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The Real Income Shares of Labor, Human and Physical Capital: Determination Method and First Results for Germany

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The Real Income Shares of Labor, Human and Physical Capital
Determination Method and First Results for Germany*

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Abstract

A method is presented that allows to separate the total labor income into parts of basic labor and human capital using annual micro data. As results yearly total income shares of physical and human capital and labor are obtained for a single country.

The method is applied to Germany using micro data of the years 1976, 1985, 1995, and 2006. The obtained average income shares are in agreement with the well known results of Mankiw, Romer and Weil [8] if only employed workers are considered.

If self-employed labor is also taken into account, the share ratios of physical and human capital and labor change to $s_K : s_H : s_L = 0.18 : 0.26 : 0.55$. This result differs considerably from the generally expected share ratios for developed countries of $1/3 : 1/3 : 1/3$.

Further on, the development of the German income shares are investigated. The observed variation is in contradiction to a constant behavior as expected from Kaldor’s stylized facts. The source could be traced to considerable changes in the qualification structure of the German work force.

Keywords: human capital, Mikrozensus, annual factor income shares, factor share development

JEL classification: D33, E25, J24

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1 Introduction

Considering only capital $K$ and labor $L$ as production factors their total income shares - $s_K$ and $s_L$ - are found to be constant. For the USA one finds $s_L : s_K \approx 2/3 : 1/3$ [5]. A similar behavior is observed for other industrial countries.

In 1992 Mankiw, Romer and Weil [8] showed that human capital $H$ is a further important production factor that needs considering. They assumed a Cobb-Douglas production function with production factors $K$, $H$, and $A-L$ (A being a Harrods-neutral technical progress that results in an enhanced $L$-wage). They analyzed economic data of 98 different countries from 1960 - 1985. They obtained ratios of income shares $s_K : s_H : s_L$ compatible with equal shares of $1/3$. Their final conclusion was that $H$ as production factor is necessary to explain cross country differences.

Human capital is considered to be the dominant driving force of economic growth. According to Le, Gibson and Oxley [7] the determination of $H$ is essential in order to establish the correlation between $H$ and growth. Folloni and Vittadini [3] have provided an extensive survey on procedures for human capital measurements. They differentiated 5 methods for obtaining a measure of $H$:

1. retrospective method:
   measure of $H$ is the present value of the costs of formation of the current stock of $H$,

2. prospective method:
   measure of $H$ is the present value of the lifelong income of the workers,

3. individual investments:
   measure of $H$ is $w(s,x)$, the wage as function of the number of schooling years ($s$) and the number of years with professional experience ($x$),

4. educational attainment level:
   possible measures of $H$ are attainment levels, like fractions of population above certain educational level (e.g. reading capability), GDP-fraction of educational costs, ratio of teachers per pupil/student,

5. $H$ as latent variable:
   there is no direct measure of $H$; instead it is assumed to be an unknown function of several qualitative and quantitative parameters.

All these different methods aim at determining measures of the aggregate $H$. The result of empirical studies of a correlation with economic growth are controversial or insignificant (see e.g.[6]). Folloni and Vittadini state[3](p. 265): “Empirical studies have demonstrated the existence of wide differences between micro analysis (micro data) and those of macro ones. Micro data find substantially positive impact of educational attainment on earnings; macro studies show very controversial results”. The origin is probably the qualitative difference between economic growth (a dynamic variable) and aggregate $H$ (a rather constant variable w.r.t. growth). Therefore the main mode of operation of $H$ is believed to work via external effects. These are difficult to determine and to valuate.

A possible alternative has been proposed by the OECD [13]: the ratio of higher educated to lower educated workers may serve as measure of $H$ of the higher educated workers. As result one could obtain a value for the human capital stock: “by weighting different segments of the workforce by the ratio of earnings at different levels of education, it is possible to derive an index of the value of average human capital stock” (p.28).

A similar approach for measuring the level of $H$ has been proposed by Mulligan and Sala-i-Martin [12]. They use the total income ratio of all workers and those without schooling as measure of the aggregate value of $H$. So the income of workers with no education serves as unit. This is based on the assumption that a person without schooling is the same always and everywhere. They further assume that the such determined $H$ is a measure of the aggregate $H$.\[1\]
B. Jeong [4] developed this method further in order to compare H across countries implying, however, more stringent assumptions, like a fixed income share of H independent of time and country.

However, also these approaches assume to determine a measure of the aggregate value of H.

In this paper - in contrast to the above approaches - H is restricted to its productively used fraction, a small part of the aggregate H. It increases the productivity of workers and results in a higher wage. This has been shown by Jacob Mincer [11] (and others later on) who demonstrated a positive relation between wage and the years of education and work time.

The productively used H is expected to show a dynamic behavior similar to economic growth: it can be increased in rather short time by activation of yet unused parts of H for production, while the aggregate H remains with little changes.

In the following a method is presented for separating the wage (W) into components of basic wage (wo) from pure labor and a surplus wage from the use of human capital in production (wh) (chapter 2.1). Yearly micro data are used to determine W from the wage of all employed workers and wo from the wage of workers with a minimum of schooling and without professional training. These workers did not invest in H and therefore are assumed to earn no surplus wage from productively used H. wh, the surplus wage, is obtained from the difference: wh = W - wo.

The fractions fo = wo / W and fh = wh / W are used to separate the usual labor share of the NNP (net national product), the total income of the economy, into the shares of basic labor and human capital. This procedure is expected to transfer the correlation between H and income from the micro to the macro level: H-investment and earnings (micro level) to the H-share of total income (sH) (macro level). The size of sH demonstrates the importance of H for the growth of NNP resp. GDP.

The value of sH itself is suited to be a measure of the productively used H in units of the NNP of the economy. Annual measures together with sK and sL will serve in understanding the development of a single economy.

Details of the determination of wages and total income shares are described in chapter 2.2, followed by a discussion of the method (chapter 2.3).

As an example the German Mikrozensus (MZ) data of 1976, 1985, 1995 and 2006 are used in order to determine the wage parameters W, wo, wh, fo and fh (chapter 3.1).

First the wage parameters of the employed workers and self-employed are determined (chapter 3.2). Then the wage parameters are used for the determinations of the NNP shares (chapter 4). At first only the L-share of the employed workers (EL) is separated into sL(EL) and sH(EL). The resulting data (chapter 4.1) are compared and found to be in agreement with the MRW-results [8].

Further on the labor and human capital shares of the self-employed workers (SE) are determined (chapter 4.2). In general they are included in the K-share. A separation from the K-share does yield the \textbf{real} sK without admixtures of L and H. It also yields the real shares sH and sL (including contributions of EL and SE).

Significant differences w.r.t. the generally expected ratio of 1/3 : 1/3 : 1/3 are observed and discussed.

The German development from 1976 - 2006 does show a considerable variation in time. This is in contradiction to a constant behavior as expected for developed countries like Germany. This behavior is investigated further using detailed micro data of different groups of qualification (chapter 4.3).

Finally the results for Germany are discussed in chapter 5.
2 Method for Separating the Wage Components of Basic Labor and Human Capital

2.1 Theoretical Background

In this chapter a method is described that allows the simultaneous determination of a basic labor wage and the surplus wage that is paid depending on the productive use of individual human capital.

Following Mankiw, Romer and Weil [8] a production function with the factors K, H, L and an exogenous technology A is considered:

\[
Y = F(K, H, AL) = r \cdot K + r_h \cdot H + Aw \cdot L
\]  

(2.1)

It should be noted that \(Y\) stands for the NNP, the total income of the production factors. The difference to the GDP is essentially due to depreciation of K and the balance of duties and subsidies of the firms. These are not accounted in the NNP in order to put all production factors onto the same footing. (Otherwise depreciation and the balance of duties and subsidies would count as additional rent of K.) In this way the NNP serves as reference value for the shares of the production factors.

