The New Keynesian Model and the Long-run Vertical Phillips Curve: Does it hold for Germany?

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Abstract

New-Keynesian macroeconomic models typically assume that any long-run trade-off between inflation and unemployment is ruled out. While this appears to be a reasonable characterization of the US economy, it is less clear that the natural rate hypothesis necessarily holds in a European country like Germany where hysteretic effects may invalidate it. Inspired by the framework developed by Farmer (2000) and Beyer and Farmer (2002), we investigate the long-run relationships between the interest rate, unemployment and inflation in West Germany from the early 1960s up to 2004 using a multivariate co-integration analysis technique. The results point to a structural break in the late 1970s. In the later time period we find for West German data a strong negative correlation between the trend components of inflation and unemployment. We show that this finding contradicts the natural rate hypothesis, introduce a version of the New Keynesian model which allows for some hysteresis and compare the effectiveness of monetary policy in these two models. In general, a policy rule with an aggressive response to a rise in unemployment performs better in a model with hysteretic characteristics than in a model without.

Keywords: Cointegration, Vector Error Correction Model, Unemployment, Phillips Curve, Hysteresis

JEL classification: B22, C32, E24

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1 Introduction

The New Keynesian model has gained widespread acceptance in recent years. Most likely this reflects the fact that this model incorporates elements from a number of mainstream macroeconomic modeling approaches, including a Keynesian transmission mechanism, the use of rational expectations popularized by New Classical models, an intertemporal optimizing framework common in RBC models, and a vertical long-run Phillips curve consistent with the natural rate hypothesis championed by monetarists. However, in spite of its strong theoretical foundations, the empirical evaluation is still in its early stages.

An important contribution in this area is a paper by Beyer and Farmer (2002) in which they present a framework for testing empirically the long-run implications of the New Keynesian model using multivariate cointegration analysis. In particular, the authors test whether the natural rate hypothesis, which is a central tenet of New Keynesian models and, for that matter, of most other modern macroeconomic models, is consistent with the data. For the United States they find that the natural rate hypothesis is actually rejected by the data, which leads them to propose an alternative aggregate supply function that allows for a non-vertical long-run Phillips curve.

In this paper, we apply Beyer and Farmer’s (2002) framework to German data. Testing the natural rate hypothesis for Germany has not only important implications for macroeconomic modeling, but also for the understanding of the causes of Germany’s persistently high unemployment rate. The natural rate hypothesis implies that attempts at managing aggregate demand conditions, in particular monetary policy actions, have at best short-run effects on the unemployment rate, and no long-run effects. Thus, the trend increase in the German unemployment rate must reflect structural factors like increasing labor market rigidities and cannot be attributed to adverse demand conditions, resulting, for example, from a tight monetary policy stance. However, if the natural rate hypothesis does not hold, this raises the prospect that demand conditions could have been a contributing factor to Germany’s unemployment problem.

Since we revisit the natural rate hypothesis within a New Keynesian theoretical framework, we provide in chapter 2 a brief outline of the standard New Keynesian model. In chapter 3, we take a first look at German data and present preliminary evidence that the natural rate hypothesis may not be consistent with Germany’s experience in the past twenty years. Next, we outline the empirical framework proposed by Beyer and Farmer (2002) and present in chapter 5 the results of the cointegration analysis. Standard mis-specification tests point towards a structural break in the macroeconomic relations occurring in 1979, and after splitting the sample period we find

\footnote{Their work builds on Farmer (2000).}
that the period after 1979 is characterized by a negative long-run correlation between unemployment and inflation. Such a long-run relation contradicts the natural rate hypothesis, and points to the possibility that disinflationary policies by the Bundesbank in the 1980s could have contributed to the trend increase in the German unemployment rate in this period. Even though a non-vertical long-run Phillips curve is inconsistent with the New Keynesian model, in chapter 6 we show that it is nevertheless consistent with a number of recent approaches in modern macroeconomics. In the final chapter 7, we modify the New Keynesian model to allow for some long-run effects of monetary policy on unemployment and investigate the implications for the effectiveness of monetary policy. We find that this modification changes the policy implications of the New Keynesian model substantially. While this is only an explorative investigation, it does suggest that the standard New Keynesian model needs some extensions to fit the long-run properties of German data better. From a policy perspective, this raises the prospect that monetary policy could make a contribution towards lowering the unemployment rate without creating inflationary pressures, even though the scope for this is likely to be limited.

2 The New Keynesian model

The New Keynesian model is derived from intertemporal optimization of rational households. The following model shows the resulting log-linearized first order conditions. According to McCallum (2001), this model represents a substantial agreement in the literature on the general, broad structure of modern macroeconomic models.\(^2\)

\[
y_t = b_0 + b_1 (R_t - E_t \Delta p_{t+1}) + E_t y_{t+1} + v^1_t
\]

\[
\Delta p_t = \beta E_t \Delta p_{t+1} + \alpha (y_t - \bar{y}_t) + v^2_t
\]

\[
R_t = (1 - \mu_3) [r + \Delta p_t + \mu_1 (\Delta p_t - \bar{\pi}) + \mu_2 (y_t - \bar{y}_t)] + \mu_3 R_{t-1} + v^3_t
\]

The variables \(y_t\) and \(p_t\) denote the logs of output and the price level, \(\bar{y}_t\) is the natural-rate value of output, \(R_t\) is a one-period interest rate, \(r\) is the equilibrium real interest rate and \(\bar{\pi}\) is the inflation target. \(E_t x_{t+1}\) is the expectation of \(x_{t+1}\) conditional on information available in \(t\). Equation (2.1) represents a forward-looking IS curve, specifying that output is a function of the real interest rate (\(b_1 < 0\)), expected future output and an exogenous

\(^2\)See McCallum (2001), p.258. A similar type of model is used, for example, by Clarida et al. (1999) to investigate optimal monetary policy. For an extensive review of this model, see also King (2000).
shock, $v^1_t$, which stands for shocks to tastes or fiscal policy.\footnote{See McCallum and Nelson (1999) for the derivation of this relation from an optimizing framework.} Equation (2.2) is a Phillips curve type relationship, stating that price adjustment is a function of expected future prices ($0 \leq \beta \leq 1$) and demand conditions ($\alpha > 0$), with $v^2_t$ representing a price shock.\footnote{See Roberts (1995) for an overview of Phillips curves used in New Keynesian models.} Finally, equation (2.3) represents a standard-Taylor type reaction function, giving the nominal interest rate as a function of the equilibrium real interest rate, inflation and demand conditions ($\mu_1, \mu_2 \geq 0$), $\mu_3$ models the degree of interest rate smoothing, and $v^3_t$ is the monetary policy shock that captures discretionary monetary policy actions.

Since this paper investigates the relationship between the interest rate, inflation and unemployment ($u_t$), output is assumed to be inversely related to unemployment through an Okun’s law type relationship, yielding the following model:

$$u_t = b^u_0 + b^u_1 (R_t - E_t \Delta p_{t+1}) + E_t u_{t+1} + v^1_t$$  \hspace{1cm} (2.4)

$$\Delta p_t = \beta E_t \Delta p_{t+1} + \alpha^u (u_t - \bar{u}_t) + v^2_t$$  \hspace{1cm} (2.5)

$$R_t = (1 - \mu_3) \left[ r + \Delta p_t + \mu_1^u (\Delta p_t - \bar{\pi}) + \mu_2^u (u_t - \bar{u}_t) \right] + \mu_3 R_{t-1} + v^3_t$$  \hspace{1cm} (2.6)

The variable $\bar{u}_t$ represents the natural rate of unemployment.

It is a salient feature of New Keynesian models that the natural rate hypothesis holds, meaning that in the long-run there is no trade-off between inflation and unemployment. To illustrate this, it is useful to note that in the model (2.1) to (2.3) the relationship between the deviation of unemployment from the natural rate and the steady-state rate of inflation is given by $u - \bar{u} = \frac{1 - \beta}{\alpha} \Delta p$. King (2000) writes that experiments with fully articulated models which lead to price adjustment equations like (2.2) and (2.5) suggest a negligible long-run trade-off at moderate inflation rates.\footnote{King (2000), p.51.} Accordingly, he notes, prominent studies of the monetary policy implications of the New IS-LM model - including that of Clarida et al. (1999) - impose the condition $\beta = 1$ on the price adjustment equation. Another example is McCallum (2001) who uses for the calibration of the model (2.1) to (2.3) the values $\beta = 0.99$ and $\alpha = 0.03$. For a steady state inflation rate of 1.5 percent this yields a long-run effect of increasing the steady state inflation rate by one percentage point of 0.125 percent on output.\footnote{Notice that McCallum, 2001 uses non-annualized interest and inflation rates in his model.} Thus, for all practical purposes the Phillips curve is vertical in the long-run in New Keynesian models.
3 A preliminary look at the data

The New Keynesian model implies that even though there is a negative relationship between inflation and unemployment at the business cycle frequency, there should be no such relationship in the long-run. In Figure 1, we plot the relation between five-year averages of both variables for West Germany. The five-year period has been chosen since it corresponds approximately to the typical length of business cycles. One would expect that over the course of a business cycle, those periods where the unemployment rate is above the natural rate are balanced by periods where the unemployment rate is below the natural rate. Since the natural rate hypothesis implies that a given natural rate is compatible with any rate of inflation, there should be no discernible relationship between the two variables. This is, after all, the essence of the natural rate hypothesis.

