savings. This last argument is true but its formal derivation requires a complete solution of the household's optimization problem thus resorting to the intertemporal budget constraint. Such a complete solution is missing in the paper and for that reason any ambiguous reaction of money demand to higher monetary uncertainty calls for a different explanation. Rather, the two countervailing effects point to the assumed utility function which departs from the commonly assumed (weak) separability of consumption and money but sees them as complements. Hence if consumption increases due to higher monetary uncertainty, this raises the marginal utility of money thus suggesting higher money holdings, too. Bossone (2014) departs from the standard general equilibrium macroeconomic model by explicitly considering different degrees of liquidity as a distinguishing feature of assets leading to different utilities assigned to them. Of relevance for his results are interactions between rational expectations and market sentiments. Pessimistic market sentiments may be such that households' preferences are directed towards "ultra liquid" assets thus raising money at the expense of expenditures on consumption goods.

The existing empirical literature on money demand is rich but almost all of these studies formulate *ad hoc* models based on story-telling or plausibility. As the focus of this article is on the theoretical part, we only briefly review the empirical money demand literature explicitly considering the role of economic risks/uncertainty for money demand.[2]

Carpenter and Lange (2003), as most studies in the empirical money demand literature, apply the cointegration method to study the long run repercussions of risk variables on money demand behavior. The authors estimate a money demand function for the U.S. economy taking into account the role of equity market risk, and find that an increase in equity risk leads to a permanent rise in the demand for M2 as risky assets are substituted for safe alternatives. In contrast to Bruggeman et al. (2003) who find for the euro area no permanent effect of stock market volatility on money demand, Carstensen (2006) argues that the observed overshooting of M3 in the euro area at the end of 2001 can partly be explained by increased stock market risk. The result by Carstensen is also in line with findings by Greiber and Lemke (2005) who argue that the consideration of a measure of aggregate preference for liquidity (measured by a common $I(1)$ factor comprising financial market returns and various volatility measures) helps to re-establish a stable cointegrating relationship for both the euro area and U.S. economy, respectively. This result is in principle confirmed by recent work by Seitz and von Landesberger (2014) for the euro area stressing the relevance of the substitution channel through which financial market uncertainty affects the demand for M3 positively. Lastly, according to Cook and Choi (2007) financial market risks, namely credit risk, help to explain structural instability of the traditional money demand model on the aggregate level in the U.S. between 1970 and 2005.

The role of inflation risk for money demand was examined by Higgins and Majin (2009) for both M1 and M2 U.S. broad money measures. These authors fit a conventional backward-looking Phillips curve model with GARCH errors to estimate the conditional variance of inflation. It is found that increased inflation uncertainty has negative impacts on the demand for M1 as concerns about higher expected inflation put low-interest bearing assets under stress leading to a shift towards higher-interest bearing components of M2. Also it is argued that M1 also includes

[2]For recent and more detailed literature surveys on empirical money demand studies see Belke and Czudaj (2010) as well as Setzer and Wolff (2013). For an overview using panel data see also Dobnik (2013) and Kumar et al. (2013).

long-term assets which agents may want to substitute for money market instruments in order to reduce the risk associated with long-term assets.

3 Theory

3.1 The theoretical model

In the following we will derive a function for both narrow money consisting of cash and sight deposits, and time deposits within a partial model of household optimization. Since the model is partial, we take the rate of inflation, all asset returns and current income as exogenously given. We focus on a representative household which maximizes lifetime consumption, and hence the demand for money forms part of the solution of the household's optimization problem. In this regard, and contrary to the typical procedure followed in DSGE models, we will derive a complete solution of the optimization problem taking the intertemporal budget constraint into account. This allows us to consider both substitution and income effects thus providing a richer variety concerning the reaction of money demand to variations of its determinants. We consider a household with a planning horizon of two periods. As in Größl and Tarassow (2015) the background setting can be conceived of as an OLG model with each generation living two periods. The assumption of a finite planning period allows us to avoid problems following from an infinite series of future incomes when integrating the intertemporal budget constraint into the derivation of a complete solution of the household optimization problem.

The young generation lives two periods and plans its optimal time path of consumption when young. The old generation finances consumption by the liquidation of accumulated wealth. It dies at the end of the second period without leaving any bequests. In accordance with Größl and Tarassow (2015) we model a stationary economy with uncertainty concerning the real rate of return of all assets. In contrast to Größl and Tarassow, we now assume that uncertainty continues to prevail in the steady state equilibrium. Contrary to the short run, however, the long run equilibrium is characterized by constant expectations and constant variances. Models that assume a risky steady state are increasing in numbers (see e.g. Coeurdacier et al., 2011; de Groot, 2013). In order to give risk an explicit representation, these approaches apply a Taylor-series of second order to the set of Euler equations, which suffers from the drawback of delivering rather intractable mathematical expressions. It is therefore not surprising that the authors have chosen to work with rather general functions which, however, make it difficult if not impossible to provide economic interpretations establishing clear relations to the household's preference structure. In order to avoid such a "black-box-setting", we follow a different route. In accordance with Größl and Fritsche (2007) and Größl and Tarassow (2015), we apply a Taylor-series already to the expected utility function delivering the certainty equivalent as a proxy for future consumption. As an important advantage, we obtain optimal results which can be traced back to the household's preference structure in a clear manner. Since we intend to focus on households' monetary demand behavior, we will not spell out the complete macroeconomic model–both in the long and short run–but instead assume that a long run equilibrium under uncertainty exists, marked by constant expectations and variances. Remaining within a partial modeling framework we furthermore treat the rates of return on assets, the rate of inflation as well as labor income as exogenous. Our representative household draws utility from present and future consumption as well as from holding narrow money and time deposits. The household starts with zero wealth and hence receives exclusively labor income in the first period, whereas nominal income in the second period consists of interest from holding time deposits as well as stock (capital). Uncertainty is assumed to hold for the real rate of return on capital as well as for future inflation where both are modeled as normally distributed random variables. Maximizing welfare then requires that the household builds expectations and evaluates possible expectation errors. Our representative household is risk-averse with constant absolute risk-aversion and maximizes the certainty equivalent of future consumption (Größl and Fritsche, 2007).

Utility is given by

$$U = u(C_t) + \nu(M1_t) + \gamma(T_t) + u(CE_t) \tag{1}$$

where C_t denotes present consumption, CE_t the certainty equivalent, T_t real time deposits defined by

$$T_t = \frac{T_t^n}{P_t} \tag{2}$$

$M1_t$ stands for real cash balances defined by

$$M1_t = \frac{M1_t^n}{P_t} \tag{3}$$

As usual, utility is assumed to be strictly concave in all its determinants. Utility maximization takes place subject to the period budget constraints, where we have used the approximation

$$\frac{1}{1+\pi_{t+1}} \approx 1 - \pi_{t+1} \tag{4}$$

The following equation specifies the budget constraint for the first period:

$$Y_t = C_t + A_t + M1_t + T_t \tag{5}$$

As the budget constraint for the second period we obtain:

$$A_t\,(1 + r_{t+1}) + T_t\,(1 + i_{t+1} - E_t\pi_{t+1}) + M1_t\,(1 - E_t\pi_{t+1}) = C_{t+1} \tag{6}$$

By assumption the household starts with zero wealth and hence its current income consists exclusively of labor income, Y_t. A_t denotes capital. Time deposits yield the safe nominal interest rate i_{t+1}. Uncertainty relates to the real rate of capital r_{t+1}, as well as to the future rate of inflation both of which are modeled as normally distributed random variables with expectation $E_t r_{t+1}$ and $E_t\pi_{t+1}$, and the variances σ_{rt}^2 and $\sigma_{\pi t}^2$, respectively, as well as the covariance $\sigma_{\pi rt}$.

As has been formally derived in Größl and Fritsche (2007), the certainty equivalent reads as

$$CE_t = E_t C_{t+1} - \frac{\alpha}{2} Var\left(C_{t+1}\right) \tag{7}$$

where

$$E_t C_{t+1} = A_t\left(1 + E_t r_{t+1}\right) + T_t\left(1 + i_{t+1} - E_t \pi_{t+1}\right) + M1_t\left(1 - E_t \pi_{t+1}\right) \tag{8}$$

$$Var\left(C_{t+1}\right) = A_t^2 \sigma_{rt}^2 + M2_t \sigma_{\pi t}^2 - 2 A_t M2_t \sigma_{\pi r t} \tag{9}$$

$$M2_t = M1_t + T_t \tag{10}$$

and where $M2_t$ represents a broad monetary aggregate similar to $M2$ in the US. Parameter α represents the Arrow-Pratt measure of absolute risk aversion and is given by

$$\alpha = -\frac{u''\left(\overline{CE}_t\right)}{u'\left(\overline{CE}_t\right)} \tag{11}$$

where barred variables represent steady state values. In our analysis we assume that utility is of the CARA type.[3] Note furthermore that in our model the steady state is not characterized by the absence of risk but rather by positive but constant risk terms. Therefore the absolute Arrow-Pratt measure of risk aversion relates to the steady state value of the certainty equivalent and not to expected consumption. The certainty equivalent differs from expected consumption by taking into account that both the real rate of return on capital as well as the rate of inflation may deviate from their long run averages as measured by their variances which will henceforth be referred to as capital market risk and inflationary risk, respectively. Together with a covariance between inflation and the real rate of return on capital, these variances determine consumption risk. Note that a positive covariance $\sigma_{\pi r t}$ signals that the real interest rate on capital and the real rate of return on cash (i.e. the rate of deflation) are negatively correlated thus rendering

[3] This assumption facilitates the computation and interpretation of results greatly. Furthermore the consideration of CRRA introduces various countervailing effects thus it does not clearly and easily yield information about the impact of the type of risk aversion.

money and capital as complements regarding the size of consumption risk. Henceforth we assume that an increase of assets always increases the certainty equivalent thus ruling out a dominating impact of consumption risk.