Further details are described in appendix A.1.

At the microeconomic level the individual worker earns a basic wage \(w_o\) plus a surplus wage \(w_h\) depending on his individual human capital engagement:

\[
W(i) = w_o + w_h(i) \quad \text{(total wage of indiv. worker i)}
\]  

(2.2)

So a worker without any human capital engagement will earn only the basic wage \(w_o\). This fact is used to determine \(w_o\): the wage of workers without schooling and professional qualification.

At the macroeconomic level the mean human capital is used:

\[
Y = r \cdot K + \sum_{i=1}^{L} w_h(i) + w_o \cdot L
\]  

(2.3)

\[
Y(s_K + s_H + s_L)
\]  

(2.4)

where \(w_h\) is the mean of the individual \(w_h(i)\), and \(s_K, s_H\) and \(s_L\) are the NNP shares. The sum in equation 2.3 is the rent obtained from productively used H.

A simultaneous determination of the total wage \(W\) and \(w_o\) yields \(w_h = W - w_o\). Also the wage fractions \(f_o = w_o / W\) and \(f_h = w_h / W\) are determined. They are used in separating the labor income into contributions from H and L.

It should be noted that the assumed Harrods neutral technology implies factor neutrality. In case of non-neutrality there will be efficiency differences between all production factors. In this case the different factor shares may have technology enhanced return rates.

2.2 Determination of Wages and Income Shares

The wages are determined from income distributions. The income is stated as monthly or annual income in the micro data. They have to be converted to hourly wages. This will yield results independent of variations in working time which depends on individual choice or general agreements.

The labor income is stated in the micro data only for the EL (employed worker). The labor income of SE (self-employed) is part of their total income that also comprises the rent of their investments. Their labor income has to be estimated.

The following procedure is used:
1. determine the wages $W$, $w_o$, $w_h$ and the wage fractions $f_o$ and $f_h$ for the EL using the income distributions from the micro data.

2. estimate the labor wages and wage fractions of the SE. Here the assumption $W_q(\text{SE}) = W_q(\text{EL})$ is used; so SE and EL of the same qualification get the same wage per hour. The following arguments apply: the self-employed wage is certainly not lower, otherwise the running of ones own firm would not make sense; it might be higher because of a higher efficiency, but this is not accessible and is ascribed to individual investment.

In case of a minimum qualification the above assumption yields

$$W_{q_{\text{min}}}(\text{EL}) = w_o(\text{EL}) = w_o(\text{SE}) = w_o,$$  \hspace{1cm} (2.6)

so the basic wages of EL and SE are identical. It is determined from the EL-data.

$W(\text{SE})$ is determined using wage distributions of the EL, weighted according to the qualification structure of the self-employed. Starting point are the wage distributions of employed workers separate for different qualifications $q$:

$$\{n_q(w_i)\}_{\text{EL}}$$

with $n_q$ being the number of entries in wage-bin $w_i$. These distributions are added with weights that reflect the differences in qualification between EL and SE:

$$\{n(w_i)\}_{\text{SE}} = \sum_q \{n_q(w_i)\}_{\text{EL}} \cdot \frac{N_q(\text{SE})}{N_q(\text{EL})},$$  \hspace{1cm} (2.8)

with $N_q$ being the number of worked hours of SE and EL. The such obtained wage distribution of SE is used for determining $W(\text{SE})$ in the same way as $W(\text{EL})$ has been obtained.

After this procedures $W$ and $w_o$ of EL and SE are available for use as input data in the calculation of the NNP shares of $K$, $H$ and $L$.

Further input data are the published annual income share of the EL. These are part of the annual economic summary data provided by statistical institutes, e.g. for Germany the VGR (Volkswirtschaftliche Gesamtrechnung) of the “Statistisches Bundesamt” [15],[17]. Part of these data are the NNP and its shares of labor and capital: $s_K(\text{VGR})$ and $s_L(\text{VGR})$, where $s_L(\text{VGR})$ comprises the sum of $s_L(\text{EL})$ and $s_H(\text{EL})$ and $s_K(\text{VGR})$ comprises $s_K$ and the sum of $s_L(\text{SE})$ and $s_H(\text{SE})$.

The shares of the EL are obtained with

$$s_L(\text{EL}) = f_o(\text{EL}) \cdot s_L(\text{VGR}) \hspace{1cm} \text{and} \hspace{1cm} (2.9)$$

$$s_H(\text{EL}) = f_h(\text{EL}) \cdot s_L(\text{VGR}),$$  \hspace{1cm} (2.10)

so $s_L(\text{VGR})$ is divided up by using the wage ratios $f_o$ and $f_h$ of the EL.

The labor income shares of the SE are determined as follows

$$s_L(\text{SE}) = s_L(\text{EL}) \cdot \frac{N(\text{SE}) \cdot h_w(\text{SE})}{N(\text{EL}) \cdot h_w(\text{EL})},$$  \hspace{1cm} (2.11)

$$s_H(\text{SE}) = s_L(\text{SE}) \cdot \frac{w_h(\text{SE})}{w_o},$$  \hspace{1cm} (2.12)

First the basic labor share $s_L(\text{SE})$ is determined by scaling $s_L(\text{EL})$ with the ratio of numbers ($N$) and weekly working hours ($h_w$) of SE to EL.

Second the human capital share $s_H(\text{SE})$ is determined from $s_L(\text{SE})$ by using the ratio of $w_h$ / $w_o$. 

Finally the real NNP shares of the production factors L, H, K are then obtained by

\[ s_L = s_L(EL) + s_L(SE) \quad (2.13) \]
\[ s_H = s_H(EL) + s_H(SE) \quad (2.14) \]
\[ s_K = 1 - s_L - s_H \quad (2.15) \]

They are called “real” because they are complete shares of the corresponding production factors and they do not contain admixtures from other production factors.

Without the separation of \( s_K \) (from the labor of the self-employed) \( s_K \) would depend on individual decisions to work as employed or self-employed. This decision is influenced by taxes and regulations. This influence on the shares of production factors is avoided by the above procedure.

Further on the income per person (IPP) of the different production factors can be derived from the real income shares:

\[ IPP_L = s_L \cdot \text{NNP}/P \quad (2.16) \]
\[ IPP_H = s_H \cdot \text{NNP}/P \quad (2.17) \]
\[ IPP_K = s_K \cdot \text{NNP}/P \quad (2.18) \]

where \( P \) stands for the population.

In an analogous way IPW can be defined, the income per worker using NNP/worker. These variables can serve as measure of productivity of the production factors in a comparison of different countries.

2.3 Discussion of the Method

The assumptions are:

- there are 3 production factors K, H, L that determine the total output of a country,
- a basic labor income \( w_0 \) is earned by persons of minimal qualification: they did not invest in H (education or professional qualification),
- persons that have invested in H do earn a surplus wage \( w_H \) that is proportional to their productively used H,
- the wage ratios \( f_o \) and \( f_h \) of the EL can be applied to the total labor share yielding \( s_L(EL) \) and \( s_H(EL) \). It is assumed that this procedure can be followed in spite of differences in the data accumulation between the GDP/NNP calculations and the micro data. In consequence this procedure transfers directly the well established correlation between H-investment and income at the micro level to the factor shares at the macro level.

The presented method allows a separation of the NNP into contribution of the production factors K, H and L. The resulting real NNP shares can be determined for a single country using annual micro data. These shares are called “real” in order to distinguish the results from generally quoted results that either refer to \( s_H \) and \( s_L \) of the EL only and to \( s_K \) that comprises admixtures of \( s_H \) and \( s_L \) of the SE.

