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The price and emission effects of a market stability reserve in a competitive allowance market

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The price and emission effects of a market stability reserve in a competitive allowance market

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Abstract

We study the impact on price and emission paths of the allowance preserving market stability reserve (MSR) to be introduced to the EU ETS in 2019 in a dynamic optimization framework. The MSR adjusts the number of allowances auctioned as a function of the size of the aggregate bank in the past, i.e. in times of a large aggregate bank it shifts the issue date of allowances into the future. We show that in a perfectly competitive allowance market such a MSR only affects price and emission paths if the baseline equilibrium is no longer feasible. If the MSR changes the allowance price path it increases in the short run but drops in the medium run relative to the baseline and is unaffected in the long run. In case of economic uncertainty, the MSR is predicted to increase price volatility.

Keywords: Market stability reserve; cap-and-trade; EU ETS reform; price volatility

JEL codes: Q58; Q54

1 Introduction

As part of a wider array of instruments, the EU Emissions Trading System (EU ETS) is the backbone of the European Union's climate policy efforts and currently covers about 45% of all greenhouse gas emissions of the 31 participating countries. Having put a price on carbon for ten years now, the EU ETS looks back on a mixed performance, pressured by institutional shortcomings and severe demand shocks (Ellerman et al., 2014). Currently, it is the general perception that at a range between 5.00-7.50 EUR in 2014, the price of emission allowances is too low (Clò et al., 2013; Nordhaus, 2011). This normative judgment is mainly based on two observations, namely that a) estimates of the social cost of carbon tend to be substantially higher (Tol, 2009; Grosjean et al., 2014; Knopf et al., 2014) and b) prices would need to be much higher to steer investment into low carbon technologies e.g. in the energy sector. The main reason for the low level in prices is seen in what has been called a large over-allocation relative to demand of emission allowances resulting in a sizable 'surplus' in the market of roughly 2.1 billion in 2013 (Burtraw et al., 2014; Knopf et al., 2014). While we will not go into assessing the validity of these claims, they induced the European Commission to introduce a reform of the EU ETS, including a market stability reserve (MSR) scheduled to start operating by 2019 (EP, 2015). This MSR adjusts the number of allowances auctioned in a particular year based on the size of the aggregate bank ('surplus') at the beginning of the previous year. If firms in total hold more than 833 million unused allowances in their accounts, the number of auctioned allowances gets reduced by an amount equal to 12% of the size of the aggregate bank. These allowances are placed in the MSR and thus temporarily set aside the market. They are then re-injected in batches of 100 million allowances per year as soon as the aggregate bank drops below a lower threshold of 400 million. This mechanism continues until the reserve is finally depleted and ceases to impact the underlying allocation schedule. As a result, given its current design, the MSR is allowance-preserving in the long run. Its objective is not to endogenize the climate target but to increase allowance prices and make them more reliable.

The European institutions (EP, 2015) expect the MSR to help "delivering the necessary investment signal to reduce CO_2 emissions in a cost-efficient manner and being a driver of low-carbon innovation" (p. 2), to "make the EU ETS more resilient to supply-demand imbalances so as to enable the ETS to function in an orderly market" (p. 3) and to "enhance synergy with other climate and energy policies" (p. 3). The first quote implicitly contains the expectation that the MSR will increase the price level in the allowance market.[1] The second quote states that the MSR is expected to improve the ability of the EU ETS to deal with demand shocks caused e.g. by an economic downturn or by overlapping policies such as renewable energy support schemes (third quote). Therefore, prices are expected to respond less to demand shocks than has been observed in the past. We conduct a positive analysis of the impact of the MSR on price and emission paths in a competitive cap-and-trade system to check whether the MSR has the potential to live up to stated expectations.

Since the MSR as a quantity-based adjustment mechanism is a novelty, there are only a few papers studying this instrument. Based on the dynamic optimization framework of cap-and-trade systems with banking introduced by Rubin (1996), extended to include uncertainty by Schennach (2000) and recently applied to the EU ETS by Ellerman et al. (2015), Fell (2015), Kollenberg & Taschini (2015) and Schopp et al. (2015) assess the impact of a MSR, but differ in several aspects from our paper. Kollenberg & Taschini (2015) and Fell (2015) consider perfectly competitive markets but do not require the MSR to be allowance pre-

[1] However, note that incentives to adopt cleaner technologies might not be monotonic in the price of emissions (Perino & Requate, 2012).