3.2 The necessary and sufficient conditions for a maximum

Inserting the equations (5), (7), (8), (9) and (10) into the utility function given by equation (1), allows us to obtain the optimal values of capital and money holdings taking the intertemporal budget constraint into account. Hence as the solution of the optimization problem we do not only obtain optimal ratios between capital, cash and time deposits but also the optimal absolute size of these variables. As necessary and sufficient conditions for optimal capital holdings we obtain:

$$u'(C_t) = \beta u'(CE_t)\frac{\partial CE_t}{\partial A_t} \tag{12}$$

$$\frac{\partial CE_t}{\partial A_t} = (1 + E_t r_{t+1}) - \alpha A_t \sigma_{rt}^2 + \alpha M2_t \sigma_{\pi rt} > 0 \tag{13}$$

As optimal narrow money holdings we obtain:

$$u'(C_t) = \beta u'(CE_t)\frac{\partial CE_t}{\partial M1_t} + \nu'(M1_t) \tag{14}$$

$$\frac{\partial CE_t}{\partial M1_t} = 1 - E_t\pi_{t+1} - \alpha M2_t \sigma_{\pi t}^2 + \alpha A_t \sigma_{\pi rt} > 0 \tag{15}$$

and as the optimal stock of time deposits we get:

$$u'(C_t) = \beta u'(CE_t)\frac{\partial CE_t}{\partial T_t} + \gamma'(T_t) \tag{16}$$

$$\frac{\partial CE_t}{\partial T_t} = (1 + i_{t+1} - E_t\pi_{t+1}) - \alpha M2_t \sigma_{\pi t}^2 + \alpha A_t \sigma_{\pi rt} > 0 \tag{17}$$

For the following analysis we will refer to the expression $\beta u'(CE_t)\frac{\partial CE_t}{\partial J}$ as the marginal utility of future consumption with respect to asset J. We observe that compared to the perfect information case the optimal relationship between consumption today and consumption tomorrow is tilted towards the present due to the impact of marginal consumption risk. In this respect we consider it as an advantage that in contrast to the expected utility approach the influence of risk is now made explicit. According to equation (13) a higher level of capital increases the utility of future consumption due to the gross real rate of return on capital and reduces it due to a higher capital market risk. The degree to which this occurs, depends positively on the initial holdings of capital and the intensity of risk aversion. A further impact of risk is rooted in a correlation between the real rate of return on capital and on broad money. If both are positively correlated, which is equivalent to a negative sign of $\sigma_{\pi r t}$, then the negative impact of risk on the marginal utility of future consumption will even be larger. In the same way, according to the equations (14) and (16) a higher level of narrow money (time deposits) reduces the utility of present consumption, increases the utility of future consumption due to a higher level of available purchasing power and reduces it as a consequence of expected inflation, inflationary risk and a possibly positive correlation between the real rates of return on capital and money. Finally, a higher stock of narrow money (time deposits) increases total utility U due to the direct utility that is rendered by both types of money. Note that since time deposits yield a (safe) nominal rate of return i_t, which is not the case for narrow money, we have to assume that for all levels ob both assets we must have:

$$\nu'(M1_t) > \gamma'(T_t) \tag{18}$$

3.3 Linearization around long run steady state values

In order to obtain explicit results we linearize the optimality conditions around a long run equilibrium which is characterized by constant expectations concerning the real rate of return on capital and the rate of inflation and a constant volatility of realized values around these expectations measured by constant variances and covariances. Linearization allows us to represent the system of optimality conditions as a system of linear equations in absolute deviations of the endogenous and exogenous variables from their long run equilibrium values as represented in matrix form by

$$\mathbf{H}\mathbf{x}=\mathbf{B}\mathbf{y} \tag{19}$$

where

$$\mathbf{x}=\begin{pmatrix} A_t-\overline{A_t} \\ M1_t-\overline{M1_t} \\ T_t-\overline{T_t} \end{pmatrix} \tag{20}$$

and

$$\mathbf{y}=\begin{pmatrix} E_t r_{t+1}-E_t\overline{r_{t+1}}=E_t\Delta r_{t+1} \\ i_{t+1}-\overline{i_{t+1}}=\Delta i_{t+1} \\ E_t\pi_{t+1}-E_t\overline{\pi_{t+1}}=E_t\Delta\pi_{t+1} \\ \sigma^2_{rt}-\overline{\sigma^2_{rt}}=\Delta\sigma^2_{rt} \\ \sigma^2_{\pi t}-\overline{\sigma^2_{\pi t}}=\Delta\sigma^2_{\pi t} \\ \sigma_{\pi rt}-\overline{\sigma_{\pi rt}}=\Delta\sigma_{\pi rt} \\ Y_t-\overline{Y_t}=\Delta Y_t \end{pmatrix} \tag{21}$$

H represents the Hessian matrix:

$$\mathbf{H}=\begin{pmatrix} a_{11}=\frac{\partial^2U(\mathbf{v})}{\partial A_t^2} & a_{12}=\frac{\partial^2U(\mathbf{v})}{\partial A_t\partial M1_t} & a_{13}=\frac{\partial^2U(\mathbf{v})}{\partial A_t\partial T_t} \\ a_{21}=\frac{\partial^2U(\mathbf{v})}{\partial M1_t\partial A_t} & a_{22}=\frac{\partial^2U(\mathbf{v})}{\partial M1_t^2} & a_{23}=\frac{\partial^2U(\mathbf{v})}{\partial M1_t\partial T_t} \\ a_{31}=\frac{\partial^2U(\mathbf{v})}{\partial T_t\partial A_t} & a_{32}=\frac{\partial^2U(\mathbf{v})}{\partial T_t\partial M1_t} & a_{33}=\frac{\partial^2U(\mathbf{v})}{\partial T_t\partial T1_t} \end{pmatrix} \tag{22}$$

and **B** the matrix of the negative value of marginal utility changes with respect to changes in the exogenous variables comprising the rates of return on assets and risk parameters:

$$\mathbf{B}=\begin{pmatrix} -b_{ij} \end{pmatrix} \tag{23}$$

is a $3 \times 7-$matrix with

$$b_{ij} = \frac{\partial^2 U(\mathbf{v})}{\partial i \partial j} \tag{24}$$

$$i = (A_t, M1_t, T_t) \tag{25}$$

$$j = \left(E_t r_{t+1}, i_{t+1}, E_t \pi_{t+1}, \sigma^2_{rt}, \sigma^2_{\pi t}, \sigma_{\pi rt}, Y_t\right) \tag{26}$$

and

$$\mathbf{v} = \left(\overline{Y_t}, \overline{A_t}, \overline{T_t}, \overline{M1_t}, E_t\overline{r_{t+1}}, E_t\overline{\pi_{t+1}}, \overline{i_{t+1}}, \overline{\sigma^2_{rt}}, \overline{\sigma^2_{\pi t}}, \overline{\sigma_{\pi rt}}\right) \tag{27}$$

For the first-order conditions to represent a maximum, the Hessian matrix has to be negative-definite, which requires that the following conditions have to met:

1. $a_{11} < 0$,
2. $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} > 0$,
3. $\det(\mathbf{H}) < 0$

where

$$a_{12} = a_{21},\ a_{13} = a_{31},\ a_{23} = a_{32} \tag{28}$$

$a_{11} = \frac{\partial^2 U(\mathbf{v})}{\partial A_t^2}$ informs us about how the marginal total utility of capital measured at its equilibrium value reacts to a further marginal increase in capital and is determined by:

$$a_{11} = \frac{\partial^2 U(\mathbf{v})}{\partial A_t^2} = u''\left(\overline{C_t}\right) + \beta u''\left(\overline{CE_t}\right)\left(\frac{\partial \overline{CE_t}}{\partial A_t}\right)^2 + \beta u'\left(\overline{CE_t}\right)\frac{\partial^2 \overline{CE_t}}{\partial A_t^2} < 0 \tag{29}$$

$$\frac{\partial \overline{CE_t}}{\partial A_t} = 1 + E_t\overline{r_{t+1}} - \alpha\overline{A_t}\overline{\sigma^2_{rt}} + \alpha\overline{M2_t}\overline{\sigma_{\pi rt}} > 0 \tag{30}$$

$$\frac{\partial^2 \overline{CE_t}}{\partial A_t^2} = -\alpha\overline{\sigma^2_{rt}} < 0 \tag{31}$$

$$\overline{C_t} = \overline{Y_t} - \overline{A_t} - \overline{M1_t} - \overline{T_t} \tag{32}$$

A marginal increase in the stock of capital yields the household marginal disutility of present consumption. If the household increases its stock of capital a bit further, this marginal disutility of present consumption increases less since $u''\left(\overline{C_t}\right) < 0$. Furthermore, a higher stock of capital leads to a higher certainty equivalent giving rise to a lower marginal utility of future consumption. Finally, the marginal certainty equivalent with respect to capital goes down due to a higher capital market risk. Hence a_{11} has a negative sign. Likewise $a_{22} = \frac{\partial^2 U(\mathbf{v})}{\partial M1_t^2}$ and $a_{33} = \frac{\partial^2 U(\mathbf{v})}{\partial T_t^2}$ inform us about how the marginal utility of narrow money (time deposits) responds to a further marginal increase in the holdings of narrow money (time deposits). With respect to the marginal utility of current consumption the reaction is the same as for capital. Differences exist with respect to the reaction of the certainty equivalent which is higher in the case of capital than in the case of broad money and which in its turn is higher for time deposits than it is for narrow money. A further difference relates to the assumption that both types of money yield direct utility. In sum we may conclude that the signs for both a_{22} and a_{33} are negative, too:

$$a_{22} = \nu''\left(\overline{M1_t}\right) + u''\left(\overline{C_t}\right) + \beta u''\left(\overline{CE_t}\right)\left(\frac{\partial \overline{CE_t}}{\partial M1_t}\right)^2 + \beta u'\left(\overline{CE_t}\right)\frac{\partial^2 \overline{CE_t}}{\partial M1_t^2} < 0 \tag{33}$$

$$\frac{\partial \overline{CE_t}}{\partial M1_t} = 1 - E_t\overline{\pi_{t+1}} - \alpha\overline{M2_t}\overline{\sigma_{\pi t}^2} + \alpha\overline{A_t}\overline{\sigma_{\pi r t}} > 0 \tag{34}$$

$$\frac{\partial^2 \overline{CE_t}}{\partial M1_t^2} = -\alpha\overline{\sigma_{\pi t}^2} < 0 \tag{35}$$

$$a_{33} = \nu''\left(\overline{T_t}\right) + u''\left(\overline{C_t}\right) + \beta u''\left(\overline{CE_t}\right)\left(\frac{\partial \overline{CE_t}}{\partial T_t}\right)^2 + \beta u'\left(\overline{CE_t}\right)\frac{\partial^2 \overline{CE_t}}{\partial T_t^2} < 0 \tag{36}$$

$$\frac{\partial \overline{CE_t}}{\partial T_t} = 1 + \overline{i_{t+1}} - E_t\overline{\pi_{t+1}} - \alpha\overline{M2_t}\overline{\sigma_{\pi t}^2} + \alpha\overline{A_t}\overline{\sigma_{\pi r t}} > 0 \tag{37}$$

$$\frac{\partial^2 \overline{CE_t}}{\partial T_t^2} = -\alpha\overline{\sigma_{\pi t}^2} < 0 \tag{38}$$

The remaining coefficients of the Hessian matrix inform us about how the marginal utility of asset i reacts upon marginal variations in the size of asset $k, k \neq i$. As a general formula we obtain:

$$a_{ik} = a_{ki} = u''\left(\overline{C_t}\right) + \beta u''\left(\overline{CE_t}\right)\left(\frac{\partial \overline{CE_t}}{\partial i}\right)\left(\frac{\partial \overline{CE_t}}{\partial k}\right) + \beta u'\left(\overline{CE_t}\right)\frac{\partial^2 \overline{CE_t}}{\partial i \partial k} \tag{39}$$

where

$$\frac{\partial^2 \overline{CE_t}}{\partial i \partial k} = \alpha \overline{\sigma_{\pi r t}}\; i = A_t,\ k = (M1_t, T_t) \tag{40}$$

and

$$\frac{\partial^2 \overline{CE_t}}{\partial i \partial k} = -\alpha \overline{\sigma^2_{\pi t}} < 0,\ i = M1_t, k = T_t \tag{41}$$

Note that

$$\frac{\partial^2 CE_t(\mathbf{v})}{\partial T_t^2} = \frac{\partial^2 CE_t(\mathbf{v})}{\partial M1_t^2} = \frac{\partial^2 CE_t(\mathbf{v})}{\partial M1_t \partial T_t} = -\alpha \overline{\sigma^2_{\pi t}} < 0 \tag{42}$$

This implies that $a_{23} = a_{32}$ takes an unambiguously negative sign whereas this is ensured for $a_{13} = a_{31}$ as well as for $a_{12} = a_{21}$ only if the real rates of return on capital and money are positively correlated. But even in the case of a negative correlation we may plausibly assume that the impact of the covariance between capital and money will not dominate, and therefore we henceforth assign a negative value to $a_{12} = a_{21}$ and $a_{13} = a_{31}$, too.