Important features of the presented method are:

- the real shares do represent the economic status of a country,
- their size demonstrates the importance of the productions factors for size and growth of total income,
- this can be considered as prove of the correlation between H and growth,
- the income shares can be considered as measures of the production factors K, H, and L using the NNP of the economy as unit,
• time series with frequencies of the micro data of the real factor shares become available. They are expected to improve the understanding of the relative importance of the production factors for the development of a country,

• they allow a yearly evaluation of costs and benefits of H-investments for the general public. Surplus taxes obtained from \( w_h \) can be compared to the costs of education, universities, etc,

• they allow a verification of the constancy of the real factor shares. This fact has been stated in Kaldor's stylized facts. It is a basic assumption in numerous theoretical considerations,

• investigations of cross country differences of annual factor incomes should improve the understanding of development differences.

Concerning the comparison of different countries: it has to be taken into account that the used technologies are different in general. This will result in different technology enhanced wages esp. of \( w_o \). Further differences do originate from differences of the minimal qualification: e.g. in countries with obligatory 8 years of schooling, there will be no persons without schooling at all, while in some development countries there can be quite some.

Concerning technology: it is considered to be freely available, however, not necessarily its use. It might not be available for some countries if the amount of \( H \) is not available that is required for its use in production. The above mentioned obligatory years of schooling establish already a kind of minimal \( H \) that will result in a higher \( w_o \) as compared to countries with a considerable amount of persons without any schooling.

3 Wage Components of Basic Labor and Human Capital for Germany

The wage parameters of employed workers are obtained from wage distributions of workers of different qualification levels. These distributions are obtained from German MZ data. This analysis is based on separate data from the years 1976, 1985, 1995, 2006.

The German MZ data contain the results from annual interviews of a representative 1% sample of the population [9]. In this work variables of individuals are evaluated. These are the weight for scaling to the total population, the type of place of living, the occupation, the position in employment, the dominant source of income, the highest level of education and professional training, the net monthly income and the usual working hours per week.

3.1 Micro Data Analysis

In the following details of the data analysis are described.

• Selection Criteria

The following selection criteria are applied:

1. only working persons at the main place of living are taken into account,
2. the statements of all the used variables have to be complete,
3. the dominant source of income is labor,
4. employed or self-employed persons are selected.

Further and more explicit details are described in appendix A.2
• Conversion of Data
The monthly net income values need some conversions for use in further analysis:

1. conversion of the net income to the gross income: including income tax, social and health
   insurances as well as contributions of the employers,
2. conversion of the monthly gross income to the wage/hour:
   \[ \text{wage/hour} = \frac{\text{monthly income}}{4 \cdot \text{hours/week}} \],
3. correction for inflation and conversion into Euro. The deflater of different years are obtained
   from the nominal and inflation corrected GDP data [18]. Reference year is 1991. For this year
   there exist 2 sets of GDP-data: for the Federal Republic alone and for the unified Germany.

Further and more explicit details are described in appendix A.3

• Unified Scale of Qualifications
The scale of the highest level of education and professional training of the yearly data has changed
in time. Therefore, a unified scale of qualification levels is defined for the different years data. It
has 7 levels of qualification. They are shown in table 5.1 for the 2006-data.

Further and more explicit details are described in appendix A.4

Further on the wages of the qualification levels in the 4 different years are also shown in the table.
The correlation between wage and qualification level resp. H-investment is quite evident.

• Evaluation of \( w_o \)
\( w_o \) is the wage of workers with minimal qualification. According to the unified scale these are the
the workers of level \( q0 \). However, a separate class of \( q0 \)-workers does exist in the 2006-data only.
The other yearly data have only information of the combined classes \( q0 \) and \( q1 \).

For the 2006-data there is a difference of 7% between the wages of class \( q0 \) and and the combined
class \( q01 \). It is used as correction factor for the years with combined \( q01 \) classes:

\[
 w_o = w(q0, q1) * \frac{w(q0)_{2006}}{w(q0, q1)_{2006}}
\]  

(3.19)

Further and more explicit details are described in appendix A.5

• Evaluation of Wage Distributions
The two wages \( W \) (all employed) and \( w_o \) (lowest qualification) have to be determined from income
distribution of the two groups. However the shapes of the two distributions differ considerably: the
wage distribution of all workers has a long tail towards high values. The standard mean from this
distribution is systematically biased to higher values. This might also result in a higher volatility of
this value in different years. This is not the case for \( w_o \). This effect is studied by using 5 different
methods (m1 - m5) for determining \( W \) and \( w_o \) simultaneously. The standard mean is method m1.
The use of a logarithmic wage scale (m2) results in a wage distribution of the q0-workers with a
tail to the low side. A mean of this distribution is biased to lower values.
Further on the means of the 10-90-percentiles of linear (m3) and logarithmic (m4) wage distributions
are determined. In this case no bias is expected. The tails have been eliminated.

Finally the median value is evaluated (m5).

These methods are applied to the yearly data for determining means resp. the medians of the wage
distributions. The resulting values are shown in figure 5.1 a.

The development over time of the wages show a rather similar shape. The standard mean of \( W(m1) \)
is considerably higher as expected from the influence of the high tail of the wage distribution. The
wage results of the other methods are close together.
The wage ratio $f_o = w_o/W$ is important for the determination of the NNP shares of basic labor and human capital. The development in time is shown in figure 5.1 b. Results of methods m1 - m4 show the same behavior and very close values. Only the median (m5) gives higher values and stronger fluctuations.

In the further analysis method m3 is selected, ascribing the means of the 10-90-percentiles linear wage distributions to $W$ and $w_o$. Further and more explicit details are described in appendix A.6.

### 3.2 Labor Wages of Employed and Self-Employed

EL wages and wage fractions are obtained from the EL income distributions. These have been obtained following the procedures of the previous chapter 3.1 for the MZ-data of 1976, 1985, 1995, and 2006. The resulting wages and the basic wage, $W$ and $w_o$, have been used to determine $w_h$, and the fractions of basic wage and human capital:

$$w_h = W - w_o,$$

$$f_o = w_o/W,$$

$$f_h = 1 - f_o.$$  \hspace{1cm} (3.20)  \hspace{1cm} (3.21)  \hspace{1cm} (3.22)

The results are shown in table 5.2.

SE wages and wage fractions are obtained following the procedures described in chapter 2.2. $w_o$ is the same for EL and SE. $W$ of the SE is not available from the micro data. It is estimated from EL income distributions for different qualifications. The weighted sum of these yields the SE wage distribution following formula 2.8. The required ratio of work times of EL and SE of different qualifications is obtained from the MZ data. The resulting SE wage distribution yields $W$(SE).

The results are shown in table 5.3.

A comparison of EL and SE wages demonstrates the better qualification structure of the SE. It should be further noted that the wages are stated per working hour. A comparison of yearly incomes does result in $\approx30\%$ higher SE-wages as the work time per week of the SE is $\approx30\%$ higher. This will manifest in higher values of the labor income shares of the self-employed.

### 3.3 Development of Labor Wages

The development of the hourly wages of employed and self-employed are shown in figure 5.2 a. The total wage $W$ shows a rather constant growth as expected from a growing NNP resp. GDP.

The wage component of human capital $w_h$ shows for the self-employed considerably higher values as compared to the employed. This originates from the higher qualification of the SE.

The basic labor wage $w_o$ is the same for employed and self-employed. It shows a similar growth as the total wage, apart from the last period: there a flattening is visible. This is compensated or taken over by an increase of $w_h$. So human capital seems to gain more importance in the last period.