Figure 1 shows that over the entire sample period from 1965 until 2004 there is indeed not much of a relationship – the $\bar{R}^2$ of the estimated regression line is zero. However, a closer look reveals that over the period from 1980 until 2004 there is a strong negative relationship between the two variables (the $\bar{R}^2$ is 0.75 and the coefficients turn out to be statistically significant even with a very low number of observations), as suggested by the traditional Phillips curve. The long-run Phillips curve in this period appears to be fairly steep, but not vertical. It also appears to be relatively stable. This is in line with the findings of Schreiber and Wolters (2005), Karanassou et al. (2003), and Franz (2005).

In the remainder of this section, we use the technique of multivariate cointegration analysis to investigate more formally whether there is a significant relationship between the trend components of inflation and unemployment. More importantly, this approach allows us also to investigate whether such a relationship can be reconciled with the New Keynesian model.

4 A framework for cointegration analysis

In this section, we outline the framework developed by Farmer (2000) and Beyer and Farmer (2002) in order to test the natural rate hypothesis before we apply it in the next section to West German data. These authors argue that if the unemployment rate, interest rate and inflation rate are non-stationary but cointegrated, a vector error correction model (VECM) can be used to study empirically the relationship between them:

$$\Delta x_t = A(L) \Delta x_{t-1} + \Pi x_{t-k} + z_t + \tilde{z}. \quad (4.1)$$

Here, $X_t$ is a vector containing the variables $R_t$, $\Delta p_t$ and $u_t$. As before, with the exception of the interest rate all variables in $x_t$ are expressed in logarithm.

\[7\text{As before, with the exception of the interest rate all variables in } x_t \text{ are expressed in logarithm.}\]
polynomial in the lag operator and models the short-run dynamics between the variables.\(^8\) The matrix \(\Pi\) is of special interest. It can be factorized so that \(\Pi = \alpha\beta'\); if the variables are cointegrated, \(\Pi\) has reduced rank \(r\), with \(r\) representing the number of cointegration vectors. The term \(\beta'x_{t-k}\) contains the cointegration relationships, while the matrix \(\alpha\) determines to what extent each variable adjusts to a given disequilibrium in the long-run relations. Finally, the vector \(z_t\) contains stationary disturbance terms, \(z \sim I(1)\), and \(\bar{z}\) collects the constants in the system.

To render the New Keynesian model suitable for cointegration analysis, Beyer and Farmer (2002) show that the model can be written as follows:

\[
A_2 E_t [x_{t+1}] + A_0 x_t + A_1 (L) x_{t-1} - \bar{v} = v_t, \\
E_t [v_{t+1}] = 0, \quad (4.2)
\]

where \(A_2\) is a matrix that describes the influence of future expectations, \(A_0\) describes the contemporaneous links and \(A_1 (L)\) is a polynomial in the lag operator. The vector \(\bar{v}\) contains the constants of the model, and \(v_t\) collects the structural disturbances, which are assumed uncorrelated.\(^9\) In a more compact form, this model can be written as,

\[
A (L) E_t [x_{t+1}] - \bar{v} = v_t. \quad (4.3)
\]

However, Beyer and Farmer (2002) argue that if all the disturbances were stationary, the model could not account for the non-stationarity of the variables reported below. To illustrate this, it is useful to consider the moving average presentation of (4.3),

\[
x_{t+1} = A (L)^{-1} (v_t + \bar{v}), \quad (4.4)
\]

where we assume for simplicity perfect foresight of agents. The matrix \(A\) in New Keynesian models is always chosen such that the resulting model is stable, in order to rule out explosive processes and to make sure that the rational expectations equilibrium is uniquely determined. This choice of \(A\) implies that if all disturbances are stationary, the variables in \(x_t\) are stationary, too.\(^10\) Hence, New Keynesian models are stationary structural models. In fact, in steady state, when \(x_t = x_{t+1} = x_{t-1} = x\) and \(v_t = 0\),

\(^8\) The lag polynomial is defined as \(A (L) = A_0 x_t + A_1 x_{t-1} + A_2 x_{t-2} \ldots\)
\(^9\) Hence, the covariance matrix \(\Sigma\) is diagonal.
\(^10\) The stationarity of the variables in \(x_t\) does not mean that New Keynesian models necessarily abstract from the trend growth rate of the economy. In fact, New Keynesian models can accommodate a trend growth in the level of these variables, but to be able to solve the model, the variables are transformed in such a way as to obtain a stationary model.
the model converges to \( x = A(1)^{-1} \bar{v} \). This solution pins down the long-run mean of the variables in the model and rules out any stochastic trends.

To introduce a source of non-stationarity into the New Keynesian model, Beyer and Farmer (2002) assume that one of the disturbances in vector \( v_t \) in the New Keynesian model is a random walk. In this case, the structural model given by equation (4.2) can be rewritten as a VECM. By differencing the equation with the non-stationary disturbance and rewriting the other two equations in differences and levels, one arrives at the following VECM representation of the New Keynesian model:

\[
\tilde{A}_2 E_t \Delta x_{t+1} + \tilde{A}_0 \Delta x_t + \tilde{A}_1 (L) \Delta x_{t-1} + \tilde{\alpha} \tilde{\beta}' x_{t-k} - \bar{w} = w_t. \tag{4.5}
\]

By appropriate ordering of the equations one can always choose the non-stationary disturbance to be in the third equation. In model (4.5), the vector of errors \( w_t \) is stationary with variance-covariance matrix \( \tilde{\Sigma} \). The disturbance vectors \( w_t \) and \( v_t \) are related by the expression:

\[
\begin{bmatrix}
  w^1_t \\
  w^2_t \\
  w^3_t
\end{bmatrix}
= 
\begin{bmatrix}
  v^1_t \\
  v^2_t \\
  v^3_t - v^3_{t-1}
\end{bmatrix}. \tag{4.6}
\]

This formulation of the New Keynesian model implies that the random walk process \( v^3_t \) leads to a non-stationary behavior of \( x_t \). This raises the possibility that some or all of the variables in \( x_t \) are cointegrated. In the VECM formulation of the model, these cointegration relationships are captured by the term \( \tilde{\alpha} \tilde{\beta}' x_{t-k} \), where \( \tilde{\alpha} \) represents the structural loading matrix and \( \tilde{\beta}' \) the matrix of structural cointegrating vectors.

As a final step, we need to supplement model (4.5) with a description of the process how expectations are formed. To this end, Beyer and Farmer (2002) assume that expectations are rational in a very weak sense by requiring only that there should be no systematic long-run biases in the mechanism generating expectations.\(^{11}\)

4.1 The aggregate demand equation in the VECM

In the VECM form of the New Keynesian model, the aggregate demand equation can be written as:

\[
E_t \left[ b_u^D (L) \Delta u_{t+1} \right] + \tilde{\alpha}^D (R_t - \Delta p_t - \bar{r}) - \bar{v}^D = v_t^D. \tag{4.7}
\]

The fact that differences of unemployment instead of the level enter this equation follows from the fact that in the forward-looking IS curve the coefficients on future and current unemployment are the same.\(^{12}\) By

\[^{11}\text{See Beyer and Farmer (2002), p.21.}\]

\[^{12}\text{Both coefficients are equal to one. See equation (2.1).}\]
including lags of the differenced unemployment variable, this model is more general than the purely forward-looking IS curve in equation (2.4). The specification in equation (4.7) would arise, for example, if habit formation is present in the utility function of agents. Hence, equation (4.7) is consistent with the New IS curve that we used in the simulation of the extended New Keynesian model.

For the cointegration analysis presented below, it is important to notice that if the disturbance term in equation (4.7) is stationary, the aggregate demand relation would give rise to a cointegration vector linking the interest rate to the inflation rate with coefficients (1; -1). This relation is also called the Fisher relation.