Since $a_{11}$is negative, the first condition for the negative-definiteness of the Hessian matrix $\mathbf{H}$ is met. Furthermore the second condition will be fulfilled if we have

$$a_{11}a_{22} - a_{12}a_{21} = a_{11}a_{22} - a_{12}^2 > 0$$

For the following analysis we assume that this condition is met, and the same is assumed for

$$\det(\mathbf{H}) = a_{33}\left(a_{11}a_{22} - a_{12}^2\right) - a_{23}\left(a_{11}a_{23} - a_{12}a_{13}\right) + a_{13}\left(a_{13}a_{23} - a_{11}a_{13}\right) < 0$$

We now turn to deriving the money demand function by examining the relationship between deviations of the optimal holdings of monetary assets from their steady state values and the deviations of the exogenous variables from their steady state values. In particular with respect to the subsequent empirical analysis it is important to emphasize that the correlations between the exogenous variables and holdings of assets which we explicitly compute for deviations from the steady state also hold for the steady state. This can easily be checked by inserting into the optimality conditions the prevailing steady state values for the endogenous and exogenous variables and calculating the impact of variations thereof on optimal asset holdings by resorting to Slutsky equations.

3.4 Determination of money demand

In our model we distinguish between alternative monetary aggregates. Narrow money is equivalent to cash and non-interest-bearing sight deposits whereas broad money includes interest-bearing time deposits as well. In the following we first consider narrow money and time deposits in turn and then arrive at drawing conclusions for broad money $M2$. Of relevance in this respect is the assumption that both narrow money and time deposits render direct utility though at different size. This together with the fact that time deposits are interest-bearing, renders narrow money and time deposits imperfect substitutes.

We start clarifying the correlation between narrow money and the exogenous variables as given by:

$$\Delta M1_t = \left(\Phi^{M1}_{E_t r_{t+1}}, \Phi^{M1}_{i_{t+1}}, \Phi^{M1}_{E_t \pi_{t+1}}, \Phi^{M1}_{\sigma^2_{rt}}, \Phi^{M1}_{\sigma^2_{\pi t}}, \Phi^{M1}_{\sigma_{\pi r t}}, \Phi^{M1}_{Y_t}\right) \begin{pmatrix} E_t \Delta r_{t+1} \\ \Delta i_{t+1} \\ E_t \Delta \pi_{t+1} \\ \Delta \sigma^2_{rt} \\ \Delta \sigma^2_{\pi t} \\ \Delta \sigma_{\pi r t} \\ \Delta Y_t \end{pmatrix} \tag{43}$$

where $\Phi_j^{M1} j = (E_t r_{t+1}, i_{t+1}, E_t\pi_{t+1}, \sigma_{rt}^2, \sigma_{\pi t}^2, \sigma_{\pi rt}, Y_t)$, informs us about how optimal holdings of narrow money and the respective exogenous variables are correlated.[4] After a few reformulations meant to facilitate the interpretation of results, we obtain

$$\Phi_j^{M1} = -\frac{1}{\det(\mathbf{H})/(a_{11}a_{33} - a_{13}a_{31})}\left[b_{M1_t j} - b_{A_t,j}\left(\frac{a_{21} - a_{23}\frac{a_{31}}{a_{33}}}{a_{11} - a_{13}\frac{a_{31}}{a_{33}}}\right) - b_{T_t j}\left(\frac{a_{23} - a_{21}\frac{a_{13}}{a_{11}}}{a_{33} - a_{31}\frac{a_{13}}{a_{11}}}\right)\right] \tag{44}$$

Assuming that $(a_{11}a_{33} - a_{13}a_{31})$ has a positive sign, the denominator of equation (44), $\det(\mathbf{H})/(a_{11}a_{33} - a_{13}a_{31})$ is negative resulting in a positive sign for the expression

$$-\frac{1}{\det(\mathbf{H})/(a_{11}a_{33} - a_{13}a_{31})}.$$

Hence the expression inside of the squared bracket determines the algebraic sign of Φ_j^{M1}. In this respect we distinguish between immediate impact effects and more indirect effects. The immediate impact effects are represented by the coefficients b_{ij}. For example $b_{M1_t j}$ informs us how the exogenous variables $j = (E_t r_{t+1}, i_{t+1}, E_t\pi_{t+1}, \sigma_{rt}^2, \sigma_{\pi t}^2, \sigma_{\pi rt}, Y_t)$ affect optimal narrow money holdings given that capital as well as time deposits have retained their initial steady state values. In the same way $b_{A_t j}$ ($b_{T_t j}$) informs us about how the exogenous variables affect capital (time deposits) given that the other assets have retained their initial equilibrium values. In this regard it is important to recall that our approach delivers a complete solution to the household's optimization problem taking the intertemporal budget constraint into account. Hence the reaction of narrow money holdings upon changes in the exogenous variables also depends on corresponding reactions of time deposits and capital. The second component of the squared bracket in equation (44) tells us how capital responds to changes in the exogenous variables now also taking more indirect effects into account which follow from reactions of time deposits to variations of capital. The response of capital taking narrow money holdings as given is revealed by the ratio $\frac{b_{A_t,j}}{a_{11} - a_{13}\frac{a_{31}}{a_{33}}}$ where the coefficient $b_{A_t,j}$ stands for the immediate impact effect thus determining the sign of

[4] For simplicity we henceforth omit the reference to deviations from the steady state.

changes in capital given that narrow money and time deposits are at their initial values. The denominator determines the strength of changes in narrow money holdings. Of crucial importance in this respect is how the total marginal utility of capital reacts to its further marginal increase. The stronger this reaction will be, the lower is a change of capital due to changes in the exogenous variables $j = (E_t r_{t+1}, i_{t+1}, E_t \pi_{t+1}, \sigma^2_{rt}, \sigma^2_{\pi t}, \sigma_{\pi rt}, Y_t)$. Of further importance for the size of the denominator is the fact that any change in capital leads to a change of the optimal size of time deposits, too, as determined by the ratio a_{31}. In order to determine the overall effects of changing capital on optimal narrow money holdings two channels have to be distinguished: The first channel operates directly through a_{21} which indicates how the marginal total utility of narrow money is affected by a changing stock of capital. The second channel operates more indirectly through the link between the impact of changing capital on the marginal utility of time deposits and the impact of changing time deposits on the marginal utility of narrow money. Henceforth we assume that these more indirect effects do not dominate.

The third component of the squared bracket in equation (44) explains how the response of time deposits to variations in the exogenous variables feed back on optimal narrow money holdings where the interpretation follows the same pattern as that for capital, where again we assume that the more indirect effects do not dominate. Hence, we subsequently assume that the signs of both $\left(\frac{a_{21}-a_{23}\frac{a_{31}}{a_{33}}}{a_{11}-a_{13}\frac{a_{31}}{a_{33}}}\right)$ and $\left(\frac{a_{23}-a_{21}\frac{a_{13}}{a_{11}}}{a_{33}-a_{31}\frac{a_{13}}{a_{11}}}\right)$ are positive.

We now turn to a detailed interpretation of the determinants of narrow money holdings and thus to the algebraic sign of each Φ^{M1}_j, $j = (E_t r_{t+1}, i_{t+1}, E_t \pi_{t+1}, \sigma^2_{rt}, \sigma^2_{\pi t}, \sigma_{\pi rt}, Y_t)$. $\Phi^{M1}_{E_t r_{t+1}}$ informs us about how narrow money holdings and the expected real rate of return on capital are correlated. The following interpretation of effects is based on equation (44) now specified by $b_{M1_t, E_t r_{t+1}}, b_{A_t, E_t r_{t+1}}, b_{T_t, E_t r_{t+1}}$ as immediate impact effects. Note that the first component of equation (44) is positive. Hence in order to establish a negative (positive) relationship between the expected real rate of return on capital, the expression inside the squared bracket has to be negative (positive). The immediate impact effect of a higher expected real rate of return on capital on narrow money is determined by:

$$b_{M1_t,E_tr_{t+1}} = \beta u''\left(\overline{CE_t}\right)\left(\frac{\partial\overline{CE_t}}{\partial M1_t}\right)\left(\frac{\partial\overline{CE_t}}{\partial E_tr_{t+1}}\right) < 0 \tag{45}$$

where $\frac{\partial\overline{CE_t}}{\partial M1_t}$ is given by equation (34). Furthermore we have

$$\frac{\partial\overline{CE_t}}{\partial E_tr_{t+1}} = A_t \tag{46}$$

We observe that the immediate impact effect of a higher expected real rate of return on capital requires a lower size of optimal narrow money holdings, which is due to a higher certainty equivalent driving the marginal future utility of consumption down. If capital and time deposits retained their initial values then we would conclude that narrow money holdings and the real rate of return on capital are unambiguously negatively correlated. However, capital and time deposits will change, too, feeding back on narrow money holdings. The reaction of capital is given by the second component of the squared bracket in equation (44). Of most interest is $b_{A_t,j}$ given by

$$b_{A,E_tr_{t+1}} = \beta u'\left(\overline{CE_{t+1}}\right)\left[\frac{\partial^2\overline{CE_t}}{\partial A_t\partial E_tr_{t+1}} - \alpha\left(\frac{\partial\overline{CE_t}}{\partial A_t}\right)\left(\frac{\partial\overline{CE_t}}{\partial E_tr_{t+1}}\right)\right] \gtreqless 0 \tag{47}$$

where $\frac{\partial\overline{CE_t}}{\partial A_t}$ is given by equation (30). Furthermore we have

$$\frac{\partial^2\overline{CE_t}}{\partial A_t\partial E_tr_{t+1}} = 1 \tag{48}$$

Note that in equation (47) we have made use of the Arrow-Pratt measure of absolute risk aversion as defined by equation (11). The first component inside the squared bracket in equation (47) represents the substitution effect: a higher expected rate of return on capital increases the marginal utility of future consumption while leaving the marginal utility of present consumption unaltered. In order to restore the optimum, capital has to increase. The second component inside the squared bracket represents the income effect: a higher expected rate of return on capital increases the certainty equivalent and hence reduces the marginal utility of future consumption, according to which a lower amount of assets and hence capital, too, is required in order to restore the optimum. If the substitution effect dominates, then $b_{A,E_t r_{t+1}}$ will take a positive sign. For this to happen the degree of risk aversion has to be sufficiently low. Given that this is the case and recalling the assumption that $\left(\frac{a_{21}-a_{23}\frac{a_{31}}{a_{33}}}{a_{11}-a_{13}\frac{a_{31}}{a_{33}}}\right)$ takes a positive sign, then the increase in the optimal stock of capital will have an additional negative effect on narrow money holdings. However, should the income effect on capital dominate, then this reaction would countervail the direct negative impact effect of a higher expected real rate of return on capital on narrow money holdings. The third component of the squared bracket in equation (44) explains how the reaction of time deposits affects optimal narrow money holdings.