serving. Both do not explicitly model the reserve as a stock and implicitly allow it to have both strictly positive and negative values at the end of the time horizon. Hence, allowances can be both created and destroyed by the MSR without limits and they can be shifted forward in time. To explain why the size of the aggregate bank might cause inefficiencies in the EU ETS, Schopp et al. (2015) assume that banking by polluting firms is restricted and that once the surplus exceeds a given threshold, speculators with a higher discount rate enter the market. Salant (2015) challenges this justification of a reserve and suggests that the observed price movements can be explained equally well by regulatory risk. Furthermore, he doubts that the MSR is a suitable fix for the challenges faced by the EU ETS as a temporal shift of allocations to the future by the reserve is not capable of reducing inefficiencies when facing substantial regulatory risk. The surplus would then in fact not be at the root of current low prices. Holt & Shobe (2015) experimentally test a MSR in the lab. They observe a marked increase in the equilibrium price. However, in their setting with eighteen rounds, re-injection of allowances was not observed (see their Figure 3, p. 18).

The remainder of the paper is organized as follows: Section 2 presents the setup and section 3 the dynamic optimization equilibrium of a competitive allowance market in the deterministic baseline case based on Rubin (1996). The MSR is introduced in Section 4 and the effects on price and emission paths are derived. Section 5 considers the impact of an MSR under uncertainty about the demand for allowances. The last section concludes. Proofs are relegated to the appendix.

2 The Model

There is a continuum of polluting firms with mass one in a perfectly competitive market for emission allowances. Each firm is characterized by an abatement cost function

$$C_i\left(u_i - e_i(t)\right) = \begin{cases} \frac{c}{2}\left(u - e_i(t)\right)^2 & if \quad e_i(t) \leq u \\ 0 & if \quad e_i(t) > u \end{cases}, \tag{1}$$

where $u > 0$ are baseline emissions in the absence of environmental regulation, $e_i(t)$ are actual emissions of firm i at time t and hence $u - e_i(t)$ is abatement. For ease of exposition we focus on firms with symmetric and quadratic abatement cost functions (with $c > 0$).[2] Each firm has an initial stock or 'bank' of emission allowances b_i^0 and $x_i(t)$ is the flow of allowances bought at time t from the market or in allowance auctions. Net sales of an individual firm can be both positive and negative at a particular point in time but in aggregate, sales of firms equal the number of allowances auctioned at time t ($\int_{i=0}^{1} x_i(t)di = S(t) \geq 0$). We abstract from free allocations of emission allowances by the regulator. Including them would only shift profits of firms up but would not alter the equilibrium emission or price paths. The price path of emission allowances $p(t)$ is exogenous to firms but endogenous to the model. Initial banks b_i^0 and time paths of individual banks $b_i(t)$ are allowed to differ between firms. We use the following equalities and notational conventions for aggregate values: $u = \int_0^1 u_i di$, $b^0 = \int_0^1 b_i^0 di$, $b(t) = \int_0^1 b_i(t)di$, $x(t) = \int_0^1 x_i(t)di$ and $e(t) = \int_0^1 e_i(t)di$.

[2]The specific functional form is not important for the gist of the results given that marginal abatement costs are increasing.

3 The Baseline Case

In the baseline case (B) the time path of auctioned allowances $S_B(t)$ is assumed to decline at a constant rate a, i.e. $S_B(t) = S_0 e^{-at}$, where S_0 is the number of allowances auctioned at $t = 0$.[3] Allowances are assumed to be scarce in the long run ($\int_0^\infty S_B(t)dt + b^0 < \int_0^\infty udt$). With unregulated emissions u constant, the scarcity of allowances is increasing over time. We assume that the initial bank is strictly positive ($b^0 > 0$) but finite and hence the market starts with a surplus.