Beyer and Farmer (2002) observe that the Fisher relation is a strong assumption to impose on the data, since there are a variety of alternative models that impose a weaker long-run restriction. To allow for this class of models, they also consider following aggregate demand function,

\[ E_t \left[ b^D_L \left( \Delta u_{t+1} \right) \right] + \alpha^D \left( R_t - \Delta p_t - \bar{r} - \beta^D_u u_t \right) - \bar{\nu}^D = v_t^D, \] (4.8)

where both the level and the differences of unemployment appear in the equation. This relation implies a cointegration vector of the form \( R - \Delta p - \beta^D_u u - \bar{r} = 0 \). Such a relation is consistent with the traditional IS curve, which postulates that there is an upward sloping relationship between the unemployment rate and the real interest rate. Moreover, Beyer and Farmer (2002) note that it is possible to derive a similar long-run relationship in overlapping generations models or in representative agent models with tax distortions that allow the real interest to vary with policy. In these models, the Fisher relation is a special case of the IS curve where the IS curve is horizontal.

### 4.2 The aggregate supply equation

In VECM form, the New Keynesian Phillips curve becomes:

\[ E_t \left[ b^S_L \left( \Delta^2 p_{t+1} \right) \right] + \alpha^S \left( u_t - \bar{u} \right) - \bar{v}^S = v_t^S. \] (4.9)

This is the VECM representation of a generalized version of the New Keynesian Phillips curve given by (2.5):

\[ \Delta p_t = \lambda_1 \left( L \right) \Delta p_{t-1} + E_t \left[ (1 - \lambda_1 \left( L \right)) \Delta p_{t+1} \right] - \alpha^S \left( u_t - \bar{u} \right) + \bar{v}^S + v_t^S. \] (4.10)

By including lags of the inflation rate, equation (4.10) has a richer specification than equation (2.5) because it adds backward looking elements to the

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13See McCallum and Nelson (1999).
price adjustment process. In fact, this specification is very similar to the widely used Furhrer and Moore (1995) specification of the New Keynesian Phillips curve.

The specification given by equation (4.10) is consistent with the natural rate hypothesis since all coefficients on the lags of inflation sum to zero. That is, we impose $\beta = 1$ on the New Keynesian model. As we have seen previously, this rules out any long-run relationship between the inflation rate and the unemployment rate. Technically, the natural rate hypothesis is imposed on the VECM form of the model by allowing only differences of the inflation rate to enter the Phillips curve. With this specification, in steady state a given unemployment rate ($u_t = \bar{u}_t$) is consistent with any constant inflation rate.

The natural rate of unemployment is, of course, an unobservable variable. For the empirical analysis, Beyer and Farmer (2002) start out by approximating this variable with a constant, $\bar{u}$. If this approximation were approximately correct, we would expect the unemployment rate $u_t$ to be stationary around a constant, provided the disturbance term $w^S_t$ is stationary. In this case, the unemployment rate would form one of the cointegration vectors in the VECM model. The resulting model would have the form,

$$
E_t \left[ b^S_{\Delta p} (L) \Delta^2 p_{t+1} \right] + \alpha^S u_t - \bar{v}^S = v^S_t. \tag{4.11}
$$

where $\bar{u}$ is part of the constants collected in $\bar{v}^S$.

However, the pronounced upward drift in the unemployment strongly suggests that the hypothesis of a stationary unemployment rate is unlikely to hold. In fact, this drift is consistent with the widely held belief that the natural rate of unemployment has been drifting over time due to structural changes in the labor market. Hence, it appears to be more appropriate to model the natural rate as a unit root process. To introduce this hypothesis into our empirical model, Beyer and Farmer (2002) assume in a second step that alternatively the natural rate follows the process,

$$
\tilde{\alpha}^S (\bar{u}_t - \bar{u}_{t-1}) = w^S_t + \tilde{\bar{w}}^S, \tag{4.12}
$$

where $w^S_t$ is an $I(0)$ variable, $\tilde{\bar{w}}^S$ is a drift parameter and $\tilde{\alpha}^S$ is the structural loading factor in the supply equation. That is, the natural rate of unemployment is modeled here as a random walk with drift. Assuming furthermore that there is no other shock hitting the aggregate supply equation so that $v^S_t$ is identically zero and abstracting from the constants in $\bar{v}^S$, we can rewrite the VECM specification of the New Keynesian Phillips curve as:

$$
E_t \left[ b^S_{\Delta p} (L) \Delta^2 p_{t+1} \right] + \alpha^S u_t = \alpha^S \bar{u}_t \tag{4.13}
$$

The left hand side of (4.13), which would constitute the observable part of our empirical model of aggregate supply, is clearly imbalanced because
of the relegation of the unobservable and non-stationary natural rate of 
unemployment to the error term. Put another way, since the 
assumption of a non-stationary natural rate of unemployment implies that \( \tilde{\alpha}^S u_t \) is non-
stationary too, we would not expect to find a cointegration relationship 
associated with the aggregate supply relation (4.13), since the only level 
variable that is included on the left hand side turns out to be non-stationary. 
Hence, our vector error correction model would have a reduced rank.

With a non-stationary natural rate of unemployment, the only way to 
arrive at an aggregate supply function with a stationary error term is taking 
differences of this relationship. In differences, equation (4.13) becomes 

\[
E_t \left[ \Delta b^S_{\Delta p} (L) \Delta^2 p_{t+1} \right] + \tilde{\alpha}^S \Delta u_t = w^S_t + \tilde{w}^S. 
\tag{4.14}
\]

In the empirical application, we account for the non-stationary error term 
by imposing a reduced rank restriction on our empirical VECM model; this 
has the effect of eliminating the level of unemployment from the aggregate 
supply equation, from which follows that we model this relation entirely in 
differences, consistent with (4.14). Finally, with a view towards the empirical 
results presented below, it needs to be emphasized that neither (4.11) nor 
(4.13) imply a cointegration relationship between the unemployment rate 
and inflation.

### 4.3 The policy rule

The policy rule in model (4.2) is given by:

\[
E_t \left[ b^P_R (L) \Delta R_{t+1} \right] + \tilde{\alpha}^P \left[ R_t - \tilde{\beta}^P_{\Delta p} \Delta p_t + \tilde{\beta}^P_u (u_t - \bar{u}_t) - \tilde{r} \right] - v^P_t = \bar{v}^P. 
\tag{4.15}
\]

In a more conventional form, this equation is equivalent to:

\[
R_t = \delta \left[ R_{t-1} + \left[ 1 - \delta \left( L \right) \right] \left[ \tilde{\beta}^P_{\Delta p} \Delta p_t - \tilde{\beta}^P_u (u_t - \bar{u}_t) + \tilde{r} \right] + v^P_t + \bar{v}^P. \tag{4.16}
\]

Here, \( \delta \left( L \right) \) is a polynomial that models the interest rate smoothing behavior of the central bank. The parameter \( \tilde{\beta}^P_{\Delta p} \) gives the long-run response of the central bank to the inflation rate, equivalent to \( (1 + \mu u^1) \) in (2.6), while \( \tilde{\beta}^P_u \) gives the response to the unemployment gap, which is equivalent to \( \mu u^2 \). The vector collecting the constants, \( \bar{v}^P \), includes also the inflation target, \( \bar{\pi} \).

Since the natural rate of unemployment is unobservable, we face a similar problem as in the preceding section. If approximating the natural rate with a constant proves to be adequate, the policy rule will give rise to a cointegration vector of the form \( R - \tilde{\beta}^P_{\Delta p} + \tilde{\beta}^P_u \Delta p - \gamma = 0 \), where \( \gamma \) is a

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constant encompassing both the steady state natural rate of interest, $\bar{r}$, and the constant natural rate of unemployment, $\bar{u}$.

On the other hand, if the natural rate of unemployment is better described as an unobservable unit root process, the policy rule will become,

$$E_t [b^P (L) \Delta R_{t+1}] + \alpha^P [R_t - \beta^P \Delta p_t + \beta^P u_t - \bar{r}] - \bar{v}^P = \bar{\alpha}^P \beta^P u_t + \bar{v}^P. \tag{4.17}$$

Similar to the aggregate supply equation (4.13), the observable part of the policy rule will again be imbalanced, and we will find no cointegration relationship because the error term on the right hand side, $\bar{\alpha}^P \beta^P u_t + \bar{v}^P$, is clearly non-stationary. Hence, in this case our New Keynesian model would yield only one cointegration vector, namely the Fisher relation resulting from the aggregate demand relationship.

However, precisely because of the unobservability of the natural rate of unemployment, it is also conceivable that the central bank would not respond to the unemployment gap, $u_t - \bar{u}_t$, but only to the actual unemployment rate, $u_t$. If this is the case, we would find a cointegration vector of the form $R - \beta^P \Delta p - \gamma = 0$ even though the natural rate of unemployment is non-stationary.