The direct impact effect of a higher expected rate of real return on capital, given unchanged levels of the other assets, is indicated by

$$b_{T_t,E_t r_{t+1}} = \beta u''\left(\overline{CE}_t\right)\left(\frac{\partial \overline{CE}_t}{\partial T_t}\right)\left(\frac{\partial \overline{CE}_t}{\partial E_t r_{t+1}}\right) < 0 \tag{49}$$

Like in the case of narrow money and based on the same reasons, the correlation will be negative. Due to this effect, narrow money holdings should be increased in order to restore the optimum. In sum we may conclude the following: If the substitution effect outweighs the income effect on capital and if this reaction is stronger than the response of time deposits, then narrow money holdings and the expected real rate of return on capital will be negatively correlated. Should the income effect dominate with respect to capital, then a negative correlation between the expected real rate of return on capital and narrow money requires that the immediate impact effect as determined by $b_{M1_t j}$ strong enough in order to outweigh the effects of smaller capital and time deposit holdings.

$\Phi^{M1}_{i_{t+1}}$ tells us how narrow money holdings and the safe nominal interest rate on time deposits are correlated. We replace in equation (44) $b_{M1_t j}, b_{A_t j}, b_{T_t j}$ by $b_{M1_t,i_{t+1}}, b_{A_t,i_{t+1}}, b_{T_t,t i_{t+1}}$ which are given by

$$b_{A_t,i_{t+1}} = \beta u'' \left(\overline{CE_t}\right) \left(\frac{\partial \overline{CE_t}}{\partial A_t}\right) \left(\frac{\partial \overline{CE_t}}{\partial i}\right) < 0 \tag{50}$$

$$b_{M1_t,i_{t+1}} = \beta u'' \left(\overline{CE_t}\right) \left(\frac{\partial \overline{CE_t}}{\partial M1_t}\right) \left(\frac{\partial \overline{CE_t}}{\partial i_{t+1}}\right) < 0 \tag{51}$$

$$b_{T_t,i_{t+1}} = \beta u' \left(\overline{CE_{t+1}}\right) \left[\frac{\partial^2 \overline{CE_t}}{\partial T_t \partial i_{t+1}} - \alpha \left(\frac{\partial \overline{CE_t}}{\partial T_t}\right) \left(\frac{\partial \overline{CE_t}}{\partial i_{t+1}}\right)\right] \gtreqless 0 \tag{52}$$

$$\frac{\partial \overline{CE_t}}{\partial i_{t+1}} = \overline{T_t} \tag{53}$$

$$\frac{\partial^2 \overline{CE_t}}{\partial T_t \partial i_{t+1}} = 1 \tag{54}$$

We see that the immediate impact effect of a higher nominal interest rate on time deposits requires a lower stock of narrow money holdings since due to a higher certainty equivalent the marginal utility of future consumption goes down. The same holds true for the immediate impact effect on capital. By contrast the immediate impact effect of a higher nominal interest rate on time deposits is ambiguous depending on the relative strength of the substitution compared to the income effect. A lower stock of capital holdings requires higher holdings of narrow money in order to restore the optimum. If the substitution effect dominates for time deposits, then its increase supports a lower holding of narrow money, and with caution we may conclude that in this case narrow money and the nominal interest rate on time deposits are negatively correlated. If by contrast the income effect outweighs the substitution effect on time deposits, then a positive correlation between narrow money and the nominal interest rate of time deposits may cannot be ruled out.

$\Phi^{M1}_{E_t\pi_{t+1}}$ reveals how narrow money holdings and expected inflation are correlated. Replacing in equation (44) $b_{M1_tj}, b_{A_tj}, b_{T_tj}$ by $b_{M1_t,E_t\pi_{t+1}}, b_{A_t,E_t\pi_{t+1}}, b_{T_t,E_t\pi_{t+1}}$, which are given by

$$b_{A_t,E_t\pi_{t+1}} = \beta u'' \left(\overline{CE_t}\right) \left(\frac{\partial \overline{CE_t}}{\partial A_t}\right) \left(\frac{\partial \overline{CE_t}}{\partial E_t\pi_{t+1}}\right) > 0 \tag{55}$$

$$b_{M1_t,E_t\pi_{t+1}} = \beta u' \left(\overline{CE_{t+1}}\right) \left[\frac{\partial^2 \overline{CE_t}}{\partial M1_t \partial E_t\pi_{t+1}} - \alpha \left(\frac{\partial \overline{CE_t}}{\partial M1_t}\right) \left(\frac{\partial \overline{CE_t}}{\partial E_t\pi_{t+1}}\right)\right] \gtreqless 0 \tag{56}$$

$$b_{T_t,E_t\pi_{t+1}} = \beta u'\left(\overline{CE_{t+1}}\right)\left[\frac{\partial^2\overline{CE_t}}{\partial T\partial E_t\pi_{t+1}} - \alpha\left(\frac{\partial\overline{CE_t}}{\partial T_t}\right)\left(\frac{\partial\overline{CE_t}}{\partial E_t\pi_{t+1}}\right)\right] \gtreqless 0 \tag{57}$$

where

$$\frac{\partial\overline{CE_t}}{\partial E_t\pi_{t+1}} = -M2_t < 0 \tag{58}$$

and

$$\frac{\partial^2\overline{CE_t}}{\partial M1_t\partial E_t\pi_{t+1}} = \frac{\partial^2\overline{CE_t}}{\partial T\partial E_t\pi_{t+1}} = -1 \tag{59}$$

we observe that the immediate impact effects of higher expected inflation on both narrow money and time deposits depend on the relative weight of the substitution effect compared to the income effect. Assume that the substitution effect dominates for both monetary assets. Then the immediate impact effect on narrow money given unchanged levels of the other assets is negative. Since the immediate impact effect on capital is positive, this will drive optimal narrow money holdings down further. On the other hand a lower stock of time deposits following the immediate impact effect of a higher expected rate of inflation drives narrow money up. With caution we may conclude that this last effect does not dominate thus rendering a negative correlation between narrow money holdings and expected inflation. Assume now that the income effect dominates the direct impact effect of higher expected inflation on narrow money. Note that the income effect is stronger for time deposits than for narrow money. In particular we observe that whenever the income effect dominates for narrow money, the same holds true for time deposits, whereas the opposite is not true. By consequence we might observe a predominance of the income effect with respect to time deposits whereas the substitution effect dominates for narrow money. In this case narrow money and expected inflation will be unambiguously negatively correlated since the immediate impact effect of higher expected inflation on capital is positive. If on the other hand the income effect dominates the immediate impact effect on both types of money, the net effect is ambiguous since now both a higher level of capital and time deposits counteract the immediate positive impact effect on narrow money.

$\Phi^{M1}_{\sigma^2_{rt}}$ tells us how narrow money balances and capital market risk are correlated. Replacing in equation (44) $b_{M1_t j}, b_{A_t j}, b_{T_t j}$ by $b_{M1_t,\sigma^2_{rt}}, b_{A_t,\sigma^2_{rt}}, b_{T_t \sigma^2_{rt}}$ as determined by

$$b_{A_t,\sigma^2_{rt}} = \beta u'\left(\overline{CE_{t+1}}\right)\left[\frac{\partial^2 \overline{CE_t}}{\partial A_t \partial \sigma^2_{rt}} - \alpha\left(\frac{\partial \overline{CE_t}}{\partial A_t}\right)\left(\frac{\partial \overline{CE_t}}{\partial \sigma^2_{rt}}\right)\right] \gtreqless 0 \tag{60}$$

$$b_{M1_t,\sigma^2_{rt}} = \beta u''\left(\overline{CE_t}\right)\left(\frac{\partial \overline{CE_t}}{\partial M1_t}\right)\left(\frac{\partial \overline{CE_t}}{\partial \sigma^2_{rt}}\right) < 0 \tag{61}$$

$$b_{T_t,\sigma^2_{rt}} = \beta u''\left(\overline{CE_t}\right)\left(\frac{\partial \overline{CE_t}}{\partial T_t}\right)\left(\frac{\partial \overline{CE_t}}{\partial \sigma^2_{rt}}\right) < 0 \tag{62}$$

where

$$\frac{\partial \overline{CE_t}}{\partial \sigma^2_{rt}} = -\frac{\alpha}{2}\overline{A^2_t} < 0 \tag{63}$$

$$\frac{\partial^2 \overline{CE_t}}{\partial A_t \partial \sigma^2_{rt}} = -2\overline{A_t} < 0 \tag{64}$$

we recognize that since a higher capital market risk reduces the certainty equivalent, the immediate impact effects on both types of money are positive. Given unchanged levels of time deposits, narrow money and capital market risk would be positively correlated. However, the positive impact effect on time deposits countervails this effect. If moreover the income effect of a higher capital market risk on capital holdings outweighed the substitution effects, then a negative correlation between narrow money holdings and capital market risk could not be ruled out. Assume that by contrast the substitution effect dominates the immediate impact effect on capital. Then there is a high likelihood for a positive correlation between narrow money and capital market risk.

$\Phi^{M1}_{\sigma^2_{\pi t}}$ represents the correlation between narrow money and inflation risk. Replacing in equation (44) $b_{M1_t j}, b_{A_t j}, b_{T_t j}$ by $b_{M1_t,\sigma^2_{\pi t}}, b_{A_t,\sigma^2_{\pi t}}, b_{T_t \sigma^2_{\pi t}}$ which are given by

$$b_{A_t,\sigma^2_{\pi_t}} = \beta u''\left(\overline{CE_t}\right)\left(\frac{\partial \overline{CE_t}}{\partial A_t}\right)\left(\frac{\partial \overline{CE_t}}{\partial \sigma^2_{\pi_t}}\right) > 0 \tag{65}$$

$$b_{M1_{t,\sigma^2_{\pi t}}} = \beta u'\left(\overline{CE_{t+1}}\right)\left[\frac{\partial^2 \overline{CE_t}}{\partial M1_t \partial \sigma^2_{\pi t}} - \alpha\left(\frac{\partial \overline{CE_t}}{\partial M1_t}\right)\left(\frac{\partial \overline{CE_t}}{\partial \sigma^2_{\pi t}}\right)\right] \gtreqless 0 \tag{66}$$

$$b_{T_{t,\sigma^2_{\pi t}}} = \beta u'\left(\overline{CE_{t+1}}\right)\left[\frac{\partial^2 \overline{CE_t}}{\partial T_t \partial \sigma^2_{\pi t}} - \alpha\left(\frac{\partial \overline{CE_t}}{\partial T_t}\right)\left(\frac{\partial \overline{CE_t}}{\partial \sigma^2_{\pi t}}\right)\right] \gtreqless 0 \tag{67}$$

where

$$\frac{\partial \overline{CE_t}}{\partial \sigma^2_{\pi t}} = -\frac{\alpha}{2}\overline{M2^2_t} < 0 \tag{68}$$

$$\frac{\partial^2 \overline{CE_t}}{\partial M1_t \partial \sigma^2_{\pi t}} = \frac{\partial^2 \overline{CE_t}}{\partial T_t \partial \sigma^2_{\pi t}} = -\alpha \overline{M2_t} < 0 \tag{69}$$

we see that the immediate impact effects of a higher inflation risk on both types of money depend on the relative weight of the substitution compared to the income effect. At least qualitatively the pattern of reactions is the same as in the case of expected inflation: If the substitution effect dominates for both assets, then due to a positive immediate impact effect on capital, narrow money holdings go down. If the substitution effect dominates for narrow money but the income effect for time deposits, then the correlation between narrow money and inflation risk is unambiguously negative. If the income effect dominates for both assets, then the net effect remains ambiguous.