The development in the last period from 1995 - 2006 is even stronger visible in the behavior of the fractions of basic labor ($f_o$) and human capital ($f_h$) as shown in figure 5.2 b. Here a change of the fractions of 0.053 is observed (see also tables 5.2, 5.3). This is a change by +19% for $f_h$, resp -7% for $f_o$; a considerable change for a period of 11 years. This strong change will enter directly into $s_L$ and $s_H$. It will be further discussed in chapter 4.3.

### 4 Total Income Shares of Basic Labor, Human and Physical Capital for Germany

Important macroeconomic parameters are the total income shares of the production factors. They are the rent of the engaged factors and can be considered as measure of its size and of its importance for the total
output of an economy. The temporal development of the income shares demonstrates the importance of the respective factors for growth.

The above determined wage components of basic labor and human capital are used to obtain the total income shares. The important assumption is that the relation of basic labor to human capital \( f_o \) and \( f_h \) - are also valid for the VGR-data, esp. for the wage of all employed workers.

A direct verification of this assumption is not possible because the Mikozensus data sample is smaller. This is due to the selection process (chapter 3.1). Further on the determination of number and income of employed workers in the VGR originates from different sources, e.g. tax-offices and statements from interrogations of representative firms. In addition the employed workers sample of the VGR comprises also apprentices, persons doing military service, working pensioners and students. These add up the number of employed workers but the increase of total income is minor.

4.1 The Income Shares of Employed Labor

The wage fractions \( f_o \) and \( f_h \) are stated in table 5.2. Following the description of chapter 2.2 the shares \( s_H(EL) \) and \( s_L(EL) \) are determined. The average shares are shown in table 5.4 together with the results of Mankiw, Romer and Weil[8] and the reanalysis of Acemoglu [1]. It should be noted that \( s_K \) in the table for this comparison still comprises the admixture of \( H \) and \( L \) of the SE.

The obtained shares are in agreement with the published ones. However, it has to be taken into account that the result of this work refers to Germany alone while the published results imply a different model with stringent assumptions and an average over about 100 countries.

4.2 Income Shares of Self-Employed Labor and Physical Capital

Following the description of chapter 2.2 the real shares \( s_K \), \( s_H \) and \( s_L \) are determined, using the wages and wage fractions of EL and SE (see tables 5.2 and 5.3).

The resulting average real shares have been added to table 5.4. The difference to the published results shows a remarkable difference esp. for \( s_K \). It drops from 0.31 to 0.18. The difference is due to \( H \) and \( L \) of the SE, which is eliminated from \( s_K \). Instead \( s_H \) and \( s_L \) are increased by the contributions from the SE.

The German real factor shares demonstrate their relative importance for the growth of the country. Most important is \( s_L \) with more than 50%. However it has to be noted that this is a combined effect of basic labor and technology which enhances productivity and the basic wage. Further on \( s_H \) is an important factor that contributes about 25%. It is higher than \( s_K \) which contributes only 18% to the total income and its growth.

4.3 Development of the Income Shares

In the period of 30 years from 1976 - 2006 the GDP and NNP have increased by 98%. The income per worker has increased by 33% [16]. The NNP per person and per worker are shown in table 5.5.

The real shares have been determined for the different years separately. They are shown in figure 5.3. Detailed numbers are also stated in table 5.5.

The development of the NNP and income shares show a different behavior in the 3 periods:

1. 1976 - 1985
   This period has been considered as reference period. The NNP/worker grows by 12.7% corresponding to a yearly growth of 1.3%. \( s_H \) stays nearly unchanged while \( s_K \) gains 0.015 at the expense of \( s_L \).

2. 1985 - 1995
   In this period the German unification took place. Overcoming the problems should dominate the economic behavior. In fact the NNP/worker grows only by 2.9% corresponding to a negligible yearly growth of 0.3%. Also \( s_K \) stays nearly unchanged. However \( s_L \) gains 0.033 on the expense of \( s_H \).
3. 1995 - 2006

In this last period one should expect that the problems of the unification have been overcome. The NNP/worker grows again by 14.7% corresponding to a yearly growth of 1.3%. So in this period the same growth rate as in the first period is observed. However the income shares show considerably different changes: $s_L$ suffers a big loss of 0.095 while $s_K$ profits enormously from a gain of 0.080 and $s_H$ from a gain of 0.015.

According to Kaldor’s findings [5] one should expect constant income shares in the time development. The lower average value of $s_K \approx 0.16$ w.r.t. Kaldor’s $1/3$ is due to the separation of the self-employed labor from the K-share. However, a constant behavior of $s_K$ is observed only in period 2, the unification period. In this period a deviation from constancy could be easily accepted. But it did not occur. On the contrary a strong increase of the K-share in periods 3 is observed. It amounts to a relative increase of nearly 50%. Also in period 1 an increase is observed that amounts to 10%. This indicates large redistributions of the factor shares from the labor sector to physical capital.

This behavior is in contradiction to the expected constancy of the income shares. It seems to violate a basic principle of the economic understanding of growth: developed countries, like Germany, are expected to be in a steady state. In this state the ratios of the production factors are constant. Growth should take place only by progress in technology without changes of the factor shares. A considerable redistribution of them, as observed here for Germany, does not fit into the generally accepted picture.

However, if a progress in technology results in large structural changes, stronger disturbances of the factor shares might occur. This could have happened in the last period when globalization and the so-called IT-revolution cause substantial economic changes. Also a massive increase of capital investments will result in a higher share. However, the increase of $s_K$ by 50% in the 3. period seems to be too high to be explainable by these effects.

Further redistribution effects are observed also between $s_H$ and $s_L$. They are smaller than the ones observed for $s_K$. But some are still notable.

The above redistribution phenomena are further investigated. The sample of employed workers is divided up in 4 groups of qualification: low (q03-group), medium (q4-group), high (q5-group) and highest (q6-group) qualification (see table 5.1). (The inclusion of self-employed would result only in a minor correction because their number is only $\approx 10\%$ of the employed and the wages of the q-groups are identical.) For the 4 qualification groups the parameters of labor participation, weekly working hours, and wage-components are determined. They are shown in figure 5.4 for the different years; table 5.6 shows the detailed numbers.

At first the amount of labor is considered, using labor participation and working time per week:

- over the full time range the general labor participation rises by +8% (fig. 5.4 (a)), while the weekly working hours decrease by -9% (fig. 5.4 (b)). So the amount of labor per person has practically not changed in the 30 years including the unification.

- the labor participation of the q6-group shows a remarkable and nearly constant growth of +140% over the full time range (fig. 5.4 (c)).

These facts demonstrate a considerable increase of the qualification of the labor force: the highest qualification group shows a very high increase going along with the decrease of jobs for the low-q group. So the productively used H should have increased accordingly.

However, the surplus wage for the productively used H, $w_H$, has decreased: by 32%, up to 1996 even by 35%. In the last period an increase to 16.1Euro/h (by 14%) is visible. This is a considerable re-increase, however, still far from the original level of 21.1Euro/h.

Further on the group of low qualification q03 has realized a loss of -40% in the labor participation and of -14% in the weekly working hours. This loss of more than 50% of the labor demand for this group has resulted in the well known high rate of long time unemployment of low qualified workers. So the non-existence of a sufficient demand for this type of labor has to be stated.
Contrariwise the income share of capital has increased by 64%, in the last period alone by 48%. So the increase in qualification is either not properly honored by the employers, or the higher qualified push the lower qualified out of their jobs: accepting jobs of lower qualification level and lower wage. Whatever the reasons are, in the end there are much more jobs for the highest qualification level and a loss of about 50% of low qualification jobs.

The consequences of these facts are:

- the loss of more than 50% of low qualified labor demand resulted in a high unemployment and a decrease of the basic wage \( w_o \) as compared to the general trend of growth,
- the q6-group has compensated the loss in labor participation,
- the wage of the q6-group is still quite low as compared to the earliest data. Their probably incomplete representation by trade unions does not allow them to enforce sufficient higher wages.
- as a consequence the employers profit from this situation: Their income share has increased by 65%.