Interestingly, the latter scenario raises the possibility that the stochastic trend in the natural rate of unemployment is transmitted to the inflation rate. In fact, this transmission channel is emphasized by Orphanides (2000) in his explanation of the increase in the trend inflation rate during the 1970s. He argues that in the 1970s many economists did not realize that the natural rate of unemployment had increased, and substantiates this by looking at real time estimates of potential output. He finds that these estimates were much more optimistic than were subsequent revisions of the same series. Hence, it is likely that the Federal Reserve Bank concluded from the increase in the actual unemployment rate that the economy suffered from a severe shortfall in demand, even though the increase in unemployment stemmed from the increase in the natural rate of unemployment. The attempt of the central bank to stimulate the economy led consequently to significant inflationary pressures. In sum, by having monetary policy respond to $u_t$ instead of the correctly specified unemployment gap, $u_t - \bar{u}_t$, this explanation accounts simultaneously for the trend increase in the unemployment rate and the inflation rate by linking both to the stochastic drift in the natural rate. However, it is hard to believe that over the long-run, the central bank would fail to recognize that the natural rate of unemployment had increased, which means this explanation might be valid for the 1970s, but probably not for the 1980s or 1990s.

Finally, we need to consider the possibility that the inflation target of the central bank, $\pi_t$, follows a stochastic trend. Like the natural rate of
unemployment, this variable is unobservable and would consequently be included in the error term. Substituting $\pi_t$ for $\pi$ in equation (2.6), and assuming for simplicity that the natural rate of unemployment is constant, would yield following VECM representation:

$$E_t \left[ b_R^P (L) \Delta R_{t+1} \right] + \alpha P \left[ R_t - \beta_{\Delta_R}^P \Delta p_t + \beta_u^P (u_t - \bar{u}) - \bar{r} \right] - \bar{v}^P = -\alpha P \beta_{\Delta_R}^P \pi_t + v_t^P. \quad (4.18)$$

Like before, a stochastic trend in the inflation target would lead to an imbalance in the observable part of the policy rule, thereby leading to a reduced rank of our vector error correction model.

To summarize, if all disturbances in the New Keynesian model were stationary, ruling out a non-stationary natural rate of unemployment and a stochastic inflation target, and given the stability of the dynamics in this model, the variables in the model will converge to means satisfying

$$R - \Delta p = 0, \text{ or } R - \Delta p - \beta_u^D u - \bar{r} = 0, \quad (4.19)$$

$$u - \bar{u} = 0, \quad (4.20)$$

$$R - \beta_{\Delta_R}^P \Delta p + \beta_u^P u - \gamma = 0 \quad (4.21)$$

However, if one of the disturbances is non-stationary, the system will have a vector error correction presentation with at most two cointegration relationships, which would correspond either to (4.19), (4.20) and/or (4.21). The preliminary evidence suggests that the disturbance term in the aggregate supply equation is non-stationary, reflecting a unit root process in the natural rate of unemployment. In this case, we would expect to find only one cointegration relationship, which would correspond to relation (4.19). However, if the central bank responds to the actual unemployment rate because it cannot observe the non-stationary natural rate, we might find in the data an additional cointegration relationship corresponding to (4.21). On the other hand, a stochastic trend in the inflation target may lead to instability in (4.21), and, consequently, to only one cointegration relationship. In section 5 we will perform a multivariate cointegration analysis for West German data, test the rank of the system, and determine whether the resulting cointegration vectors are consistent with those derived from the New Keynesian model.

5 Multivariate cointegration analysis

In a first step (section 5.1), we estimate a vector autoregression (VAR) model for the period from 1965 until 2004 and use Chow sample-split tests
to test for a structural break in the model. The preliminary evidence on the long-run Phillips curve suggests that such a break has occurred in the late 1970s or early 1980s. Since a stable long-run Phillips curve appears to be present only in the latter sample period, finding formal evidence for a structural break around this time is crucial for the argument that the natural rate hypothesis may not hold for Germany in the past twenty years. In a second step, we are using univariate unit root tests to determine whether the unemployment rate, the interest rate and inflation in West Germany are non-stationary (section 5.2). In a third step, we present the results of the cointegration analysis for Germany (sections 5.3 and 5.4).

5.1 Testing for a structural break

To test for structural breaks, we estimate a VAR model for quarterly data for the full sample period, from 1965:2 until 2004:4. It consists of the 3-month interest rate, the West German unemployment rate and the inflation rate, computed on the basis of the West German consumer price index. On the basis of the Bayesian information criterion we choose a lag length of two. For the full sample period, this model shows some sign of misspecification, in particular in the interest rate equation (Table 1)

We test for structural breaks using CUSUM statistics and Chow tests. Both the CUSUM and CUSUM square statistics in figures 2 and 3 indicate that there are signs of structural instability in the interest rate equation around 1980. On a system level, this is confirmed by the Chow test: Figure 4 in the appendix shows clearly that the break statistic is significant in the early and late seventies. Since the early 1980s however, the relationship seems to be stable. This is consistent with the impression from Figure 8 that the relationship between the three variables changed around 1980. To investigate this further, we split the sample in 1979:4 and recompute the CUSUM, CUSUM square and sample-split Chow test statistics for the period until 2004, which now show few signs of instability (Figures 5, 6, and 16).

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17 Since a structural break in the time series may lead unit root tests to conclude that the time series are non-stationary, we compute first the structural break tests using a VAR model that is robust with respect to the stationary properties of the time series. For testing the lag length, we consider Bayesian information criterion.
18 All time series were originally obtained from Datastream. The corresponding Datastream codes are BD3MTH..R, WGTOTUN%E and WGCPE. For the period from 1999 onwards, the EURIBOR (instead of the FIBOR) was used as a measure of the 3-months interest rate.
19 The empirical analysis has been conducted using EViews 4.1 and JMulTi 4.02, for the latter programme see Lütkepohl (2004) for details.
20 Figure 4 in the appendix shows the bootstrapped p-values for a sample-split Chow test. The dashed resp. dotted line indicates the p-values of 5% and 10%. See Andrews (1994) and Andrews and Ploberger (1994) for the test. The critical values were obtained from the bootstrapping procedure described in Candelon and Lütkepohl (2001).
7). Residual tests also indicate a somewhat better specification of the two sub-periods (Table 1). In particular, the models for the two sub-periods show no signs of heteroscedasticity, in contrast to the model for the full period, and the rejection of non-normality in the interest rate equation is also less pronounced. In sum, splitting the sample in 1979:4 leads to stable and reasonably well-specified models for the sub-periods.

Before turning to the results for the cointegration analysis, it is noteworthy that the choice of a breakpoint in the fourth quarter of 1979 is also consistent with the observation of Clarida et al. (1998) that policy rules in the G3 countries changed after 1979. These authors note that after nearly a decade of high inflation, a number of important central banks, including the Bundesbank, began in 1979 a concerted effort to reign in inflation.21 As a result, after 1979 they raised interest rates sufficiently to increase the real interest rate in response to the inflationary pressure emanating from the second oil price shock, while before 1979 they allowed the real interest rate to decline following an increase in inflation. This change in policy is also visible in Figure 9, which shows the annualized real short-term interest rate in Germany. Consistent with a shift towards a more aggressive policy in fighting inflation, the real short-term interest increased markedly after 1979 and remained high throughout the 1980s. Finally, a break in the policy function of the Bundesbank is also consistent with the results from the CUSUM tests (Figures 2 and 3), which shows that the interest rate equation in the VAR is instable.

Assuming a break in 1979, Table 1 shows that the resulting VAR models for the sub-sample periods are probably slightly better specified than the model estimated for the entire period even if the sub-sample models still display signs of misspecification. However – as several tests indicate – the model is stable within the sub-sample periods.22 Below, we will show that we find stable cointegration vectors in both sub-sample periods. This allows us to employ multivariate cointegration analysis to investigate whether the natural rate hypothesis holds.

### 5.2 Univariate unit root tests

In this section, we employ conventional ADF tests to test the null hypothesis that the time series have a unit root. We compute the tests for the two sub-sample periods, since the structural break in the full sample period could be mistaken by the unit root tests as signs of non-stationarity. The lag length is chosen based on the Bayesian information criterion, and the results are shown in Table 2. The ADF tests indicate that all variables are integrated of order one, with the possible exception of the inflation rate in the early sample period.

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22Detailed results are available from the authors on request.
5.3 Results for the period 1965-1979

In a first step, we test the cointegration rank of the model using the maximum likelihood procedure suggested by [Johansen (1988)](#26). Table 3 reports the values of the $\lambda$-trace statistic testing the null hypothesis of no cointegration relationship, at most one and at most two cointegration relationships.

At the ten percent significance level – and indeed close to the five per cent level –, there is evidence for one cointegration relationship. The existence of one cointegration vector implies that two of the long-run relationships resulting from the New Keynesian model have non-stationary disturbances.