Each firm solves the following optimization problem

$$min_{e_i(t),x_i(t)} \quad \int_{t=0}^{\infty} \left[e^{-rt} C\left(u - e_i(t)\right) + p(t)x_i(t)\right] dt \tag{2}$$

$$s.t.: \quad \dot{b}_i(t) = x_i(t) - e_i(t) \tag{3}$$

$$b_i(t) \geq 0 \qquad \forall t. \tag{4}$$

The time horizon is assumed to be infinite since the end of the EU ETS - or rather the period of continuous banking of allowances - is not yet determined. Allowances of both Phase II (2008-2012) and Phase III (2013-2020) can be transferred to Phase IV (2021-2030) and the rules for a potential Phase V have not yet been specified. Explicit EU emission targets for 2040 and 2050 exist and climate policy is expected to be in place for the rest of the century. Any uncertainty about the continuation of the EU ETS, or rather the ability to bank emissions, is thus assumed to be captured by the discount rate r. Assuming a fixed time horizon might shut down the MSR before all allowances have been re-injected into the market and hence setting a specific time horizon does affect the environmental and economic performance of the intervention. Given that such a date would be arbitrarily chosen, we refrain from doing so.

[3]In fact, the EU ETS exhibits a "linear reduction factor" that reduces the annual cap on emissions to be in line with the reduction target for 2030. Given that we use an infinite time horizon, a linear representation is not appropriate.

The present-value Hamiltonian for firm i's optimization problem is

$$H = e^{-rt}\left[C\left(u - e_i(t)\right) + p(t)x_i(t)\right] + \lambda_i(t)\left[x_i(t) - e_i(t)\right], \tag{5}$$

where $\lambda_i(t)$ is the co-state on the state equation (3). The Lagrangian is

$$L = e^{-rt}\left[C\left(u - e_i(t)\right) + p(t)x_i(t)\right] + \lambda_i(t)\left[x_i(t) - e_i(t)\right] - \mu_i(t)b_i(t), \tag{6}$$

where $\mu_i(t)$ is the multiplier function of the non-borrowing constraint (4).

The corresponding necessary conditions for an optimal solution yield

$$\begin{aligned}
\dot{b}_i(t) &= x_i(t) - e_i(t), && (7)\\
\dot{\lambda}_i(t) &= \mu_i(t), && (8)\\
\mu_i(t)b_i(t) = 0, \quad \mu_i(t) &\geq 0, \quad b_i(t) \geq 0, && (9)\\
-e^{-rt}C'\left(u - e_i(t)\right) = -e^{-rt}c\left(u - e_i(t)\right) &= \lambda_i(t), && (10)\\
-e^{-rt}p(t) &= \lambda_i(t). && (11)
\end{aligned}$$

Note that the initial endowment of firm i, b_i^0, does not explicitly feature in conditions (7) to (11). It shifts profits up or down because it affects the net number of allowances bought by firm i, but not the equilibrium emission profile. Since initial endowments are the only dimension in which firms differ and optimal emission paths do not depend on them, the emission paths of all firms will be the same in equilibrium. They can hence be represented by the path of aggregate emissions $e^*(t) = \int_{j=0}^{1} e_j^*(t)dj$, where an asterisks indicates equilibrium values.

The price path is characterized by conditions (8), (9) and (11), which yield

$$\frac{\dot{p}(t)}{p(t)} = \begin{cases} r & if \quad b(t) > 0 \\ r - \frac{e^{rt}\mu(t)}{p(t)} & if \quad b(t) = 0. \end{cases} \tag{12}$$

While the price of allowances rises at the rate of interest r, firms are indifferent as to when they acquire or sell emission allowances as long as they can realize the optimal emission path (condition 9). Because banking of allowances is allowed, the equilibrium price will never increase at a rate above r. Firms would otherwise want to purchase more allowances today to sell them in the future, i.e. bank additional allowances. However, if in equilibrium the allowance price rises at a rate less than r, firms would like to borrow allowances. In most real world emission trading schemes, there are restrictions on borrowing.[4] Hence, market forces cannot prevent the allowance price from rising at a rate below the rate of interest. However, this will only happen if the constraint on borrowing is binding (here: $b(t) = 0$).

Using conditions (7) to (11) and defining time τ_B as the instant when the aggregate bank equals zero for the first time ($\tau_B = \inf\{t : b^*(t) = 0\}$) this can be simplified to the following set of equilibrium conditions:

$$
\begin{aligned}
p_B^0 e^{rt} &= C'(u - e_B^*(t)) = c(u - e_B^*(t)) \quad \forall t < \tau_B, &(13)\\
\dot{e}_B^*(t) &= \frac{rC'(u - e_B^*(t))}{C''(u - e_B^*(t))} = r(u - e_B^*(t)) \quad \forall t < \tau_B, &(14)\\
\int_{t=0}^{\tau_B} e_B^*(t)dt &= b^0 + \frac{S_0}{a}(1 - e^{-a\tau_B}), &(15)\\
\int_{s=0}^{t} e_B^*(s)ds &\leq b^0 + \frac{S_0}{a}(1 - e^{-at}) \quad \forall t < \tau_B, &(16)
\end{aligned}
$$