$\Phi^{M1}_{\sigma^2_{\pi rt}}$ represents the correlation between narrow money and the covariance between inflation and the real rate of return on capital. Replacing in equation (44) $b_{M1_{tj}}, b_{A_{tj}}, b_{T_{tj}}$ by $b_{M1_{t,\sigma^2_{\pi rt}}}, b_{A_{t,\sigma^2_{\pi rt}}}, b_{T_{t\sigma^2_{\pi rt}}}$ as given by

$$b_{A_{t,\sigma_{\pi r_t}}} = \beta u'\left(\overline{CE_{t+1}}\right)\left[\frac{\partial^2 \overline{CE_t}}{\partial A_t \partial \sigma_{\pi rt}} - \alpha\left(\frac{\partial \overline{CE_t}}{\partial A_t}\right)\left(\frac{\partial \overline{CE_t}}{\partial \sigma_{\pi rt}}\right)\right] \gtreqless 0 \tag{70}$$

$$b_{M1_{t,\sigma_{\pi r_t}}} = \beta u'\left(\overline{CE_{t+1}}\right)\left[\frac{\partial^2 \overline{CE_t}}{\partial M1_t \partial \sigma_{\pi rt}} - \alpha\left(\frac{\partial \overline{CE_t}}{\partial M1_t}\right)\left(\frac{\partial \overline{CE_t}}{\partial \sigma_{\pi rt}}\right)\right] \gtreqless 0 \tag{71}$$

$$b_{T_{t,\sigma_{\pi r_t}}} = \beta u'\left(\overline{CE_{t+1}}\right)\left[\frac{\partial^2 \overline{CE_t}}{\partial T_t \partial \sigma_{\pi rt}} - \alpha\left(\frac{\partial \overline{CE_t}}{\partial T_t}\right)\left(\frac{\partial \overline{CE_t}}{\partial \sigma_{\pi rt}}\right)\right] \gtreqless 0 \tag{72}$$

where

$$\frac{\partial \overline{CE_t}}{\partial \sigma_{\pi rt}} = \alpha \overline{M2_t A_t} > 0 \tag{73}$$

$$\frac{\partial^2 \overline{CE_t}}{\partial A_t \partial \sigma_{\pi rt}} = \alpha \overline{M2_t} > 0 \tag{74}$$

$$\frac{\partial^2 \overline{CE_t}}{\partial M1_t \partial \sigma_{\pi rt}} = \frac{\partial^2 \overline{CE_t}}{\partial T_t \partial \sigma_{\pi rt}} = \alpha \overline{A_t} > 0 \tag{75}$$

we observe that the immediate impact effects for all three types of assets depend on the relative weight of the substitution compared to the income effect. If the substitution effect dominates for narrow money but the income effect for time deposits and capital, then the optimal stock of narrow money will be positively correlated with $\sigma_{\pi rt}$. If the substitution effect dominates for all three assets, a positive correlation between the covariance and narrow money requires that the direct impact effect $b_{M1_t,\sigma^2_{\pi rt}}$ is strong enough.

$\Phi^{M1}_{Y_t}$ represents the correlation between narrow money balances and present labor income. Replacing in equation (44) $b_{M1_tj}, b_{A_tj}, b_{T_tj}$ by $b_{M1_t,Y_t}, b_{A_tY_t}, b_{T_tY_t}$ as given by

$$b_{A_t,Y_t} = b_{M1_t,Y_t} = b_{T_t,Y_t} = -u''\left(C_t\right) > 0 \tag{76}$$

we see that all three immediate impact effects are the same. If current labor income rises, this allows a higher level of consumption driving the marginal utility of consumption down. Since marginal future consumption remains unaltered the same has to happen for the marginal utility of current consumption, which has to be achieved by a higher level of all three assets thus that current consumption retains its initial level. In sum only if the immediate impact effect b_{M1_t,Y_t} is sufficiently large, will the correlation between narrow money and labor income be positive.

In close correspondence to narrow money we obtain results for time deposits concerning the impact of the real rate of return on capital and present labor income according to the general formula

$$\Phi^T_j = -\frac{1}{det\left(\mathbf{H}\right)/\left(a_{11}a_{22} - a_{12}a_{21}\right)}\left[b_{Tj} - b_{A_t,j}\left(\frac{a_{31} - a_{32}\frac{a_{21}}{a_{22}}}{a_{22} - a_{12}\frac{a_{21}}{a_{22}}}\right) - b_{M1_tj}\left(\frac{a_{32} - a_{31}\frac{a_{12}}{a_{22}}}{a_{33} - a_{31}\frac{a_{13}}{a_{11}}}\right)\right] \tag{77}$$

However, concerning the impact of expected inflation, inflation risk as well as the covariance, we have already learned that a predominance of the income effect has a higher likelihood for time deposits than in the case of narrow money. Consider the case that the income effect of higher expected inflation (inflation risk) dominates for time deposits but not for narrow money. In this case there is a high likelihood for a positive correlation between time deposits and expected inflation (inflation risk). A further distinction concerns the impact of the safe nominal interest rate on time deposits. Replacing in equation (44) $b_{T_t,j}, b_{A_t,j}, b_{M1_t,j}$ by $b_{T_t,i_{t+1}}, b_{A_t,i_{t+1}}, b_{M1_t,i_{t+1}}$ as given by the equations (52), (50), (51), we observe that if the substitution effect dominates the immediate impact effect of a higher nominal interest rate on time deposits, then both variables will be unambiguously positively correlated. A predominance of the income effect by contrast will establish a negative correlation between time deposits and its safe nominal rate of return only if this immediate impact effect is sufficiently strong.

Summarizing results we may state that a complete solution of the optimization problem taking the intertemporal budget constraint into account, provides a rich variety of possible correlations between narrow money, time deposits and the exogenous variables. Of importance in this respect is the relationship between substitution and income effects which in its turn is affected by the degree of risk aversion. Whenever the household is highly risk averse, it is highly likely that income effects dominate. In this case there is also a high likelihood that our model delivers results which oppose those given by approaches resting on Euler equations.

We also see that due to different direct utilities of both types of monetary assets and different rates of real returns, narrow money and time deposits are imperfect substitutes which complicates the calculation of net effects of changes in the exogenous variables on broad money $M2$. To see this, we have a look at how broad money is correlated with the exogenous variables according to the equations (43) and (77):

$$\begin{aligned}\Phi_j^{M2} = -\frac{1}{\mathbf{H}} \big(& b_{M1_{t,j}} \left[(a_{11}a_{33} - a_{31}a_{13}) - (a_{11}a_{32} - a_{12}a_{31}) \right] \\ & b_{T_{t,j}} \left[(a_{11}a_{22} - a_{21}a_{12}) - (a_{23}a_{11} - a_{21}a_{13}) \right] \\ & b_{A_t,j} \left[(a_{21}a_{32} - a_{22}a_{31}) - (a_{23}a_{11}a_{21}a_{13}) \right] \Big) \end{aligned} \tag{78}$$

It is plausible to assume, and we have done so in calculating the algebraic sign of the Hessian determinant, that the signs of the first and second squared bracket of equation (78) are positive. Unclear remains the algebraic sign of the third squared bracket. Hence we may conclude that whenever narrow money and time deposit change in the same direction, then the net effect on broad money depends on the reaction of capital holdings according to the immediate impact effect $b_{A_t,j}$ and its effects on both monetary assets. If the impact of capital holdings is not sufficiently strong, then we will obtain clear net effects for broad money. This is the case for a higher expected real rate of return on capital, for a higher level of income, for a higher capital market risk. However, the same is not true for a change in the nominal interest rate on time deposits, a change in expected inflation and a change in inflation risk. If the nominal interest rate rises and the substitution effect outweighs the income effect on time deposits, then given unchanged capital holdings, households will rise time deposits but lower narrow money. The impact of a lower stock of capital should not change the fact that both moneys reveal opposing reactions. Hence in this case broad money will reveal a positive correlation with the nominal interest rate only if a positive correlation with time deposits is sufficiently strong. Concerning expected inflation and inflation risk, a dominance of the income effect appears to be more likely for time deposits than for narrow money. Hence it may well be the case that time deposits increase with rising expected inflation and a higher inflation risk, whereas narrow money holdings go down. In this case again the net effect on broad money remains unclear. If the income effects dominate for both types of assets then we may expect positive correlations between broad money and expected inflation as well as inflation risks if the immediate impact effects are strong enough to outweigh the effect of rising capital holdings.

4 Empirical methodology

Our aim is to build a parsimonious econometric model with which to evaluate the effect of both inflation risk and capital market risk on households' money demand in the aggregate. To this end we estimate a single-equation error-correction model as applied recently e.g. by Cook and Choi (2007); Dreger and Wolters (2015). The cointegration approach allows us to model the long run as well as short run dynamics in a flexible manner considering jointly both non-stationary and stationary variables. For the conduct of monetary policy it is particularly important to analyze the short to medium run response of money demand to specific shocks as well (Ball, 2012). Also the cointegration approach allows us to avoid the often problematic estimation of some sort of forward-looking structural model in terms of deviations from a yet to be determined steady-state by means of univariate *ad hoc* filtering which may result in spurious cycles (Phillips and Jin, 2015) and severe estimation bias (Garratt et al., 2012, p. 29).

4.1 The ARDL approach to cointegration

We estimate a Autoregressive Distributed Lag (ARDL) Model of order p and q. For presentation purpose only, the model can be formulated in levels in a simplified form without any deterministics and a single regressor x_t as follows

$$m_t = \sum_{j=1}^{p} \phi_j m_{t-j} + \sum_{j=0}^{q} \theta_j x_{t-j} + u_t, \qquad u_t \sim N(0, \sigma^2),\ t = 1, ..., T \tag{79}$$

which can be re-written as an unrestricted conditional error-correction model (ARDL-ECM)

$$\Delta m_t = \rho m_{t-1} + \theta x_{t-1} + \sum_{j=1}^{p-1} \gamma_j \Delta m_{t-j} + \sum_{j=0}^{q-1} \psi_j \Delta x_{t-j} + u_t \tag{80}$$

where $\rho = -(1 - \sum_{j=1}^{p} \phi_j)$, $\theta = \sum_{j=0}^{q} \theta_j$. The error correction term representing stationary deviations from the long run relationship between m_t and x_t is given by $\widehat{\zeta}_t = m_t - \widehat{\beta} x_t$ where $\widehat{\beta} = -\frac{\widehat{\theta}}{\widehat{\rho}}$ and $\widehat{\rho}$ and $\widehat{\theta}$ are the OLS estimates obtained from eq. (80). Note that the consideration of contemporaneous values of the first differences of the exogenous variable Δx_t allows for correlation between the regression errors and first-differenced regressors and ensures efficient estimates (Shin, 1994).

The validity of this approach when the underlying variables are $I(1)$ has been shown by Pesaran and Shin (1998). Furthermore, the ARDL-ECM-based estimates of the long run coefficients are super consistent and valid inferences via the Wald- or F-Test on the long run parameters can be made using standard normal asymptotic theory as long as the regressors are weakly exogenous (Hassler and Wolters, 2006).[5] Inference on $\widehat{\beta}$ can be conducted by means of the Delta method, as described in Pesaran and Shin (1998), or as conducted in this study, by means of bootstrap methods (Efron and Tibshirani, 1993, ch. 5). Additional $I(0)$ series and deterministic variables can be added without causing further issues for estimation and inference.