The estimated cointegration vector resulting from the empirical model has the following form:

$$R - 0.56\Delta p + 0.23u = 0.$$  \hspace{1cm} (5.1)

This cointegration vector is consistent with the policy rule [4.21](#4). This suggests that both the aggregate demand and the aggregate supply equations have non-stationary disturbances. The non-stationarity of the aggregate supply equation in particular does not come as a surprise, since already Figure 1 showed that there is no long-run correlation between the unemployment rate and inflation for the first sub-sample. Moreover, Figure 8 in the appendix shows that the unemployment rate increased over time in the first sample period, which points to a stochastic trend in the natural rate of unemployment. Above, we argued that in the case where the natural rate hypothesis holds, but the natural rate is non-stationary, we would not expect to find a cointegration relationship associated with the aggregate supply relation. Consequently, our finding of a reduced rank of our empirical model supports the natural rate hypothesis. Regarding the non-stationarity of the aggregate demand relation, this may be related to the fact that the demise of the Bretton Woods system led to a large real appreciation of the German currency foreign demand, which caused foreign demand for German goods to decline considerably. This regime shift may have induced a non-stationary behavior of aggregate demand.

The interpretation of (5.1) as a policy rule is supported by the fact that the estimated coefficients have the expected signs and are of plausible magnitude: according to (5.1) the Bundesbank responded to an increase in the unemployment rate by lowering the short-term interest, thereby seeking to stabilize the economy. In response to an increase in inflation, the Bundesbank increased the interest rate, which is consistent with an attempt to contain the inflationary pressures, but the estimated coefficient is smaller than one. Thus, the Bundesbank allowed the real short-term interest to decline when inflation increased. This result confirms [Clarida et al. (1998)](#25) observation that G3 central banks before 1979 did not respond strongly

\[ \text{We used the 'simple 2-stage' (S2S) procedure as described in Lütkepohl (2004).} \]
to inflationary pressures. Moreover, Clarida et al. (1999) have shown that with $\beta P$, the rational expectations equilibrium is not uniquely determined. Hence, monetary policy is unable to ensure that inflation converges in the long-run to its inflation target, and the inflation rate may permanently increase. The resulting trend increase in inflation set the stage for a more aggressive response of central banks in the late 1970s to the second oil price shock, thereby trying to avoid past mistakes and reverse inflationary pressures.

The results from the cointegration analysis suggest also that explanation in Orphanides (2000) of the simultaneous increase in the unemployment rate and the inflation rate due to a misjudgment of the central bank of the supply potential may be relevant not only for the United States but also for Germany. After all, our findings that the natural rate of unemployment followed a unit root process and that the Bundesbank responded to the actual unemployment rate instead of the unemployment gap are consistent with this view. Moreover, taking into account that the natural rate of unemployment was fairly stable throughout the 1960s, it is indeed conceivable that the Bundesbank did not realize that the natural rate increased permanently in the 1970s. If the Bundesbank mistakenly assumed that the natural rate remained constant, it would have interpreted the increase in the unemployment rate as indicating a large unemployment gap ($u_t - \bar{u} > 0$) and eased policy in an ultimately unsuccessful attempt to restore full employment. This easing would have slowed down the adjustment of the unemployment rate to the natural rate, but at the cost of permanently increasing inflation. Hence, the increase in the natural rate would have led to a simultaneous increase in the trend rates of inflation and unemployment.

5.4 Results for the period 1979-2004

The results for the rank test for the second sub-sample period are displayed in Table 4. We find strong evidence for a rank of two giving rise to the assumption of two cointegrating vectors in the system.

With two cointegration vectors, it is necessary to impose one identifying restriction on each vector to obtain estimates of just identified cointegration vectors. Following Beyer and Farmer (2002), we impose the two zero restrictions as shown in Table 5, where $\beta_1$ and $\beta_2$ are freely estimated parameters. This yields the following two cointegrating vectors:

$$R - 1.96\Delta p = 0,$$  \hspace{1cm} (5.2)

$$u + 0.64\Delta p = 0.$$  \hspace{1cm} (5.3)

\footnote{It may be useful to recall here that if the Bundesbank had responded to the unemployment gap, the non-stationarity of the natural rate would have meant that no stable cointegration vector corresponding to the policy rule would exist.}
The existence of two cointegration vectors implies that one of the disturbances in the New Keynesian model is non-stationary. If either the aggregate demand disturbance or the policy rule disturbance were non-stationary, one of the stationary long-run relationships implied by the New Keynesian model would be the long-run relationship resulting from the New Keynesian Phillips curve, equation (4.20). According to this equation, the unemployment rate would be a stationary. Since this is clearly rejected by the data, it follows that it is the aggregate supply disturbance term, which is non-stationary.

The analysis so far suggests that the natural rate of unemployment followed a random walk in both sub-sample periods. While the natural rate hypothesis is consistent with the empirical facts of the first sub-sample period, we still need to determine whether this also the case for the second period. In particular, it is still an open question whether the estimated cointegration vectors given by (5.2) and (5.3) are consistent with the long-run relations resulting from the aggregate demand equation, (4.19), and the policy rule, (4.21).

If the strong form of the aggregate demand equation holds, equations (4.19) and (4.21) would imply the following two cointegration vectors (neglecting constants):

\[ R - \Delta p = 0, \tag{5.4} \]
\[ u + \frac{1 - \tilde{\beta}_u}{\beta_u} \Delta p = 0. \tag{5.5} \]

Beginning with equation (5.4), this relation shows that in New Keynesian model the Fisher relation holds. However, when we test whether the Fisher relation is one of the cointegration vectors in (5.2) and (5.3), this is clearly rejected by the data at the five percent significance level. Regarding equation (5.5), since monetary policy responded after 1979 strongly to an increase in inflation, the parameter \( \tilde{\beta}_\Delta \) in the policy rule is likely to be greater than one.\(^{25}\) Since \( \beta_u \) is greater than zero, this means the New Keynesian model predicts the term \( (1 - \tilde{\beta}_\Delta) / \beta_u \) to be smaller than zero. However, in equation (5.3) this coefficient is positive. Hence, this version of the New Keynesian model does not fit the data.

As an alternative, we consider the weaker form of the aggregate demand equation, which allows for a long-run relationship between the real interest rate and the unemployment rate. In this case, the New Keynesian model would yield the following cointegration vectors:

\(^{25}\text{Clarida et al. (1998) estimate the parameters in the policy rule of the Bundesbank after 1979 and find that the parameter } \tilde{\beta}_\Delta \text{ is approximately 1.3 while } \beta_u \text{ is approximately 0.25. See Clarida et al. (1998), p. 1045.}
With $\tilde{\beta}_{\Delta p}$, $\tilde{\beta}_u^P$ and $\tilde{\beta}_u^D$ all larger than zero, the estimated cointegration vector (5.3) is now consistent with (5.6). However, equation (5.7) still cannot account for the positive coefficient on the inflation variable in (5.3) if $\tilde{\beta}_{\Delta p}^P > 1$. It follows that even the weaker version of the New Keynesian model does not provide an adequate description of the long-run relations in the German data.

In sum, the difficulties to reconcile the New Keynesian Phillips curve with our estimated cointegration vectors stem from the fact that one of the estimated vectors contains a negative long-run relationship between inflation and the unemployment rate. We have shown above that the New Keynesian Phillips curve specifically rules out any long-run relationship between these two variables. The other two equations in the model also do not give rise to such a relation, because empirical estimates of the Taylor rule by Clarida et al. (1998) suggest that $\tilde{\beta}_{\Delta p}^P$ typically takes a value of approximately 1.5. Inserting this into (5.7) shows that this model gives at best rise to a weak positive long-run relationship between unemployment and inflation, but not to the strong negative relationship that we observe in the data.

Even if the New Keynesian model were consistent with the estimated cointegration vectors, it would still be difficult to explain with this model why the unemployment rate increased over most of the second sub-sample period, while simultaneously the inflation rate declined. For the first sub-sample period, we showed that the New Keynesian model could account for a simultaneous increase in both variables, but in the second period, they have been moving in opposite directions.

It is, of course, possible that the natural rate of unemployment has continued to drift upwards, while at the same time the Bundesbank may have chosen to disinflate the economy, for reasons unrelated to the trend increase in unemployment. For example, the Bundesbank might have chosen to reverse the increase in trend inflation it had brought about in the 1970s. This view of events probably represents the main stream view, but it is nevertheless based on the somewhat unattractive assumption that the correlation in the trend components we observe in the data is nothing but a coincidence. In addition, this explanation would correspond to the case where the inflation target in the policy rule is not constant but follows a stochastic trend. As shown above, in this case we would not expect to find a cointegration
relationship corresponding to the policy rule. That is, with two independent drifts in the unemployment rate and the inflation rate, we would expect to find only one cointegration vector, which would represent the aggregate demand relation. However, this interpretation of events in the 1980s is contradicted by our finding of two cointegration vectors.