[4]Note that while explicit borrowing is not permitted in the EU ETS, some implicit borrowing is feasible, because allowances for the next year are issued in part before the compliance date of the previous year (Chevallier, 2012; Bertrand, 2014; Kollenberg & Taschini, 2015). In Phase II, where almost all allowances were issued for free at the beginning of each year, borrowing was effectively feasible up to one year's allocation. However, in Phase III, an increasing share of allowances is auctioned and auctions are spread over the entire year shifting the issue date of a substantial share of a year's inflow of allowances past the compliance date for the previous year (mid-May). Hence, the borrowing constraint has moved closer to zero within the EU ETS. Moreover, this mechanism is not available between phases, i.e. allowances of Phase III could not be used for compliance in Phase II. In what follows, we assume that borrowing is not feasible in the EU ETS, but discuss implications of relaxing this assumption.

$$e_B^*(t) \quad = \quad S_0 e^{-at} \quad \forall t \geq \tau_B. \tag{17}$$

Conditions (13) and (14) determine the optimal price and emission paths up to a constant shift parameter. The latter can be specified by using condition (15) that follows from the bank being empty at τ_B. Condition (16) ensures feasibility of the equilibrium path up to τ_B and condition (17) determines emissions at and after τ_B. Once the bank has been depleted, firms will never again have an incentive to bank, i.e. in the deterministic baseline case the banking phase is unique.[5]

4 The Market Stability Reserve

We now investigate when and how the introduction of a market stability reserve (MSR) affects the equilibrium price and emission paths compared to the baseline case. Generally speaking, a MSR is a set of rules that, conditional on the state of the aggregate bank of allowances, $b(t)$, postpones the auctioning of some allowances to a later point compared to the baseline. In contrast to (hard and soft) price collars that have been discussed in the literature at some length (Murray et al., 2009; Grüll & Taschini, 2011; Fell et al., 2012) there is no direct link to the price of allowances. The auctioning profile $S_{MSR}(b(t),t)$ is thus no longer exogenous but a function of the aggregate bank $b(t)$. No allowances are created or destroyed in the process, i.e.

$$\int_{t=0}^{\infty} S_{MSR}(b(t),t)\, dt = \int_{s=0}^{\infty} S_B(t) dt. \tag{18}$$

Note that Fell (2015) and Kollenberg & Taschini (2015) differ in their representation of the MSR. They do not require the MSR to be allowance preserving, which is the main driver for the differences in results. Since we assume the MSR

[5] See also Schennach (2000) and Ellerman et al. (2015).

to shift the issue date of some allowances into the future but never to bring it forward, it holds that $\int_{s=0}^{t} S_{MSR}(b(s),s)\,ds \leq \int_{s=0}^{t} S_B(s)ds$ for all t and for all feasible emission profiles $e(t)$. Moreover, the MSR is assumed to reduce the number of auctioned allowances only at times of a strictly positive aggregate bank, i.e.

$$S_{MSR}(b(t),t) \geq S_B(t) \quad if \quad b(t) = 0 \quad \forall t, e(t). \tag{19}$$

The MSR might be seeded with an initial stock of allowances R^0 as will be the case for the EU ETS, because currently backloaded allowances will be put directly into the reserve. In line with the rules of the MSR to be introduced to the EU ETS, this does not change the total number of allowances available in the long run, i.e. $b^0 = b^0_{MSR} + R^0$.

Then, the optimization problem of firm i under the MSR is identical to (2)-(4), because firms operate in a perfectly competitive market and do not take into account how their individual behavior affects aggregate outcomes, i.e. $e(t)$ and also $S_{MSR}(b(t),t)$. The MSR only affects the set of feasible aggregate emission paths. The corresponding equilibrium conditions are then:

$$p^0_{MSR}e^{rt} = C'(u - e^*_{MSR}(t)) = c\,(u - e^*_{MSR}(t)) \quad \forall t < \tau_{MSR} \tag{20}$$

$$\dot{e}^*_{MSR}(t) = \frac{rC'\left(u - e^*_{MSR}(t)\right)}{C''\left(u - e^*_{MSR}(t)\right)} = r\,(u - e^*_{MSR}(t)) \quad \forall t < \tau_{MSR} \tag{21}$$