5 Discussion of the Results for Germany

The method has been applied to German data of the years 1976, 1985, 1995, 2006. The German MZ-data of these years have been used to obtain \( W \) and \( w_o \) of EL and SE. Data of the German “Volkswirtschaftliche Gesamtrechnung” (VGR) of the Statistisches Bundesamt are then evaluated to obtain the real total income shares of the production factors of physical and human capital and labor. The average shares (1976 - 2006) for Germany yield

\[
s_K : s_{H_p} : s_{L_o} = 0.18 : 0.26 : 0.56.
\]

These numbers differ considerably from the generally assumed equal shares of 1/3. It is caused by the separation of the self-employed labor from the original capital share of the VGR. Without this separation

\[
s_K : s_{H_p} : s_{L_o} = 0.31 : 0.21 : 0.48
\]

is obtained. This is in agreement with the published results of Mankiw, Romer and Weil and Acemoglu:

\[
s_K : s_{H_p} : s_{L_o} = 0.30 : 0.28 : 0.42 \quad 98 \text{ countries (1960-1985)} \quad [8]
\]

\[
= 0.36 : 0.26 : 0.38 \quad 107 \text{ countries (1960-2000)} \quad [1]
\]

The difference to the German data can be attributed to the different model, more stringent assumptions and the evaluation of about 100 different countries.

The German factor shares of equation 5.23 demonstrate their relative importance for the growth of the country. Remarkable is the considerably higher share of H w.r.t. K. The very high value of \( s_L \) demonstrates the importance of technology which contributes to this share in form of technology enhanced wages.

The development of the income shares from 1976 to 2006 does not show a constant behavior as expected from Kaldor’s stylized fact for developed countries. It could apply to the human capital share. But definitely not to the shares of capital and labor. Instead there is a sudden increase of the capital share in the period from 1995 to 2006 by 0.080, a 48% change. The labor share decreases accordingly. This is in complete contradiction with an expected constant behavior.

An explanation for this feature can be a strong structural change. Evidence is coming from a more detailed analysis of MZ data: the development of parameters like labor participation and wages from
human capital for groups of different qualification. Over the whole 30-years period an enormous shift of qualification towards a higher level is encountered. This fact manifests in a 140% increase of the group of highest qualification (degree of university or “Fachhochschule”). This goes along with a constant overall work participation while workers with low qualification suffer from a 50% loss of jobs.

However, the highest qualified workers did not profit from this development. Instead the surplus wage of productively used \( H \) decreased by more than 30% in spite of a growth of GDP and total income by nearly 30%. As a result the employers have profited much more than the qualified workers by an increase of their income share by about 65%.

In sum there is the positive effect of the large increase of workers with highest qualification, which is certainly an increase of the productively used \( H \). But it does not show up in \( s_H \) because of a decreasing wage for the productive \( H \). This highly questionable effect is certainly no incentive for investments in higher education. Therefore this fact should be transmitted to government and/or the employers with the request for a change.

The markable changes of income shares within the last 10 years, and its probable origin, the strong structural changes, opens a box of new questions:

- to government: how to handle and overcome the resulting unwanted consequences
- to growth theory: how to implement effects like the IT-revolution, or globalization effects.
- to economic science: is there a method to identify the origin and estimate its effect on economy and population, and are there early indicators?

At present there are no answers to these questions. They can be addressed in future investigations that would involve more detailed analysis of micro data as well as detailed time series with yearly frequency of e.g. the income shares, labor participation, etc.

Finally the results for Germany should be compared to those of other countries, e.g. in Europe. The analysis of differences and similarities might improve the understanding of different developments.
Figure 5.1: Wages from different methods: $W$, $w_o$ (a) and $f_o$ (b).
Methods: $m1$: standard mean (full circle), $m2$: log-mean (open circle), $m3$: 10-90 percentiles, linear mean (full square), $m4$: 10-90 percentiles, log mean (open square), $m5$: median (star)

Figure 5.2: Wages (a) and wage fractions (b) of employed (EL) and self-employed (SE) for the years 1976, 1985, 1995 and 2006
Figure 5.3: Total income shares of basic labor ($s_L$), human capital ($s_H$), the sum ($s_L + s_H$) and physical capital ($s_K$) for the years 1976, 1985, 1995 and 2006. Also shown is the labor share as provided by the VGR: $s_L(VGR) = s_L(EL) + s_H(EL)$.
Figure 5.4: Development of labor participation (a), working time per week (b) and the wage components $w_h$ and $w_o$ (c) for different qualification levels (as marked in the diagrams; for interpretation see text, resp. table 5.1).
<table>
<thead>
<tr>
<th>Level</th>
<th>Qualification</th>
<th>persons [1000] in 2006</th>
<th>wage/hour [Euro]</th>
</tr>
</thead>
<tbody>
<tr>
<td>q0</td>
<td>w/o qualification</td>
<td>482</td>
<td>11.5</td>
</tr>
<tr>
<td>q1</td>
<td>basic schooling finished</td>
<td>1820</td>
<td>9.6 10.8 11.9 12.5</td>
</tr>
<tr>
<td>q2</td>
<td>higher schooling finished</td>
<td>1526</td>
<td>13.4 12.0 11.1 12.5</td>
</tr>
<tr>
<td>q3</td>
<td>learning by doing education</td>
<td>360</td>
<td>13.5 15.5 14.7 13.6</td>
</tr>
<tr>
<td>q4</td>
<td>apprenticeship</td>
<td>16109</td>
<td>12.6 13.5 13.7 15.3</td>
</tr>
<tr>
<td>q5</td>
<td>technician, “Meister”</td>
<td>1899</td>
<td>18.7 20.8 19.5 20.0</td>
</tr>
<tr>
<td>q6</td>
<td>degree of university or “Fachhochschule”</td>
<td>4612</td>
<td>33.6 29.9 27.1 28.4</td>
</tr>
</tbody>
</table>

Table 5.1: Unified levels of qualification, professional training, frequencies (2006-data), and wage per hour in different years. Wages are corrected for inflation and converted to Euro. The wages of minimal qualification q0 is included in q1 for the years 1976 - 1995.

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>wage/hour [Euro]</td>
</tr>
<tr>
<td>(\text{W})</td>
<td>12.9 14.7 15.2 17.0</td>
</tr>
<tr>
<td>(w_o)</td>
<td>8.9 10.1 11.0 11.4</td>
</tr>
<tr>
<td>(w_h)</td>
<td>4.0 4.6 4.2 5.6</td>
</tr>
<tr>
<td>(f_o)</td>
<td>0.686 0.684 0.723 0.670</td>
</tr>
<tr>
<td>(f_h)</td>
<td>0.314 0.316 0.277 0.330</td>
</tr>
</tbody>
</table>

Table 5.2: Wages and fractions of basic wage and human capital of the employed workers. Results are corrected for inflation and converted to Euro.

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>wage/hour [Euro]</td>
</tr>
<tr>
<td>(\text{W})</td>
<td>15.1 17.3 17.5 18.9</td>
</tr>
<tr>
<td>(w_o)</td>
<td>8.9 10.1 11.0 11.4</td>
</tr>
<tr>
<td>(w_h)</td>
<td>6.2 7.2 6.5 7.5</td>
</tr>
<tr>
<td>(f_o)</td>
<td>0.590 0.581 0.629 0.605</td>
</tr>
<tr>
<td>(f_h)</td>
<td>0.410 0.419 0.371 0.395</td>
</tr>
</tbody>
</table>

Table 5.3: Wages and fractions of basic wage and human capital of the self-employed. Wages are corrected for inflation and converted to Euro.
The 3. row contains the results of this work using only EL for the factor shares. In this case $s_K$ comprises also labor contributions of the SE. This allows a direct comparison to the published results of Mankiw, Romer and Weil [8] and Acemoglu [1]. In the last row the results of this work for the real factor shares are stated.