6 A long-run Phillips curve

In this section, we adopt a proposal by Beyer and Farmer (2002) and replace the natural rate hypothesis (4.20) with the following long-run relation:

\[ u - \bar{u} - \tilde{\beta} S \Delta p = 0. \]  

(6.1)

These two authors find that the natural rate hypothesis does not hold for U.S. data either, owing to a positive correlation between the trend components of inflation and unemployment, and propose as an alternative the aggregate supply equation given by (6.1) with a positively sloped long-run Phillips curve (\( \tilde{\beta} S > 0 \)). Like our results for Germany, they find evidence for a structural break in 1979 in the policy equation. For the time period from 1980 until 1999, they find two cointegration vectors, and conclude that one of those vectors corresponds to the upward sloping long-run Phillips curve (6.1) and the other to the policy equation. Hence, the non-stationarity in this model is induced by a non-stationary disturbance term in the aggregate demand equation.

In general, our results for Germany are similar to those of Beyer and Farmer (2002), but in contrast to the United States, the correlation between the trend components of unemployment and inflation is negative in Germany and not positive. Hence, we modify (6.1) as follows,

\[ u - \bar{u} + \tilde{\beta} S \Delta p = 0. \]  

(6.2)

In this model, the long-run Phillips curve has a negative slope, just like the traditional Phillips curve.

Before we explore the theoretical reasoning behind such a relation, we need to show that it is consistent with the data. Like Beyer and Farmer (2002), we assume that the disturbance term in the aggregate demand equation is non-stationary.\(^{26}\) Combining the policy equation (4.21) with (5.7) yields the following long-run relationships implied by the modified New Keynesian model:

\[ R - \left( \tilde{\beta}_P^P + \tilde{\beta}_P^U \tilde{\beta}_U^S \right) \Delta p = 0 \]  

(6.3)

\(^{26}\)We also considered the case where the disturbance term in the policy equation is non-stationary, but this turned out to be inconsistent with the data.
In this case, the cointegration vector (5.3) can be interpreted as an estimate of the long-run Phillips curve (6.4), yielding a slope parameter of -0.64. Moreover, cointegration vector (5.2) implies $\beta_{\Delta p}^P + \beta_{\Delta p}^S \beta_{\Delta p}^S = 1.96$. Inserting in this equation our estimate for $\beta_{\Delta p}^S$ and assuming that $\beta_{\Delta p}^P$ is approximately 1.3, we obtain a value of approximately 0.7 for $\beta_{\Delta p}^P$. This value is close to the one typically found in the estimation of policy rules.\textsuperscript{27} Hence, this model appears to be consistent with the data. Importantly, this finding implies that the natural rate hypothesis has to be abandoned to obtain a version of the New Keynesian model that is consistent with the long-run trends in German data.

If the natural rate hypothesis does not hold, this raises the possibility that demand conditions have a lasting effect on the German unemployment rate. In particular, our estimate of the long-run Phillips curve suggests that the reduction in the inflation rate in the 1980s was accompanied by a permanent increase in the unemployment rate. Average inflation decreased from approximately 5 percent in the 1970s to approximately 3 percent in the 1980s and to 2.5 percent in the 1990s. Assuming that this reduction in trend inflation is the result of the Bundesbank’s determination to lower average inflation, our estimate of the long-run Phillips curve implies that in the 1980s this would have been accompanied by a permanent increase in the unemployment rate of 2.3 percentage points, and a further increase of 0.6 percentage points in the 1990s. This would explain about a quarter of the increase in average unemployment from 3 percent in the 1970s to 8 percent in the 1980s and to 9 percent in the 1990s.

Conventional wisdom holds that the trend increase in unemployment has structural causes. Since our finding that weak macroeconomic conditions play a role for the trend increase in unemployment contradicts conventional wisdom, we try to bolster our case with additional evidence. To this end, we plot in Figure 10 the relationship between the vacancy and the unemployment rate for West Germany, the so-called Beveridge curve. At any moment, the Beveridge curve is a downward sloping curve since it is easier to fill a vacancy when there are more unemployed workers to choose from. The upper left area can be described as a fast growing economy with many employment opportunities whereas the lower right area reflects a recession state with few employment opportunities and high unemployment. In a frictionless labor market, the Beveridge curve would coincide with the axes of the diagram.\textsuperscript{28} The more frictions there are in the labor market, the more

\textsuperscript{27}See e.g. Clarida et al. (1998).

\textsuperscript{28}In a frictionless labor market, there would be no unemployed workers if vacancies were available, and there would be no vacancies if unemployed workers were available to fill these positions.
the Beveridge curve shifts outward. Since an increase in structural unemployment typically means that the labor market has become less efficient, one would expect that an increase in the structural unemployment rate coincides with an outward shift in the Beveridge curve.\textsuperscript{29} However, Figure 10 shows that the Beveridge curve in Germany has been remarkably stable in the period from 1970 to the early 1980s, which is exactly the period when the unemployment rate increased from 1 percent to 9 percent.\textsuperscript{30}

To summarize, our empirical findings suggest that the disinflation in the first half of the 1980s is likely to have contributed to the permanent increase in the unemployment rate that occurred in this time period. However, the further increases in trend-unemployment in the remainder of the 1980s and 1990s are probably unrelated to demand conditions, since the trend-inflation rate changed little in this period. Instead, the strong outward shifts in the Beveridge curve in this period suggest that other factors – structural factors or the interaction of macroeconomic shocks and institutions – are responsible.

However, the question remains what theories are able to explain a stable long-run Phillips in West Germany. There are several possible explanations for that phenomenon:

1. One possible explanation of the long-run Phillips curve we observe in the data draws on what \textit{Greenwald and Stiglitz} (1995) call the ‘second strand of New Keynesian literature’. The key ingredients of these models are risk averse firms, a credit allocation mechanism with risk averse banks, the existence of asymmetric information and real wage rigidity in the labor market. This model can give rise to very persistent effects of demand conditions on unemployment, with aggregate supply ultimately becoming dependent on aggregate demand.

2. \textit{Akerlof et al.} (2000) offered an alternative explanation for our empirical finding. Based on microeconometric evidence these authors argue that the long-run Phillips curve may be nonlinear. They build a macroeconomic model in which agents at low rates of inflation display near-rationality, meaning that in the wage setting process they either ignore inflation entirely, or they fail to appreciate that inflation increases the nominal demand for their services, and consequently demanding higher wages would not reduce their competitiveness. Hence, they are prepared to accept lower wage increases than they otherwise would. In this case, at low rates of inflation wages are set lower relative to nominal demand than predicted in models with fully rational agents, and the economy can operate at a higher level of real activity.

\textsuperscript{29}See also \textit{Bleakley and Fuhrer} (1997) on the factors determining the Beveridge curve.
\textsuperscript{30}See \textit{Solow} (2000), who summarizes the evidence on the Beveridge curve in Germany and France for a similar interpretation.
This means that at low rates of inflation, the unemployment rate will be below its natural rate defined as the unemployment rate resulting from an environment with fully rational agents. However, if inflation approaches zero, the near-rational effect disappears, and the unemployment rate returns to the natural rate, which is also the case when inflation increases.

3. Ball (1999) offers another explanation for the link between disinflation and higher unemployment. Like in asymmetric information models, in his model aggregate demand conditions can have long-run effects on the unemployment rate. He argues that these effects arise due to hysteresis effects.31 The response of monetary policy to a recession and the accompanying disinflation is decisive for the path of unemployment following the recession. Ball shows empirically for the recessions in the early 1980s that countries like the United States, which have been successful in maintaining low unemployment, have eased monetary policy in a recession and reflated the economy once the recession has ended, bringing the unemployment rate back to its pre-recession levels. Other countries like Germany, for example, have maintained a tight monetary policy stance during the recession and refused to reflate the economy after the recession in order to disinflate the economy even further. However, by keeping the unemployment rate high for a long period of time, Ball argues that this made it possible for hysteresis effects to take hold, causing the natural rate of unemployment to increase. This effect is due to the long-term unemployed becoming increasingly unemployable in the labor market, either because their human capital deteriorates, or because employers view them suspiciously, or because they loose attachment to the labor force. To illustrate Ball’s argument, in Figure 11, we plotted unemployment and inflation figures for the US and West Germany and shaded periods of sustained falling inflation as ‘disinflation’ periods.32 The periods of falling inflation clearly lasted longer in Germany than in the US which holds until the mid-1990s. In sum, by drawing out the disinflation over a long period of time, countries like Germany had to pay a high price for a lower inflation rate by incurring a permanently higher unemployment rate.33