$$\int_{t=0}^{\tau_{MSR}} e^*_{MSR}(t)dt = b^0_{MSR} + \frac{S_0}{a}(1 - e^{-a\tau_{MSR}}) + R^0 - R(\tau_{MSR}) \tag{22}$$

$$\int_{s=0}^{t} e^*_{MSR}(s)ds \leq b^0_{MSR} + \frac{S_0}{a}(1 - e^{-at}) + R^0 - R(t) \quad \forall t < \tau_{MSR} \tag{23}$$

$$e^*_{MSR}(\tau_{MSR}) = S_0 e^{-a\tau_{MSR}} - \dot{R}(\tau_{MSR}), \tag{24}$$

where $R(t) \geq 0$ represents the number of allowances held in the reserve at point t and τ_{MSR} is defined analogously to τ_B, i.e. it is the point in time when the aggregate bank under the MSR is depleted for the first time ($\tau_{MSR} = \inf\{t : b^*_{MSR}(t) = 0\}$). Note that in contrast to condition 17 condition (24) does not hold for all $t > \tau_{MSR}$. Conditional on the specific rules governing the MSR, the banking phase might no longer be unique, which will indeed be the case for the EU ETS as we show below.

The equilibrium conditions give rise to the following proposition (see also Fell, 2015; Salant, 2015; Schopp et al., 2015).

Proposition 1 *If the equilibrium emission path of the baseline case, $e_B^*(t)$, is feasible under the market stability reserve, i.e.*

$$\int_{s=0}^{t} e_B^*(s)ds \quad \leq \quad b_{MSR}^0 + \int_{s=0}^{t} S_{MSR}\left(e_B^*(s), s\right) ds \quad \forall t \leq \tau_B, \tag{25}$$

both the equilibrium price and emission paths are unaffected by the market stability reserve, i.e.

$$e_{MSR}^*(t) = e_B^*(t) \quad \forall t, \tag{26}$$

$$p_{MSR}^*(t) = p_B^*(t) \quad \forall t. \tag{27}$$

Given a particular structure of the market stability reserve, it is possible to derive conditions for its design parameters to meet the requirements of proposition 1. Proposition 1 requires feasibility. If unrestricted borrowing is allowed, this ceases to be a binding constraint.

Proposition 2 *If the cap-and-trade scheme allows unrestricted borrowing, then a market stability reserve that leaves the total number of allowances issued unaffected, has no effect on equilibrium emission or price paths, regardless of how it shifts the auctioning of allowances over time.*

However, if borrowing is restricted, feasibility can impose a binding constraint. To identify how the MSR impacts equilibrium outcomes, it is necessary to specify the rules governing the MSR. Looking at the EU ETS, the following rules have been adopted (EP, 2015): (a) if the aggregate bank exceeds $\bar{b}$ =833 million allowances at the beginning of a given year, then the number of allowances auctioned in the next year will be reduced by 12% of the size of that bank and held-back allowances will be placed in the reserve, (b) if the aggregate bank drops below $\underline{b}$ =400 million allowances at the beginning of a given year, then I =100 million allowances will be taken from the MSR and are additionally auctioned in the following year unless there are less than 100 million allowances left in the MSR. If the latter is the case, all remaining allowances in the MSR are auctioned off.[6] In the present model, these rules can be represented as follows:

$$\dot{R}(t) = \begin{cases} \gamma b_{MSR}(t) & \text{if} \quad b_{MSR}(t) > \bar{b}(t) = 833 million \\ -I & \text{if} \quad b_{MSR}(t) < \underline{b}(t) = 400 million \\ & \quad\;\; \text{and} \quad R(t) > 0 \\ 0 & \text{if} \quad otherwise \end{cases} . \tag{28}$$

where γ is the percentage of the aggregate bank that determines the number of allowances withheld from auctioning (here: 12%).[7] The MSR will be seeded with about 1.5 billion allowances from the so-called "backloading" and other special reserves (EP, 2015).