Table 5.5: Development of NNP per person and per worker [18] as well as the real shares of physical and human capital and labor from yearly German data of 1976, 1985, 1995 and 2006. NNP-data are in Euro and corrected for inflation. The changes in the time periods are stated as well.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>mean labor participation [workers/population]</td>
<td>0.301</td>
<td>0.330</td>
<td>0.327</td>
<td>0.326</td>
<td>8.3</td>
</tr>
<tr>
<td>mean working time [h/week]</td>
<td>40.0</td>
<td>39.1</td>
<td>37.2</td>
<td>36.3</td>
<td>-9.3</td>
</tr>
<tr>
<td>W</td>
<td>12.9</td>
<td>14.7</td>
<td>15.3</td>
<td>17.0</td>
<td>31.6</td>
</tr>
<tr>
<td>(w_o)</td>
<td>8.9</td>
<td>10.1</td>
<td>11.0</td>
<td>11.4</td>
<td>28.4</td>
</tr>
<tr>
<td>(w_h)</td>
<td>4.1</td>
<td>4.6</td>
<td>4.2</td>
<td>5.6</td>
<td>38.7</td>
</tr>
<tr>
<td>labor participation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>q03</td>
<td>0.085</td>
<td>0.071</td>
<td>0.040</td>
<td>0.051</td>
<td>-40.1</td>
</tr>
<tr>
<td>q4</td>
<td>0.174</td>
<td>0.198</td>
<td>0.214</td>
<td>0.196</td>
<td>12.3</td>
</tr>
<tr>
<td>q5</td>
<td>0.018</td>
<td>0.024</td>
<td>0.025</td>
<td>0.023</td>
<td>27.1</td>
</tr>
<tr>
<td>q6</td>
<td>0.023</td>
<td>0.037</td>
<td>0.048</td>
<td>0.056</td>
<td>139.9</td>
</tr>
<tr>
<td>working time</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>q03</td>
<td>39.4</td>
<td>37.9</td>
<td>35.2</td>
<td>34.0</td>
<td>-13.6</td>
</tr>
<tr>
<td>q4</td>
<td>40.2</td>
<td>39.1</td>
<td>37.1</td>
<td>36.1</td>
<td>-10.1</td>
</tr>
<tr>
<td>q5</td>
<td>41.0</td>
<td>40.4</td>
<td>38.5</td>
<td>37.7</td>
<td>-8.1</td>
</tr>
<tr>
<td>q6</td>
<td>40.5</td>
<td>40.5</td>
<td>38.7</td>
<td>38.7</td>
<td>-4.3</td>
</tr>
<tr>
<td>(w_h)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>q03</td>
<td>1.4</td>
<td>1.3</td>
<td>0.9</td>
<td>1.0</td>
<td>-26.2</td>
</tr>
<tr>
<td>q4</td>
<td>3.8</td>
<td>3.4</td>
<td>2.7</td>
<td>3.9</td>
<td>3.8</td>
</tr>
<tr>
<td>q5</td>
<td>9.8</td>
<td>10.8</td>
<td>8.5</td>
<td>8.6</td>
<td>-12.9</td>
</tr>
<tr>
<td>q6</td>
<td>24.8</td>
<td>20.0</td>
<td>16.1</td>
<td>16.9</td>
<td>-31.7</td>
</tr>
</tbody>
</table>

Table 5.6: Developments of parameters for groups of different qualification: from low (q03) to the highest (q6) level (see text for further explanation). The last column contains the total change from 1976 to 2006.
A Appendix

A.1 GDP and Total Income

GDP and total income are closely related. For Germany both are determined by the “Statistisches Bundesamt” in the yearly VGR ("Volkswirtschaftliche Gesamtrechnung") [14]. The relation is given by

\[
\begin{align*}
\text{GDP} & \quad 100\% \quad (2006) \\
\pm \text{ net factor income from abroad} & +2.0\% \\
- \text{ depreciation} & -14.5\% \\
\pm \text{ indirect taxes(-) und subsidies(+) } & - 9.6\% \\
= \quad \text{NNP} \equiv \text{total income} & \quad 78.2\%
\end{align*}
\]

\[
\begin{align*}
\text{NNP} & \quad \text{total income} \\
\begin{array}{ccc}
\text{employed labor income share} & \text{s}_L(\text{VGR}) & 63.9\% \quad (\text{NNP}) \\
\text{self-employed and firms income shares} & \text{s}_K(\text{VGR}) & 36.1\% \quad (\text{NNP})
\end{array}
\end{align*}
\]

The \text{NNP} (net national product) is the total income of the economy. Its difference to the GDP is rather constant being \approx 22\%: the variation in time from 1970 - 2011 is less than \pm 0.5\%. It is the basis for the annual income shares of capital and labor. The VGR separates the \text{NNP} into the labor income share \text{s}_L(\text{VGR}) comprising only the labor income of employed workers, and the capital share \text{s}_K(\text{VGR}) comprising the income share of enterprises as well as investment and labor share of self-employed workers.

A.2 Mikrozensus Selection

The method (see chapter 2) is based on a simultaneous determinations of incomes of all employed workers and of those with a minimal qualification. This requires identical conditions under which incomes are determined. Therefore the following selection criteria are applied to the personal Mikrozensus (MZ) data:

1. persons at their main living location (this criterion avoids double counting of people),
2. individual persons who are working (only working persons have a labor income);
3. complete statements w.r.t. income, working hours per week, highest degree in school and professional education (these data are necessary for further evaluation);
4. employed workers: excluded are persons without a labor income, self-employed (their income stems from labor as well as investments), students (they are still in education), apprentices and persons in military or civil duty services (they obtain a subsistence contribution but no labor income);
5. persons whose income stem mainly from labor: no unemployed, no persons with further income from e.g. pensions, rents from capital investments.

Selections 1 and 2 are the same as used by the Statistisches Bundesamt in published income tables [10].

The effect of the selection criteria 3 - 5 on the number of workers is shown in table A.7. The strongest effect is due to criteria 4 and 5.

\textbf{Criterion 4} eliminates about 15\% of the workers: about 10\% are self-employed, 5\% are apprentices, students or persons in civil or military duty services.

\textbf{Criterion 5} eliminates further 7.5\% of the employed workers: these are persons who have further income sources. Their labor incomes are only a minor source of their total incomes.
In total the selection criteria eliminate 23.5% of the working population. However, the remaining 76.5% comprise the essential fraction of the labor incomes of the employed workers.

It should be noted that for the later estimation of the labor income of the self-employed (chapter 3.2) the criterion 4 is changed accordingly.

A.3 Details of Income Conversions

Income scale: The income is stated in the MZ data as net monthly income. These statements are given in an ordinal scale that differs for different years. For the 2006-data it is

\{ < 150, -300, -500, -700, -900, \ldots, -7500, -18000, > 18000 \} \text{ Euro.}

The lowest and the highest bins are unlimited. They have been converted to confined bins by fixing the range to $50 < \text{income} < 28000$. Income values outside the range are changed to the minimal/maximal value. The lower bound has practical reasons: it allows the evaluation of logarithmic distributions. The upper bound limits extreme incomes in order to reduce their influence on a mean.

Income distributions are evaluated in using the bin centers of the ordinal scale.

Gross income: The net income has to be converted into gross income. This is necessary for a later conversion of wage ratios into shares of the total income which is the sum of gross incomes.