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31 Hysteresis as an explanation for persistently high European unemployment has been introduced by Blanchard and Summers (1986).
32 A disinflation period was characterized by a negative change in inflation in the ‘smoothed’ (7 quarters moving average) time series.
33 Ball’s model implies that as time passes, tight monetary policy becomes less effective in reducing inflation, because the long-term unemployed become less of a threat to other workers in the competition for jobs, and therefore exert less downward pressure on wages. This suggests that a gradual approach to disinflation is not only costly, but also inefficient. Nordhaus (1999), p. 245, summarizes the lessons from Ball’s model for disinflation as follows: ‘I would label his approach the Powell-Ball doctrine for economic stabiliza-
A New Keynesian model with hysteresis

This paper has argued that monetary policy might have contributed to the trend increase in German unemployment; the issue that remains to be resolved is whether monetary policy can also be used to permanently lower unemployment in countries like Germany. It needs to be emphasized here that the empirical evidence presented in this paper suggests that to the extent that tight monetary conditions did lead to a lasting increase in unemployment, this happened mostly in the 1980s. This result arises mainly because the task of reducing the trend inflation rate to acceptable levels was essentially completed by the late-1980s. Thus, a negatively sloped long-run Phillips curve cannot account for much of the increase in trend unemployment since then, since the reduction in trend inflation in this period was marginal. Moreover, given the currently low levels of inflation, the unemployment costs of disinflation are unlikely to play a significant role in the future either. Hence, the issue is not so much how to engineer a disinflation without incurring high costs in terms of permanent unemployment, because Germany went through this phase already almost twenty years ago; rather, the issue is whether monetary policy can contribute in some way to a permanent reduction in unemployment once unemployment has shifted upwards.

An important implication of the preceding theoretical discussion is that simply pursuing an expansionary policy to increase the trend rate of inflation is unlikely to lead to a permanent reduction in unemployment, because in two of the three models discussed here a low inflation rate in itself is not the cause of high unemployment. In particular, in the asymmetric information models and the hysteresis model a traditional Phillips curve relation would not arise in the data because there is an inherent trade-off between unemployment and inflation, but because a poorly conducted monetary policy can have negative long-run real effects. That is, the long-run aggregate supply curve may be vertical, but its location is endogenous to macroeconomic policy, and sustained tight demand conditions may shift this curve inwards. In these models, to be successful in reducing unemployment permanently, monetary policy has to reflate the economy without triggering inflationary pressures, since otherwise higher in-
Inflation would force the central bank eventually to change course and deflate the economy again, thereby reversing previous employment gains again. If the expansionary stance cannot be sustained for a long period of time, there is no hope that firms will shift their supply curve outwards or that hysteresis will work in reverse.

Regarding the hysteresis approach, Ball (1999) provides empirical and theoretical evidence that monetary policy can be successful in raising employment permanently with only modest inflationary costs. From a theoretical standpoint of view, it is essential that inflation expectations have a backward-looking component for this to happen.\footnote{For a formal exposition, see Buiter (1987).}

In this case, an expansionary policy does not lead to an immediate upward revision of inflation expectations, and monetary policy may be able to reduce unemployment over a sustained period of time without triggering strong inflationary pressures. With hysteresis at work, the higher employment level resulting from the monetary stimulus may become permanent. Since this increases the productive capacity of the economy, this tends to dampen the inflationary pressures resulting from the expansionary policy and a permanent increase in employment can be achieved at modest inflationary costs. In the following, we are going to investigate the effectiveness of monetary policy in a model with hysteresis in more detail.

In this model, we extend the standard New Keynesian model introduced above by including hysteresis effects. This way, we hope to obtain a first insight whether adding one of the mechanisms explaining a non-vertical Phillips curve could change the effectiveness of monetary policy in New Keynesian models markedly. As a base model we use the model (2.1) to (2.3). To add more realistic dynamics we follow a suggestion by McCallum (2001) and modify the IS and the price adjustment equations. To introduce more persistence into the output equation, he proposes adopting a household utility function in which current utility depends on the ratio \( \frac{C_t}{C_{t-1}} \), where \( C_t \) denotes per capita consumption. This specification introduces habit persistence into the behavior of optimizing agents via the parameter \( h \). The larger \( h \) is, the more agents will hesitate to change their consumption level from that of the previous period. Regarding the price adjustment equation, he suggests to replace equation (2.2) with

\[
\Delta p_t = (1 - \phi) E_t \Delta p_{t+1} + \phi \Delta p_{t-1} + \alpha (y_t - \bar{y}_t) + v_t, \tag{7.1}
\]

which is the so-called Fuhrer-Moore specification of the price adjustment process. In contrast to equation (2.5), the Fuhrer-Moore specification includes in addition to the forward-looking component also a backward-looking component of the expectations formation process. As noted above, this is an important element if monetary policy is to have long-run real effects in a
model with hysteresis. Of course, this is not the reason why modern Phillips curve models often include such a backward-looking component. They do so because a purely forward-looking Phillips curve like (2.5) is found to yield sticky prices while the inflation rate displays little persistence, which is contradicted by the persistence of inflation observed in the data.\footnote{For a discussion of the empirical shortcomings of the price adjustment equation (2.5), see also \cite{Estrella98} and \cite{Mankiw01}.} This shortcoming can be remedied by including a backward-looking component in the expectations formation process, like the Fuhrer-Moore specification does. Finally, to add some realism to the policy rule, McCallum replaces the inflation rate in (2.3) with expected inflation, $E_{t-1} \Delta p_t$. \cite{McCallum01} simulates this model by assuming that the model’s parameters are $b_1 = -0.4$, $\beta = 0.99$, $\alpha = 0.03$, $\mu_1 = 0.5$, $\mu_2 = 0.5$, and $\mu_3 = 0.8$. In addition, in the IS curve with habit persistence he sets $h = 0.8$, and in the Fuhrer-Moore price adjustment equation he sets $\phi = 0.5$. The stochastic process for potential output is specified as a near random walk process,

$$\tilde{y}_t = 0.95\tilde{y}_{t-1} + \varepsilon_{t}^{Pot}$$ \hspace{1cm} (7.2)

Simulating this model yields the impulse-response functions shown in Figure 12 (solid lines).\footnote{We are grateful to Bennett T. McCallum for making his Matlab program available to us.} In the first row the solid lines show the response of the economy to a monetary policy shock.\footnote{In Figure 12, $\Delta p$ denotes inflation and $\bar{y}$ potential output.} This shock increases the interest rate on impact by one percentage point, and within the next ten quarters it returns to its base line.\footnote{Neither the interest rate nor the inflation rate are annualized in Figure 12.} The monetary policy shock leads to a negative output response and a reduction in inflation, with both series displaying a hump shaped response consistent with evidence from VAR models. The second row displays the response to an IS shock. Without habit persistence, this shock would increase output on impact by one percent, but due to the habit persistence effect output only increases by approximately 0.5 percent. The effects of this shock on output dissipate within one year. The short-run Phillips curve is fairly flat, so this shock has only a small impact on inflation. The policy response is also fairly small, due to the small inflation response and the interest rate smoothing in the policy rule. The third row shows the response to a price shock. This shock leads to a strong increase in inflation which lasts for about two years. In response, monetary policy tightens substantially and maintains this stance for a long time. The tight monetary policy stance leads to a deep and long recession.\footnote{It should be noted here that due to the relative simple structure of the model used here, the price shock in itself would not lead to a negative output response. In contrast, with a constant nominal interest rate the increase in inflation would lead to a decline in the real interest rate, thereby stimulating output.} In the fourth row the
response to a negative technology shock is shown, which lowers potential output on impact by one percent. Since potential output is stationary in this model, the effect of the technology shock dissipates eventually. With actual output only slowly adjusting to the fall in potential output, the output gap in the price adjustment equation is positive and inflation increases. Consequently, monetary policy becomes tighter, too.

To introduce hysteresis into this model, we follow a suggestion by Mankiw (2001) and re-specify the equation for potential output as follows:

\[ \tilde{y}_t = 0.85\tilde{y}_{t-1} + 0.1y_t + \varepsilon_{Pot} \]  

(7.3)

We will use this specification in place of the specification of equation (7.2) used in the simulation of the New Keynesian model without hysteresis. In equation (7.3), we preserve the near-random walk specification of potential output common in New Keynesian models, but add a small hysteresis effect by including past actual output as a determinant of potential output. This way, potential output tends to adjust towards the level of actual output. This specification represents a short cut to modeling hysteresis, since we omit the microfoundations that would give rise to hysteresis effects, but it captures nevertheless the essential feature of these models to make the natural rate of output dependent on the actual level of output. Moreover, this specification has the advantage that it preserves the linear structure of the New Keynesian model. Finally, it should be noted that in equation (7.3) we keep the hysteresis parameter small in size in order to show that already a small modification of the standard New Keynesian model can have major implications for the conduct of monetary policy.