In order to determine the optimal lag length of the ARDL-ECM and to obtain a sparse specification, we apply a type of general-to-specific modeling approach as well as automatic outlier detection, as described in detail in the Appendix in Section B.1. Given the small sample size, we provide the bootstrap estimation results of the error-correction adjustment term $\widehat{\rho}$ and the long run coefficients $\widehat{\beta(\cdot)}$ jointly with bootstrap standard errors, the R^2 and the *Hit Rate*. The latter refers to the fraction of correctly predicted changes (negative or positive) of the response variable. A battery of standard specification tests on serial correlation, heteroskedasticity, normality, functional form and parameter stability are performed on the final specification estimated.

[5] An alternative approach could be a system framework in the spirit of a Vector Error Correction Model. However, the system approach is more sensitive to (mis-)specification issues and thus induces further issues. Of course, this does not rule out the use of the VECM framework *per se*. However, the ARDL single-equation framework allows for the mixture of both $I(1)$ as well as $I(0)$ variables in the long run relationship, but the VECM framework is less flexible with respect to this problem.

4.2 The bounds test for long run relations and dynamic multipliers

Classical cointegration methods require all the underlying variables to follow integrated stochastic processes of the same order. The unit-root pre-testing introduces additional uncertainty into the estimation process. Recently Pesaran et al. (2001) have suggested a bounds testing methodology to test for the existence of a long run relationship which is applicable irrespective of whether the underlying regressors are $I(0)$, $I(1)$ or mutually cointegrated.

Consider again eq. (80): The null hypothesis of no long run relationship is stated as H_0^{PSS} : $\rho = \theta = 0$ and can be tested by using a Wald test for which the asymptotic distribution of the test statistics is non-standard. Instead of exact critical values for an arbitrary mix of $I(0)$ and $I(1)$ variables, Pesaran et al. (2001) provide two sets of critical values: one which assumes that all regressors are $I(1)$, and the other one assuming that all series are $I(0)$. If the computed test-statistics falls below the $I(0)$ bound, one can conclude that the variables are $I(0)$, and hence no long run relationship exists. If the statistics exceeds the $I(1)$ bound a long run relationship between the variables exists. The test is inconclusive if the statistics falls within the upper and lower bounds, and some knowledge about the order of integration of the underlying variables will be needed. To improve the power and size of the PSS test under potential heteroskedasticity, we apply a bootstrap version of the PSS test.[6]

We also compute the cumulative dynamic multiplier effects of x_t on m_t in order to study the short and medium term dynamics. The dynamic multipliers for horizon h can be evaluated as follows (see Shin et al., 2014, for details on the exact 1computation):

$$M_h = \sum_{j=0}^{h} \frac{\partial m_{t+j}}{\partial x_t} \text{ , } h = 0, 1, 2, ... \tag{81}$$

Note that, by construction, and $h \to \infty$, $M_h \to \beta$, where β is the long run coefficient.

[6]Furthermore, it was just recently shown by Cavaliere et al. (2014) in a multivariate framework that in the presence of heteroskedasticity in the innovations process, the wild bootstrap approach typically outperforms the *i.i.d.* bootstrap analogue. We expect that this also holds in the univariate context. In the Appendix in Section B.2 the corresponding bootstrap algorithm is described.

4.3 Tests for constancy of the cointegration space

Lastly we test for constancy of the cointegration (β-)parameters as suggested by Hansen and Johansen (1999). The authors propose a LM-type test which examines a recursive sequence of test statistics. Under the null of β-constancy the asymptotic distribution of the test statistics is non-standard and depends on the number of cointegrating relations which is set to one hre. Fortunately, the limiting distributions are independent of nuisance parameters and simple to approximate by simulation.

4.4 Econometric specifications

We proceed with the determination and estimation of possible long run relationships. Re-write the error-correction term in the multiple regression form:

$$m_t = \beta' \mathbf{x}_t + \zeta_t, \ t = 1, ..., T \tag{82}$$

where m_t is the level of real money demand and $\mathbf{x}_t = [x_{1t}, x_{1t}, ..., x_{kt}]'$ is the k-dimensional vector of $I(0)$ and $I(1)$ regressors. The following six long run model specifications are estimated where the vectors of regressors are:[7]

$$
\begin{aligned}
\mathbf{x}_t^{(1)} &= [y_t \quad i_t \quad R_t]' \\
\mathbf{x}_t^{(2)} &= [y_t \quad i_t \quad R_t \quad \pi_t]' \\
\mathbf{x}_t^{(3)} &= [y_t \quad i_t \quad R_t \quad \pi_t \quad \sigma^2_{\pi_t}]' \\
\mathbf{x}_t^{(4)} &= [y_t \quad i_t \quad R_t \quad \pi_t \quad \sigma^2_{r_t}]' \\
\mathbf{x}_t^{(5)} &= [y_t \quad i_t \quad R_t \quad \pi_t \quad \sigma^2_{\pi_t} \quad \sigma^2_{r_t}]' \\
\mathbf{x}_t^{(6)} &= [y_t \quad i_t \quad R_t \quad \pi^e_t \quad \sigma^2_{\pi_t} \quad \sigma^2_{r_t}]' \,.
\end{aligned}
$$

Our monetary aggregate (m_t) comprises the U.S. households' sector demand for M2, as e.g. in Cook and Choi (2007) who also examine the determinants of sectoral money holdings. The benchmark Model 1 includes the standard set of explanatory variables namely real households' disposable income (y_t), the own rate of M2 (i_t) and the real return on capital (R_t) which is defined as the cumulated sum of real stock market returns.[8] Step by step CPI price inflation (π_t) is included before we replace this measure by the well-known survey based inflation expectation measure of the University of Michigan (π^e) as a robustness check in Model 6.

Central to our argument is the consideration of both inflation risk and capital market risk for money demand. First, following Stock and Watson (2007), we approximate inflation risk ($\sigma^2_{\pi t}$) as the time-varying variance of the permanent component of CPI inflation estimated by means of a univariate unobserved component model with stochastic volatility (UC-SV).[9] This approach

[7]As can be seen, the expectational variables, $E_t(x_{t+1})$, were replaced by its realized values, x_t for the following reasons. First, in some cases it may be hard to find a reasonable proxy capturing expectations. Using realized future values imposes the rational expectation assumption, and also requires the GMM method which is not studied for the cointegration analysis. Furthermore, if the respective variables follow a random-walk process, then current values are as good as any realization of the respective variable as a predictor. Also in stock market practice, historical realizations play an important role for analyzing relationships as well as forecasting exercises. Lastly, as long as expectational errors are stationary, at least the long run analysis should be invariant to the use of realized instead of expected values (Carstensen, 2006, p. 396).

[8]We had to replace the stationary real stock market return series (which is clearly $I(0)$) by its cumulated $I(1)$ counterpart to obtain a reasonable specification and economic meaningful parameter estimates. Note, this is closely in line with the modeling strategy as in Pesaran et al. (2000) who replace the stationary log-change in oil prices as modeled in Johansen and Juselius (1992) by its cumulated and hence non-stationary series.

[9]We thank Peter Summers for providing his `gretl` code to us.

models heteroskedasticity in inflation explicitly and might be preferred to e.g. standard (S)VAR models based on the restrictive assumption of homoskedasticity (Chua et al., 2011). We again refer to the Data Appendix for details on this model. The studies by Wright (2011) and Dovern et al. (2012) have applied this model to inflation series before. Among others Grimme et al. (2014) interpret the permanent component as a measure of inflation uncertainty.

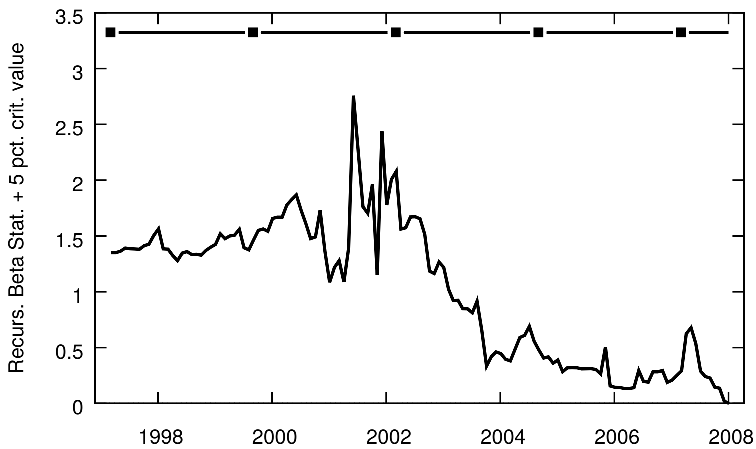
Second, capital market risk (σ^2_{rt}) is measured by the well known VXO implied volatility index based on trading of S&P 100. This index is a key measure of market expectations of near-term volatility, and was also used by Bloom (2009) for evaluating the repercussions of uncertainty shocks for the business cycle. A detailed summary of our data sources and of the transformations applied to each series can be found in the Data Appendix, and all time series are depicted in Figure 3 in the Appendix.

5 Estimation results

Our sample covers the period 1986M1–2007M12 at monthly frequency. Our choice to end the sample in 2007M12 is made in light of both theoretical and practical concerns. Monetary policy since the GFC has been characterized by a ZLB environment in which a combination of quantitative easing and forward guidance has emerged as the preferred policy. Hofmann and Bogdanova (2012) stress that no systematic relationship can be discerned between the majority of standard macroeconomic indicators at this time. Several models comprising a longer sample ending 2014 were estimated but our experience indicated multiple structural breaks which are complicated to model in a satisfactory way. However, a thorough modeling of structural breaks goes beyond this paper which just seeks to illustrate the role and relevance of capital market risk and inflation risk for money demand. Details of the data sources including series codes and any required transformations are collected in the Data Appendix.

5.1 Model specification test results and in-sample fit

Table 1 provides the estimation and specification test results for all six models. Among all models there is only Model 6 which passes the tests on remaining serial correlation, correctly specified functional form and the PSS bounds test for a long run relationship (for both the restricted intercept and restricted trend case, respectively). Additionally we do not find statistical evidence for a structural break in the long run relationship for Model 6 as depicted in Figure 1. While the R^2 is only 0.2 for Model 1 it increases to 0.43 for Model 6. Furthermore, the bootstrap mean value of the *Hit Rate* is 77% for Model 6 in contrast to the benchmark case (69%). These first results may indicate that the joint consideration of both capital market and inflation risk help to restore a plausible and stable households' money demand function for the U.S. economy.



NOTE: A recursive estimation of the Cointegrated Vector Autoregressive Model testing for stability of the cointegration relationship as suggested by Hansen and Johansen (1999). The optimal lag length is determined by means of the AIC criteria based on a VAR with 3 lags in levels. A single cointegrating vector is assumed. The 5% critical value is reported.

Figure 1: Recursive Test for Constancy of the Cointegration Space of Model 6. Sample: 1986m1 – 2007m12.