The difference between gross and net income comprises income tax, social insurances and contributions of employers. The conversion of net to gross income is performed by determining the conversion for the different bounds of the ordinal scale. In detail the following procedure is used:

1. the income is multiplied by 12 in order to obtain the yearly income,

2. the income tax is calculated using the tax law of the different years [2],

3. contributions to social insurance as well as contributions to it from employers are obtained from the VGR-tables [19] of the different years,

4. the resulting gross income is divided by 12 in order to obtain the gross monthly income.

The resulting gross scale is used for the evaluation of the income distribution.

Inflation correction: Incomes of different years can be compared only if corrections for inflation are applied. The different deflators are determined from the tables of nominal and deflated GDP values of the years [18]. For Germany there are 2 different tables for the time before and after the unification. They overlap for the year 1991. Therefore 1991 is used as reference year. The obtained deflators are listed in table A.10.

Income per hour: The weekly working hours have changed with the years. In addition the tendency to work part time has increased. Therefore the monthly income is converted to income per hour by using the MZ statements on the weekly working hours and assuming 4 weeks per month. The final distribution of income per hour has equidistant bins with the overflow accumulated in the last bin.

Conversion to Euro: The incomes obtained so far are stated in different units: DM and Euro. All DM-values are converted to Euro using the ratio $\text{Euro/DM} = 1.95583$. 

20
A.4 The Unified Scale of Qualification Levels

When investigating incomes of workers of different qualification a scale of qualification has to be defined. This scale has to be the same for all years.

In the Microzensus data the educational and professional qualification is stated separately as highest degree reached in schooling as well as in professional qualification. As a consequence all persons are counted twice in the 2 qualification scales. For the 2006-data the different degrees are listed in table A.8. There are 6 levels for schooling and 11 levels for professional qualification (university and “Fachhochschul” degrees are listed as professional qualification; schooling degree of “Abitur” or “Fachhochschulreife” are indispensable for them).

Figure A.5 shows the frequency (a) as well as the average net monthly income (b) for different qualifications. The income distribution (b) shows 2 independent qualification scales: schooling (0 - 6) and professional degree (10 - 20). For either scale the income rises with increasing qualification. They have to be combined to a single scale without any double counting. This unified scale which is valid for all years is obtained after the following procedure:

- the degrees of the former GDR are combined with appropriate degrees of the German federal republic,
- the professional qualification is given priority over the schooling qualification: it shows a stronger dynamic with the professional degree and minimal schooling degrees are indispensable for the higher professional degrees,
- qualifications with too few persons have been combined in order to avoid qualification groups with too low population.

The final qualification scale is shown in table A.9. 7 different groups (q0 - q6) are specified together with the relative population. The also stated net monthly income shows a considerable increase with the qualification level.

Further on combined groups are specified: the low qualification group q03 comprises q0 - q3, and the group q01 comprising q0 and q1.

The group q01 has been formed in order to overcome a problem of the data from 1976 - 1995. In these data there is no differentiation between q0 and q1. However in the 2006-data q0 and q1 are separate qualification groups that show an income difference of 7%. This difference is used to obtain $w_{o} = w(q0)$ for all years:

$$w_{o} = w(q0) \frac{W(q0)_{2006}}{W(q01)_{2006}}.$$

A.5 $w_{o}$ Determination

$w_{o}$ is determined as income of workers with minimal qualification. In Germany these are persons who have passed the obligatory years of schooling (about 8). But they finished schooling without a degree.

As shown in equation 2.1 a Harrods neutral technology is involved that results in a technology enhanced wage. The technology is country specific, in this case specific for Germany.

In addition also $w_{h}$, the surplus income from H engagement, might include effects of current technology. This has to be taken into account in comparisons of $w_{o}$ and $w_{h}$ or of income shares of different countries.

An alternative determination of $w_{o}$ is the result of a Mincer regression [11]:

$$\ln w = \alpha + \rho \cdot s + \beta_{0}x + \beta_{1}x^{2} + \varepsilon,$$

with the variable $s$ (number of school years) and $x$ (years of working). The value of $\alpha$, obtained from the regression, is assumed to be the logarithm of the wage for a worker with 0 years of schooling. Arguments against this method are:
1. in many developed countries there are no persons without any schooling; therefore one has to rely on the linearity of the above Mincer equation.

2. the type of schools differ considerably; therefore the parameter $\rho$ and with it the extrapolation to 0 school years is determined by the population of the different types of schools,

3. the German MZ data do not contain statements about the years of schooling; there are only statements on the highest degree reached. An estimate of the years of schooling is not excluded, but it has not been investigated.

Further on the linearity requirement would be an additional rather strong assumption. This is avoided by using the wage of workers with minimal qualification.

### A.6 Wage Determination from Income Distributions

The two wages $W$ and $w_\alpha$ are determined from income distributions of two groups: the incomes of all employed ($W$) and the incomes of workers with minimal qualification ($w_\alpha$). However, the distributions of the two groups differ considerably as shown in figure A.6 with linear (left: a,c) and log. (right: b,d) income scales.

Comparing the linear distributions one observes a long tail of the $W$-distribution (a). This is absent in the $w_\alpha$-distribution (c). This fact will result in a mean that is systematically biased by the extreme high values of the $W$-distribution. This is also a potential source of a higher volatility of this value in different years. The logarithmic distributions show the reverse effect: a stronger tail to the low side of the $w_\alpha$-distribution.

Both effects introduce a bias of the important wage ratios $f_\alpha$ and $f_h$. Therefore different methods have been studied in order to determine representative values of $W$ and $w_\alpha$ from the linear and the logarithmic distributions. The difference of the results between linear and logarithmic distributions ($\Delta$) will be a kind of measure for the sensitivity to the tails of the distributions: to the low side (log) or the high side (linear).

The following methods have been evaluated: the standard mean using the full distributions, the mean of the 10-90-percentiles of the distributions, and the median (the median is identical for linear and logarithmic distribution). The results are shown in table A.11 for 4 years from 1976 to 2006. The important number in this table is $\Delta$. It demonstrates the effect of high and low tails.

The standard means yield rather large differences $\Delta$: $\approx 20\%$ of the mean. However, the wage ratio $f_\alpha$ shows only small differences. This originates from the tails of the distributions:

- linear distribution: systematic larger value for $W$ and normal value for $w_\alpha$;
- logarithmic distribution: systematic lower value for $w_\alpha$ and normal value for $W$;

For the wage ratio $f_\alpha = w_\alpha / W$ the systematic effects go into the same direction yielding small differences.

But the value of $f_\alpha$ itself is systematically low.

The means from the 10-90-percentile distribution yield smaller values of $\Delta$: $\approx 7\%$ of the mean. The wage ratio $f_\alpha$ shows negligible differences. This results from the absence of high and low tails in the 10-90-percentile distributions.

The median values are the same for linear and logarithmic distributions: $\Delta = 0.0$. However the values show a stronger fluctuation over the years as compared to the other methods. This is esp. visible for $f_\alpha$, shown in figure 5.1 (b). In this figure $f_\alpha$(median) fluctuates considerably up and down from 1976 - 2006; the other $f_\alpha$-values show a nearly linear rise up to 1995 and a sharp fall in the last period. A reason for the $f_\alpha$-behavior of the median is the ordinal scale of the income distributions. A too coarse binning in the range of the median value results in larger fluctuations as compared to average values.