In Figure 12, we plot the impulse response functions of the extended New Keynesian model together with the results for the hysteretic specification of this model (dotted lines). Regarding the monetary policy shock, Figure 12 shows clearly that adding hysteresis does not change much the properties of the New Keynesian model. From this follows that even if hysteresis is present, an expansionary monetary policy in itself would not be effective in reducing unemployment permanently, because the boom created by a stimulating monetary policy shock would not be persistent enough to allow large hysteresis effects to set in. Like monetary policy shocks, neither IS nor technology shocks would be effective in permanently reducing unemployment, since the output response in both cases is again not persistent enough for hysteresis to have significant effects. It needs to be emphasized here that we obtain these results even though the model used here includes already all the elements typically used in New Keynesian models to enhance the persistence of variables.

However, our simulation exercise shows that the results for the price shock in the model with hysteresis differ substantially from those found in the non-hysteretic model. In particular, in the case of the price shock the
recession induced by the sustained monetary policy tightening in response to the increase in inflation is deep and long enough for significant hysteresis effects to take hold. Figure 12 shows that after five years about one third of the peak effect of the monetary tightening on output is still present in the output series. This result is consistent with Ball’s hypothesis that a disinflation drawn out over a long period of time can have significant adverse effects on real variables if hysteresis is present.

These results suggest that an opportunistic monetary policy, which stimulates the economy in the presence of a negative price shock, could be effective in lowering the unemployment rate permanently. A negative price shock lowers the inflation rate for a relatively long time, which offers monetary policy the opportunity to pursue a sustained expansionary stance without triggering inflationary pressures, thereby being able to engineer a boom long enough for hysteresis to work in reverse. However, the response to a positive price shock, which leads to an increase in inflation, would have to be asymmetric. That is, monetary policy would have to respond either with a sharp but short tightening of policy to reign in the inflationary pressures without causing a long recession, or it would have to respond to a positive price shock in a much weaker manner than to a negative shock, thereby avoiding a deep recession in the first place. As long as the commitment of the central bank to the inflation target is credible, such a response would not lead to a permanently higher inflation rate following the price shock. This asymmetric response is essential for monetary policy to have a permanent effect on output, because if the distribution of price shocks is symmetric in the sense that over time as many negative and positive shocks occur, a symmetric policy response implies that the positive and negative long-run effects of monetary policy actions would cancel each other out.

However, even though these results point to some potential of monetary policy to contribute to the objective of lowering unemployment in Germany, it is worth noting that the New Keynesian model with hysteresis would not give rise to the negative long-run relationship between inflation and unemployment which we observe in the data. The reason for this is that in this model only price shocks lead to persistent effects of monetary policy, and these shocks push unemployment and inflation into the same direction, thereby giving rise to a positive and not a negative long-run relationship between these two variables. To obtain a negative long-run relationship – as we observe in the data –, the effects of aggregate demand disturbances would have to be considerably more persistent than they are in the present model. Since we have already included habit persistence in the IS curve to make the effects of IS and monetary policy shocks more persistent, additional mechanisms inducing even more persistent would be needed. Including capital accumulation into the model might lead to some additional persistence, but this is an area for further research.
8 Conclusion

In sum, in this paper we showed that the New Keynesian model has difficulties accounting for the long-run correlations that we observe in the German data. In particular, we find that the natural rate hypothesis central to New Keynesian models is inconsistent with the negative long-run correlation between inflation and unemployment that is clearly present in the 1980s and 1990s. There are, however, a number of approaches in modern macroeconomics, which could give rise to such a correlation. Since in all these models non-linearities play an important role, they deviate from the New Keynesian model in a significant way, since the latter is inherently linear. Interestingly, the inclusion of non-linearities represents also a return to the past, since already the earliest Keynesian models included such asymmetries in the form of downward but not upward rigid nominal wages. This suggests the possibility that present day New Keynesian models may be missing an important aspect of earlier Keynesian models that may be crucial for explaining the German experience in the 1980s. Even though these asymmetries are difficult to model, it might be nevertheless worthwhile to pursue this avenue to gain a better understanding of the limits and potential of monetary policy in European economies that suffer from persistently high unemployment.
References


References


Appendix

Table 1: VAR specification statistics

<table>
<thead>
<tr>
<th>Sample</th>
<th>Equ.</th>
<th>Lags</th>
<th>Port.</th>
<th>AR 1-5</th>
<th>Norm.</th>
<th>Het.</th>
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<tbody>
<tr>
<td>1965:2-2004:4</td>
<td>i</td>
<td>2</td>
<td></td>
<td>108.2 [0.00]</td>
<td>33.9 [0.01]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>u</td>
<td></td>
<td></td>
<td>0.1 [0.95]</td>
<td>11.7 [0.76]</td>
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<tr>
<td></td>
<td>Δp</td>
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<td></td>
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<tr>
<td></td>
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<td>83.7 [0.00]</td>
<td>258.5 [0.00]</td>
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<td>1965:2-1979:3</td>
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<td></td>
<td>6.92 [0.03]</td>
<td>10.7 [0.82]</td>
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<tr>
<td></td>
<td>u</td>
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<td>1.0 [0.61]</td>
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<td>system</td>
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<td>31.9 [0.00]</td>
<td>203.9 [0.11]</td>
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</tr>
<tr>
<td>1979:4-2004:4</td>
<td>i</td>
<td>2</td>
<td></td>
<td>31.9 [0.00]</td>
<td>18.3 [0.30]</td>
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<tr>
<td></td>
<td>u</td>
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<td>14.1 [0.59]</td>
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</tr>
<tr>
<td></td>
<td>Δp</td>
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Table 2: Results from ADF tests

<table>
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<tr>
<th>Sample</th>
<th>Variable</th>
<th>Specification</th>
<th>ADF t-stat.</th>
<th>Integration</th>
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</thead>
<tbody>
<tr>
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<td>-2.9</td>
<td>I(1)</td>
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<tr>
<td></td>
<td>u</td>
<td>1,c,t</td>
<td>-2.9</td>
<td>I(1)</td>
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<td>Δp</td>
<td>0,c,t</td>
<td>-4.0*</td>
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<tr>
<td>1979:4 - 2004:4</td>
<td>i</td>
<td>1,c,t</td>
<td>-2.7</td>
<td>I(1)</td>
</tr>
<tr>
<td></td>
<td>u</td>
<td>1,c,t</td>
<td>-3.4</td>
<td>I(1)</td>
</tr>
<tr>
<td></td>
<td>Δp</td>
<td>4,c</td>
<td>-2.2</td>
<td>I(1)</td>
</tr>
</tbody>
</table>

Asterisks denote: * = significant at 5% level; ** = significant at 1% level. A time trend (t) is included in the regression if the time series appears to be trending over time, otherwise only a constant (c) is allowed for. The lag length was choosen according to the minimum of BIC.
Table 3: Cointegration test statistics: sample 1965Q2 – 1979Q3

<table>
<thead>
<tr>
<th>$H_0$ of rank test</th>
<th>Trace test statistic</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r \leq 0$</td>
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<td>0.059</td>
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<tr>
<td>$r \leq 1$</td>
<td>10.1</td>
<td>0.634</td>
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<tr>
<td>$r \leq 2$</td>
<td>2.4</td>
<td>0.697</td>
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</table>

Results of Johansen (1988) test. A constant is included in the cointegration relationship. The test statistics have been computed using JMulTi 4.02.

Table 4: Cointegration test statistics: sample 1979Q4 – 2004Q4

<table>
<thead>
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<th>$H_0$ of rank test</th>
<th>Trace test statistic</th>
<th>P-value</th>
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<td>$r \leq 1$</td>
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<td>0.048</td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td>5.7</td>
<td>0.223</td>
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</table>

Results of Johansen (1988) test. A constant is included in the cointegration relationship. The test statistics have been computed using JMulTi 4.02.

Table 5: Restrictions on the cointegration vectors

<table>
<thead>
<tr>
<th>i</th>
<th>u</th>
<th>$\Delta p$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\beta_1$</td>
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<tr>
<td>Vector 1</td>
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<td>0</td>
</tr>
<tr>
<td>Vector 2</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

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Figure 1: Unemployment and inflation in West Germany
Figure 2: Results of CUSUM test: full sample
Figure 3: Results of CUSUM square test: full sample
Figure 4: Results of sample-split Chow test: full sample
Figure 5: Results of CUSUM test: 1979Q4 – 2004Q4
Figure 6: Results of CUSUM square test: 1979Q4 – 2004Q4
Figure 7: Results of sample-split Chow test: 1979Q4 – 2004Q4
Figure 8: The time series and their trend components
Figure 9: The real short-term interest rate in Germany

Figure 10: The Beveridge curve for West Germany
Figure 11: Disinflation and unemployment
Figure 12: Impulse-response functions for the New Keynesian models