The error-correction speed evolves quite slowly as only about 6% of the long run disequilibrium are reduced each month which is in line with findings e.g. by Higgins and Majin (2009, p. 1326) and Dreger and Wolters (2015, see Table 2 there). The long run income elasticity of money demand is about $\beta(y) = 1.26$ for Model 6 and significant at the 1% level. The mis-specification of Models 2, 3 and 4 is obvious as the income elasticity approaches 2 for these models which is implausibly high. Note, that an income elasticity of money demand (slightly) above unity is often interpreted as signaling omitted wealth effects (Coenen and Vega, 2001), and were also found for the U.S by Greiber and Lemke (2005) and Dreger and Wolters (2015).

We find a positive and significant long run effect of the own rate ($\beta(i)$) on money holdings for Model 6 such that a percentage point increase in the own rate yields a long run increase in money demand by about 2.8%. An increase in cumulated real stock market returns by a percentage point ($\beta(R)$), results in a significant (at the 1% level) decrease in money holdings in the long run by about 2.4%. With regard to expected inflation, the results indicate a significant and negative long run semi-elasticity of money demand which is $\beta(\pi^e) = -0.043$ (significant at the 5% level), implying that a percentage point increase in inflation is associated with a reduction in the stock of money holdings by about 4% in the long run. Again recall that in correspondence with our theoretical analysis this suggests the dominance of substitution effects.

5.2 A change in capital market risk

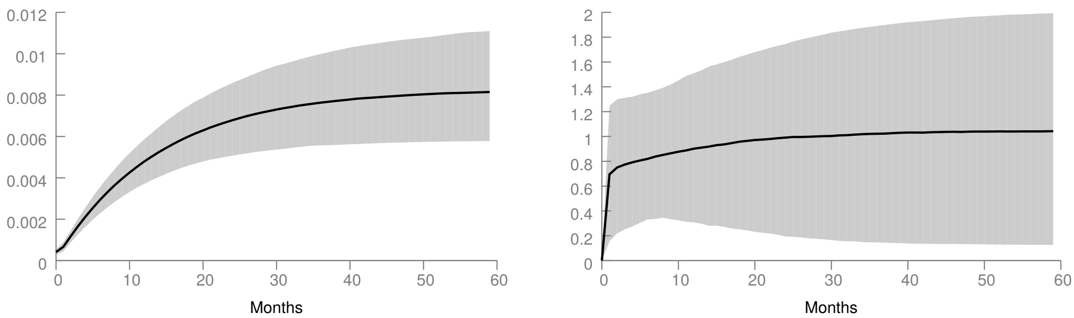
The results indicate that U.S. households adapt their money holdings in response to changes in capital market risk in the long run. The estimated long run multiplier of Model 6 is $\beta(\sigma_r) = 0.008$ saying that a unit increase in the VXO measure results in an increase in money holdings by about 0.8% (significant at the 1% level) which is substantial given that the standard deviation of the change in the VXO index is 3.7 units.

Figure 2(a) depicts the cumulative dynamic multiplier of a change in capital market risk. A permanent unit increase in capital market risk reveals an instantaneous and persistently positive reaction of money demand, which is in line with our theoretical model, and which confirms Greiber and Lemke (2005) who also find a positive long run relationship between their capital market risk measure and *economy-wide* M2. It should be mentioned that also Cook and Choi (2007, Table 7) find a positive long run response of M2 money demand of the U.S. household sector to a positive change in default risk but a negative one to an increase in stock market liquidity risk based on a quarterly sample covering the period from 1970q1 to 2005q4.[10]

5.3 A change in inflation risk

U.S. households demand more safe assets in form of higher money holdings in response to a positive change in inflation risk in the long run as reported in Table 1 for Model 6. The corresponding long run coefficient $\beta(\sigma_\pi)$ is significant at the 5% level, and the point estimate implies that a 0.01 unit increase in inflation risk results in a 1.04% increase in money demand. Again, the effect is substantial given a standard deviation of 0.007 of the change in inflation risk, $\Delta\sigma_\pi$.

The dynamic multiplier analysis also reveals an instantaneous and persistent increase in money demand holdings as reported in Figure 2(b). The positive short run dynamics confirm the previous findings by Higgins and Majin (2009) who considered inflation risk only as part of the transitory model component but neglected its eventual permanent effect on money demand. Providing an interpretation of the empirical results in accordance with our theoretical model, we observe that contrary to obviously strong substitution effects for income and real interest rate variables, the evidence points to dominating income effects for inflation risk in our sample.



(a) Capital market risk shock: $\sigma_r \to m$

(b) Inflation risk shock: $\sigma_\pi \to m$

NOTE: The 90% Efron percentiles are based on a wild bootstrap method using 1999 iterations. The black line refers to the median value.

Figure 2: Cumulative dynamic multipliers of money demand to different shocks. Results based on estimation of Model 6. Sample: 1986m1 – 2007m12.

[10]The default risk is defined as the spread between yields on BAA-rated bonds and the AAA-rated bond. Their stock market liquidity measure refers to the work by Amihud (2002).

(A) Estimation Results

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
ρ	-0.043	-0.021	-0.024	-0.034	-0.049	-0.065
$CI(\rho)$	$(-0.077/$	$(-0.042/$	$(-0.048/$	$(-0.051/$	$(-0.069/$	$(-0.088/)$
	$-0.021)$	$-0.008)$	$-0.008)$	$-0.022)$	$-0.034)$	$-0.047)$
$\beta(y)$	0.886^{***}	2.088	1.895	2.110^{***}	1.260^{***}	1.268^{***}
	(0.297)	(2.264)	(4.103)	(0.456)	(0.307)	(0.276)
$\beta(i)$	0.025	0.130	0.113	0.044^{**}	0.028^{**}	0.028^{***}
	(0.016)	(0.228)	(0.244)	(0.020)	(0.013)	(0.011)
$\beta(R)$	-0.008	-0.029	-0.027	-0.035^{***}	-0.025^{***}	-0.024^{***}
	(0.006)	(0.032)	(0.053)	(0.009)	(0.005)	(0.005)
$\beta(\pi)$		-0.151	-0.132	-0.039	-0.031^{*}	
		(0.308)	(0.309)	(0.024)	(0.016)	
$\beta(\sigma_\pi)$			0.076		1.365^{**}	1.048^{*}
			(5.341)		(0.667)	(0.587)
$\beta(\sigma_r)$				0.014^{***}	0.011^{***}	0.008^{***}
				(0.004)	(0.003)	(0.002)
$\beta(\pi^e)$						-0.043^{**}
						(0.021)
R^2	0.20	0.37	0.35	0.46	0.48	0.43
	(0.13/0.28)	(0.27/0.45)	(0.27/0.44)	(0.36/0.54)	(0.38/0.56)	(0.32/0.53)
Hit Rate	0.69	0.76	0.75	0.80	0.79	0.77
	(0.64/0.74)	(0.71/0.80)	(0.71/0.79)	(0.76/0.84)	(0.75/0.83)	(0.73/0.81)

(B) Diagnostic Statistics

$F_{SC(1)}$	0.387	0.710	0.748	0.532	0.390	0.103
$F_{SC(6)}$	0.011	0.141	0.011	0.211	0.199	0.097
χ^2_H	0.000	0.000	0.000	0.000	0.000	0.000
χ^2_N	0.000	0.000	0.000	0.000	0.000	0.000
F_{FF}	0.010	0.002	0.056	0.000	0.003	0.203
$FPSS^b_{rC}$	0.299	0.095	0.286	0.005	0.004	0.004
$FPSS^b_{rT}$	0.033	0.284	0.119	0.003	0.001	0.002

Note: For both ρ and β the bootstrap median values are reported. For ρ, R^2 and the Hit Rate the 90 pct. bootstrap confidence intervals while for β the bootstrap standard errors are reported in rounded parentheses. $***$, $**$ and $*$ denote the 1 pct., 5 pct. and 10 pct. rejection probabilities. All results are based on 1999 stable bootstrap iterations. The optimal lag length of the ARDL(p,q) model as well potential impulse dummies are determined by an automatic algorithm as described in the Appendix. $F_{SC(1)}$, $F_{SC(6)}$, χ^2_H, χ^2_N and F_{FF} denote the p-values for the tests of no serial correlation of order 1(6), White's test of homoskedasticity, the Doornik-Hansen test of residual normality and Ramsey's RESET test of the correct functional form. $FPSS^b_{rC/rT}$ reports the bootstrap p-values of the Pesaran et al. (2001) F-test on cointegration (1999 iterations) with restricted intercept (trend), rC (rT).

Table 1: Estimated Long-Run Coefficients and Specification Test Results. Period: 1986m1 to 2007m12.

6 Concluding remarks

We investigated the demand for narrow as well as broad money both within a theoretical as well as empirical framework. In doing so our primary focus was directed to the impact of inflation and stock market risks. In our theoretical analysis we distinguished between a stochastic stationary state implying constant expectations and variances of the real rate of return on capital and inflation, and deviations from this long run equilibrium marked by varying expectations and risks. Two differences compared to standard DSGE models stand out: First, risk parameters enter the household's objective function directly which is a due consequence of using the certainty equivalent instead of expected utility. This procedure enabled us to give risk parameters an explicit representation in the Euler equations even after linearization around the steady state. Second, the demand for money in our model is the result of a complete solution to the household optimization problem taking the intertemporal budget constraint into account. This implies that the impact of rates of return as well as risk parameters on money demand do not only depend on substitution effects but also on income effects. Most notably both effects proved to be countervailing leading to ambiguous results with the household's degree of risk aversion playing a relevant role. In particular we were not able to rule out a higher demand for narrow money and time deposits due to higher inflation risks, which has to be expected whenever households are highly risk averse.

We applied an error-correction model to study the effects of inflation risk and capital market risk on U.S. households' money demand over the period 1986M1–2007M12. Inflation risk is measured by the standard deviation of the permanent CPI price inflation component using the unobserved-component-stochastic-volatility model proposed by Stock and Watson (2007) while capital market risk is approximated by the well-known VXO implied volatility measure. Both inflation risk and capital market risk enter the long run money demand relationship significantly implying that the empirical steady-state is not characterized by a fixed-point with full certainty as higher moments of shocks play a role as described in our theoretical model. This result questions the frequent theoretical assumption that the (deterministic) steady-state incorporates no information about the stochastic nature of the economic environment. Future theoretical research should consider this perspective if it wants to build more realistic models which are closer in line with empirical evidence. In line with the literature, we obtain a positive reaction of households' money demand to a positive capital market risk change. By contrast concerning a higher inflation risk, we find evidence for a positive reaction in demand for M2 money holdings which is due to the domination of the income effect.

Our paper provides a foundation for continuing research and we conclude by mentioning three promising avenues. First, by incorporating the theoretical into a complete macroeconomic model it would be interesting to study the general-equilibrium effects. This model could then be estimated by means of a system modeling framework. Second, for further analysis of the recent financial crisis one would need to apply a time-varying estimation framework to control for structural changes in the effects as it is plausible to assume that the households' degree of risk-aversion is state-dependent.

The policy implications of our model and the estimation results are as follows: As the cost of investing in stocks and bonds has declined and households hold broader sets of monetary assets, it can be argued that money holdings may have become more sensitive to financial as well as inflation risk (Cook and Choi, 2007). It might be of particular interest for policy makers that a positive correlation between inflation risks and money holdings by the household sector moderates a threat to price stability due to higher inflation expectations. Our results also support the argument in favor of including financial stability measures into a central bank's objective function, as the stabilization of financial markets can be seen an additional pillar for ensuring price stability (Cronin et al., 2011).