The finally accepted method for the determination of $W$ and $w_\alpha$ is the mean of the 10-90-percentile linear distributions: it has a low dependence on tails to the high or low side (as compared to the standard mean),
it is less sensitive to the binning of the income distribution (as compared to the median), it is preferable to the evaluation of the logarithmic distributions, if subsamples (like groups of different qualification level) are analyzed: the weighted means of the subsamples do not add up to the total mean in case of a logarithmic distributions.
Figure A.5: Frequency (top) and net monthly income (bottom) of different qualification levels from school and professional education. The numbers of the horizontal scale refer qualification groups of table A.8. The number 0 refers to persons having neither a degree from schooling nor from professional training. The number 24 refers to the overall average.
Figure A.6: Income distributions (2006-data) with linear (a,c) and log. (b,d) income scale: (a,b): all employed worker (c,d): minimal qualification: q01 and q0 (see q-groups of table A.9).
Table A.7: Results for Selections 3 - 5 (2006-data)

<table>
<thead>
<tr>
<th>Selection</th>
<th>Persons [1000]</th>
<th>Selected Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 + 2: preselection</td>
<td>35 063</td>
<td>100%</td>
</tr>
<tr>
<td>3. complete statements</td>
<td>34 596</td>
<td>98.7</td>
</tr>
<tr>
<td>4. employed</td>
<td>29 436</td>
<td>84.0</td>
</tr>
<tr>
<td>5. mainly labor income</td>
<td>26 812</td>
<td>76.5</td>
</tr>
</tbody>
</table>

Table A.8: Qualification groups of 2006 data: highest degree obtained in education and professional training.
The codes refer to the horizontal scale in figure A.5 showing population and net income of the groups.

<table>
<thead>
<tr>
<th>code</th>
<th>schooling degree</th>
<th>code</th>
<th>professional degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>no schooling level</td>
<td>10.</td>
<td>no professional level</td>
</tr>
<tr>
<td>2.</td>
<td>basic school</td>
<td>11.</td>
<td>learning-by-doing training</td>
</tr>
<tr>
<td>4.</td>
<td>“Realschule”</td>
<td>13.</td>
<td>apprenticeship</td>
</tr>
<tr>
<td>7.</td>
<td></td>
<td>16.</td>
<td>“Fachschule der DDR”</td>
</tr>
<tr>
<td>8.</td>
<td></td>
<td>17.</td>
<td>“Verwaltungs-Fachhochschule”</td>
</tr>
<tr>
<td>9.</td>
<td></td>
<td>18.</td>
<td>“Fachhochschule”</td>
</tr>
<tr>
<td>10.</td>
<td></td>
<td>19.</td>
<td>university</td>
</tr>
<tr>
<td>11.</td>
<td></td>
<td>20.</td>
<td>PhD</td>
</tr>
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</table>

Table A.9: Unified qualification levels, population and net monthly income of 2006 data. For q3 - q6 no school degrees are required explicitly, but there are implicit requirements.

<table>
<thead>
<tr>
<th>level</th>
<th>qualification</th>
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<tbody>
<tr>
<td>q0</td>
<td>w/o qualification level</td>
</tr>
<tr>
<td>q1</td>
<td>no prof.level, basic school degree</td>
</tr>
<tr>
<td>q2</td>
<td>no prof.level, higher school degree</td>
</tr>
<tr>
<td>q3</td>
<td>learning-by-doing training</td>
</tr>
<tr>
<td>q4</td>
<td>apprenticeship</td>
</tr>
<tr>
<td>q5</td>
<td>technician, “Meister”</td>
</tr>
<tr>
<td>q6</td>
<td>university or ”Fachhochschule” degree</td>
</tr>
<tr>
<td>q03</td>
<td>low qualified: levels q0 - q3</td>
</tr>
<tr>
<td>q01</td>
<td>min.qualification: q0 - q1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>persons [1000]</th>
<th>income [Euro/h]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>q0</td>
<td>482</td>
<td>14.0</td>
</tr>
<tr>
<td>q1</td>
<td>1820</td>
<td>15.3</td>
<td></td>
</tr>
<tr>
<td>q2</td>
<td>1526</td>
<td>15.3</td>
<td></td>
</tr>
<tr>
<td>q3</td>
<td>360</td>
<td>16.6</td>
<td></td>
</tr>
<tr>
<td>q4</td>
<td>16109</td>
<td>18.7</td>
<td></td>
</tr>
<tr>
<td>q5</td>
<td>1899</td>
<td>24.5</td>
<td></td>
</tr>
<tr>
<td>q6</td>
<td>4612</td>
<td>34.7</td>
<td></td>
</tr>
<tr>
<td>q03</td>
<td>4188</td>
<td>15.3</td>
<td></td>
</tr>
<tr>
<td>q01</td>
<td>2302</td>
<td>15.0</td>
<td></td>
</tr>
</tbody>
</table>

Table A.10: Inflation correction for the years 1976 - 2006.

<table>
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<th></th>
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<th></th>
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<td>Deflater</td>
<td>1.593</td>
<td>1.163</td>
<td>0.873</td>
<td>0.817</td>
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</table>

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Table A.11: Results of different methods for mean income of all employed workers and those with minimal qualification and the wage ratio $f_o$. Results from linear and logarithmic distributions as well as the difference $\Delta$. 

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>standard mean</td>
<td>linear $W$</td>
<td>15.6</td>
<td>16.9</td>
<td>17.1</td>
<td>19.2</td>
</tr>
<tr>
<td></td>
<td>log. $W$</td>
<td>12.4</td>
<td>13.9</td>
<td>14.3</td>
<td>15.8</td>
</tr>
<tr>
<td></td>
<td>$\Delta$</td>
<td>3.2</td>
<td>3.0</td>
<td>2.8</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>linear $w_o$</td>
<td>9.6</td>
<td>10.8</td>
<td>11.6</td>
<td>12.4</td>
</tr>
<tr>
<td></td>
<td>log. $w_o$</td>
<td>8.0</td>
<td>9.1</td>
<td>9.8</td>
<td>9.8</td>
</tr>
<tr>
<td></td>
<td>$\Delta$</td>
<td>1.6</td>
<td>1.7</td>
<td>1.8</td>
<td>2.6</td>
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<tr>
<td>mean of 10-90-percentile</td>
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<td>14.7</td>
<td>15.3</td>
<td>17.0</td>
</tr>
<tr>
<td></td>
<td>log. $W$</td>
<td>12.2</td>
<td>13.8</td>
<td>14.4</td>
<td>15.8</td>
</tr>
<tr>
<td></td>
<td>$\Delta$</td>
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<td>0.9</td>
<td>0.9</td>
<td>1.2</td>
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<td></td>
<td>linear $w_o$</td>
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<td>10.1</td>
<td>11.0</td>
<td>11.4</td>
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<tr>
<td></td>
<td>log. $w_o$</td>
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<td>9.5</td>
<td>10.4</td>
<td>10.5</td>
</tr>
<tr>
<td></td>
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<td>0.6</td>
<td>0.7</td>
<td>0.9</td>
</tr>
<tr>
<td>median</td>
<td>linear $W$</td>
<td>12.0</td>
<td>13.3</td>
<td>13.9</td>
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</tr>
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<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>$\Delta$</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>linear $w_o$</td>
<td>9.2</td>
<td>9.6</td>
<td>10.6</td>
<td>10.7</td>
</tr>
<tr>
<td></td>
<td>log. $w_o$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>$\Delta$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>linear $f_o$</td>
<td>0.69</td>
<td>0.68</td>
<td>0.72</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>log. $f_o$</td>
<td>0.68</td>
<td>0.69</td>
<td>0.72</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>$\Delta$</td>
<td>0.007</td>
<td>-0.003</td>
<td>0.004</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>linear $f_o$</td>
<td>0.77</td>
<td>0.72</td>
<td>0.76</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>log. $f_o$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>$\Delta$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
References


[18] ibid. Tabelle 1.4: Bruttoinlandsprodukt, Bruttonationaleinkommen, Volkseinkommen (Pro-Kopf-Angaben), Wiesbaden, 2011