Data Appendix

All, except two series, were collected from the Federal Reserve Economic Data Service. The variables are defined as follows:

Real money demand, m_t, is the difference between M2 money stock (FRED: M2, SA) and the sum of demand for money by the firm sector which consists of the sum of time and saving deposits held by nonfinancial corporate business (FRED: NCBTSDQ027S, SA) and nonfinancial noncorporate business (FRED: NNBTTDQ027S, SA) as well as money market mutual fund shares of both the nonfinancial corporate business (FRED: NCBMASQ027S, SA) and nonfinancial noncorporate business (FRED: NNBMFTQ027S, SA). The resulting nominal series is seasonally adjusted by X-13-ARIMA before deflated by the CPI price deflator (FRED: CPIAUCSL, SA) and logged.

Real disposable income, y_t, is the log of real disposable income (FRED: DPIC96, SA).

Price inflation, π_t, is measured by the year-over-year percentage change in the CPI price deflator (FRED: CPIAUCSL, SA). A second measure of price inflation is the median expected price change next 12 months, π_t^e, stemming from the Surveys of Consumers (FRED: MICH, NSA).

The own rate, i_t, refers to the own rate of M2 (FRED: M2OWN, NSA).

The real stock market rate of return, r_t, is the nominal rate of return (year-over-year) of the S&P 500 Stock Price Index (seasonally adjusted by X-13-ARIMA.) plus dividends on S&P 500 (both data are available at: `http://www.econ.yale.edu/~shiller/data/ie_data.xls`) The nominal rate of return is deflated by the year-over-year CPI inflation rate, π_t.

Capital market risk, σ^2_{rt}, is measured by the VXO CBOE Market Statistics (FRED: VXOCLS, NSA).

The inflation risk measure, $\sigma^2_{\pi t}$, was estimated by means of the Stock and Watson (2007) UCSV model. The setup of the UCSV model is as follows: It is assumed that the series of interest, x_t, can be decomposed into a permanent and transitory component with time-varying volatility. Allowing for time-variations is based on the empirical fact that parameter shifts in the estimated variances of the components have occurred over time for the U.S. economy (Stock and Watson, 2007). The dynamics of inflation closely follow an integrated moving-average process which can be re-written as an unobserved component model. It is assumed that x_t is driven by a stochastic trend, τ_t, with serially uncorrelated innovations η_t. The stochastic trend is driven by another white noise innovation ϵ_t:

$$x_t = \tau_t + \eta_t \tag{83}$$

$$\tau_t = \tau_{t-1} + \epsilon_t \; . \tag{84}$$

Both innovations η_t and ϵ_t are *i.i.d* normally distributed. Furthermore, the logarithms of the variances of both the transitory part, $\sigma^2_{\eta,t}$ ($\eta_t \sim N(0,\sigma^2_{\eta,t})$), as well as permanent part, $\sigma^2_{\epsilon,t}$ ($\epsilon_t \sim N(0,\sigma^2_{\epsilon,t})$), evolve as separate random-walks according to:

$$log\ \sigma^2_{\eta,t} = log\ \sigma^2_{\eta,t-1} + \nu_{\eta,t} \tag{85}$$

$$log\ \sigma^2_{\epsilon,t} = log\ \sigma^2_{\epsilon,t-1} + \nu_{\epsilon,t} \; . \tag{86}$$

The innovations to the variances, $\nu_t = (\nu_{\eta,t}, \nu_{\epsilon,t})'$, are *i.i.d.* $N(0,\gamma I_2)$ and orthogonal to each other. The parameter γ controls the smoothness of the stochastic volatilities $\sigma^2_{*,t}$. The model is estimated using the Gibbs sampling approach. We fit the UCSV(0.2) model to our CPI price inflation time series, π_t using a prior for the initial condition of $\gamma = 0.2$.[11]

[11] This prior was also used by Stock and Watson (2007) for GDP inflation. We found that the results were robust against different prior values.

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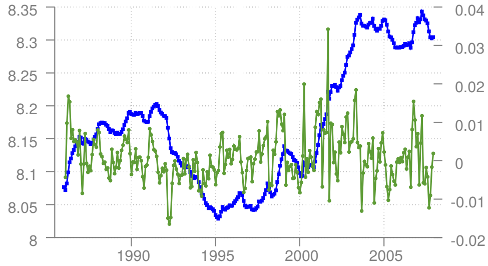
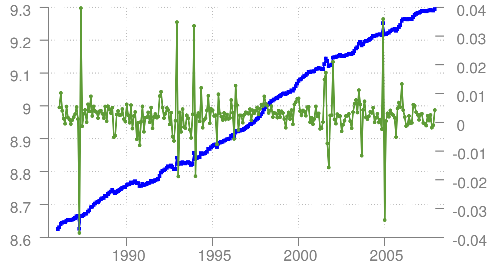
Appendix

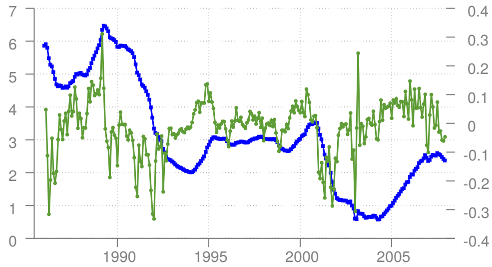
A Figures



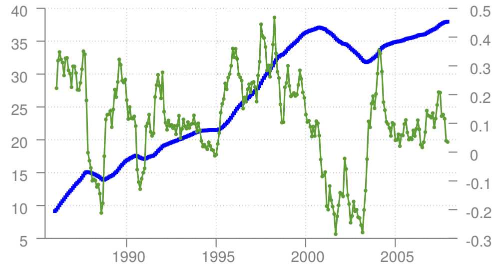
(a) m: Log of Real M2 (HH sector)



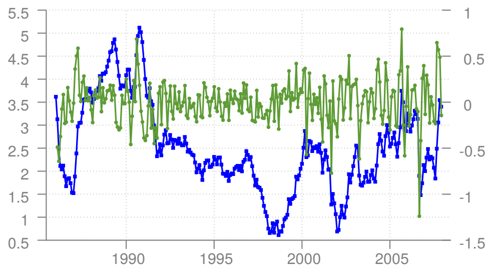
(b) y: Log of Real Disposable Income



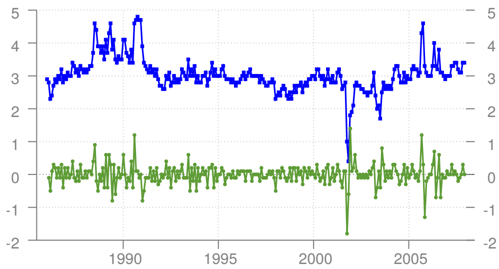
(c) i: Own Rate of M2



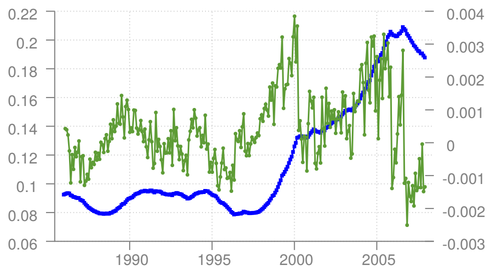
(d) R: Cumulated Real Stock Market Returns



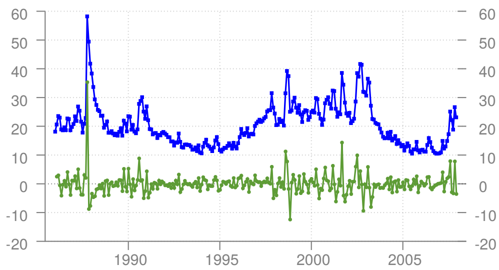
(e) π: CPI Price Inflation



(f) π^e: Expected Price Inflation



(g) σ_π^2: Inflation Risk



(h) σ_r^2: Capital Market Risk

Figure 3: Time series plots of the level variables (blue line) and its corresponding first difference. If a second left-hand y-axis is given, it refers to the first difference variable. Sample: 1986m1 – 2007m12.

B Routines

B.1 Notes on the General-To-Specific Algorithm and Outlier Detection Procedure

The following algorithm is applied to determine the lag order of the ARDL(p,q) model as well as the need for impulse dummy variables:

1. Estimate the ARDL(p,q) and set the lag length to $p = q = k$ where k is an integer value and $k = 1..4$. The BIC information criteria is used to select the lag length which minimizes the BIC criteria. The maximum lag order tested is $k = 4$. The optimal lag order is denoted by ARDL(p^*,q^*).

2. Store the residuals $\widehat{u}$ of the estimated ARDL(p^*,q^*) model. Create impulse dummies taking unit for observations for which $\widehat{u}_t >= 2\sigma(\widehat{u})$, otherwise zero, where $\sigma(\widehat{u})$ refers to the estimated standard deviation.

3. Re-estimate the ARDL(p^*,q^*) model including all dummy variables determined in the step before. Sequentially eliminate the dummy variables with a p-value greater 0.1, until all remaining dummy variables have a p-value not greater than 0.1.

B.2 Notes on Pesaran-Shin-Smith Wild Bootstrap Test on Cointegration

The bootstrap estimator of the cointegration relationship, denoted $\widehat{PSS}_b$ in what follows, iterates over the following steps:

1. Estimate model (87) under null hypothesis $H_0 : \rho = \theta = 0$ using OLS yielding the estimates $\widehat{\gamma}_1^r, ..., \widehat{\gamma}_{p-1}^r$ and $\widehat{\phi}_1^r, ..., \widehat{\phi}_{p-1}^r$ together with the corresponding residuals $\widehat{u}_t$:

$$\Delta y_t = \rho y_{t-1} + \theta x_{t-1} + \sum_{j=1}^{p-1} \gamma_j \Delta y_{t-j} + \sum_{j=1}^{q-1} \phi_j \Delta x_{t-j} + u_t \qquad t = 1, ..., T \tag{87}$$

   where the initial values, $y_{1-p}, ..., y_0$ and $x_{1-q}, ..., x_0$, are taken to be fixed in the statistical analysis.

2. Construct the bootstrap sample, $\{y_t^*\}$, recursively from the first step with the T bootstrap errors u_t^*, generated using the re-centered residuals, $\widehat{u}_t^c := \widehat{u}_t - T^{-1}\sum_{i=1}^{T}\widehat{u}_t$, for the wild bootstrap, where for each $t = 1, ..., T$, $u_t^* := \widehat{u}_t^c w_t$, where w_t, $t = 1, ..., T$, is an i.i.d. $N(0, 1)$ sequence.

3. Using the bootstrap sample, $\{y_t^*\}$, estimate model (87) under the alternative $H_1 : \rho \neq \theta \neq 0$ using OLS. Check that the error-correction term $\rho <= 0.0001$ and that stability is ensured. If the condition is fulfilled, proceed with the next step, otherwise go back to step 2 and draw from another set of residuals.

4. Using the bootstrap sample, $\{y_t^*\}$, compute the bootstrap PSS test statistics, $\widehat{PSS}_b$.

5. Repeat steps 1 to 4 B times.

6. The bootstrap p-value is computed as $F_{PSS}^b = \#\{\widehat{PSS}_b \geq \widehat{PSS}\}/B$ where $\widehat{PSS}$ is the observed value of the statistics.