Given the rules for the MSR, the following proposition holds:

Proposition 3 *If the equilibrium emission path of the baseline case, $e_B^*(t)$, is not feasible under the market stability reserve, then:*

- *in the short run (for all $t \leq \tau_{MSR}$, with $\tau_{MSR} < \tau_B$) the equilibrium emission path with the MSR is lower and the equilibrium price path higher than in the baseline case*
- *in the medium run (for an interval that starts at some point in time $t_{cross} \in$*

[6]In the continuous time version of the problem used here, the problem of less than 100 million allowances remaining in the reserve does not arise since the MSR can stop issuing allowances at any point in time and not only at full integer intervals.

[7]Note that the time lag that exists in the MSR of the EU ETS has been ignored for convenience.

$[\tau_{MSR}, \tau_B]$ and ends at $\bar{t}_3$) the equilibrium emission path with the MSR is higher and the equilibrium price path lower than in the baseline case

- *in the long run (for all $t > \bar{t}_3$) emission and price paths with and without a MSR are identical*
- *there are exactly two separate banking phases $[0, \tau_{MSR}]$ and $[\underline{t}_3, \bar{t}_3]$ with $\tau_{MSR} < \underline{t}_3 < t_3 < \bar{t}_3$.*

Figure 1 illustrates proposition 3 by presenting the equilibrium paths of emissions, prices, banks and the MSR for the scenarios with and without a MSR. Parameter values are chosen to broadly represent the case of the EU ETS in 2019.[8] Taking a look at panel (a) of figure 1, it is evident that the MSR shifts the allocation path (solid black line) relative to the baseline case without MSR (solid gray line). The MSR creates discrete jumps in the allocation path when its activity changes from reducing auctioned volumes to inactivity at t_1 as the aggregate bank undercuts the upper threshold ($\bar{b}$), from inactivity to injections at t_2 as the lower threshold $\underline{b}$ is undercut and back from injections to inactivity as soon as the reserve is completely empty (t_3). Due to being emissions preserving, the areas between the depicted allocation paths sum up to zero and once the MSR is empty ($t \geq t_3$), those paths are identical in the two scenarios. The MSR-induced reduction of auctioned volumes in the short run leads to a faster depletion of the aggregate bank (solid black line in panel (c)) relative to the baseline scenario (solid gray line) as well as the build-up of the reserve stock (dashed black line). This reduction of the aggregate bank implies that firms cannot realize their optimal baseline emission path. Firms respond by increasing their abatement efforts in order to counter

[8]Parameter values used: $a = 0.022$, $c = 0.05044$ (see Landis, 2015, Table 4), $S_0 = 1.9$ billion, $u = 1.9$ billion, $b^0 = 3$ billion, $R^0 = 1.5$ billion, $r = 0.1$, $\bar{b} = 833$ million, $\underline{b} = 400$ million, $\gamma = 0.12$, $I = 100$ million.

at least some of the additional temporary scarcity induced by the MSR (see also Holt & Shobe, 2015). The dotted black line in panel (c) shows this elevated and stretched outside placement of stored allowances when compared to the baseline. As laid out in proposition 3, relative higher abatement during the first banking phase for all $t \in [0; \tau_{MSR}]$ leads to a higher path for the price of allowances (solid black line in panel (b)) in the short run. However, due to the non-borrowing constraint binding earlier with the MSR than without ($\tau_{MSR} < \tau_B$), the initially higher price path under the MSR starts to rise at a lower rate earlier than in the baseline case. While the baseline price path keeps rising at r until τ_B, the smaller slope of the price path with a MSR leads to an intersection with the baseline path (solid gray line) at t_{cross} and stays below until the end of the second banking phase at $\bar{t}_3$. With the MSR still injecting after τ_{MSR}, this reversal in price levels leads to higher emissions in the medium run (see panel (a)). Given the allowance preserving nature of the reserve, firms anticipate the sudden downward shift of the allocation path at $t = t_3$. As a consequence of banking still being allowed, they smooth this shift by accumulating a small bank (bank remains below the lower trigger level) for all $t \in (\underline{t}_3; \bar{t}_3)$ (solid black line in panel (d)). Due to this smoothing, emission levels under the MSR approach those of the baseline scenario (panel (a)) with allowance prices under the MSR rising again at r (panel (b)), until the aggregate bank is depleted for good and price and emission levels with and without the MSR are the same. This second banking phase fits an additional kink into the equilibrium price path when compared to the baseline scenario. After $t = \bar{t}_3$, both the baseline and MSR-scenario are behaving identically again, as the effects of the reserve vanish and the system returns to its baseline dynamics under an aggregate bank of zero.

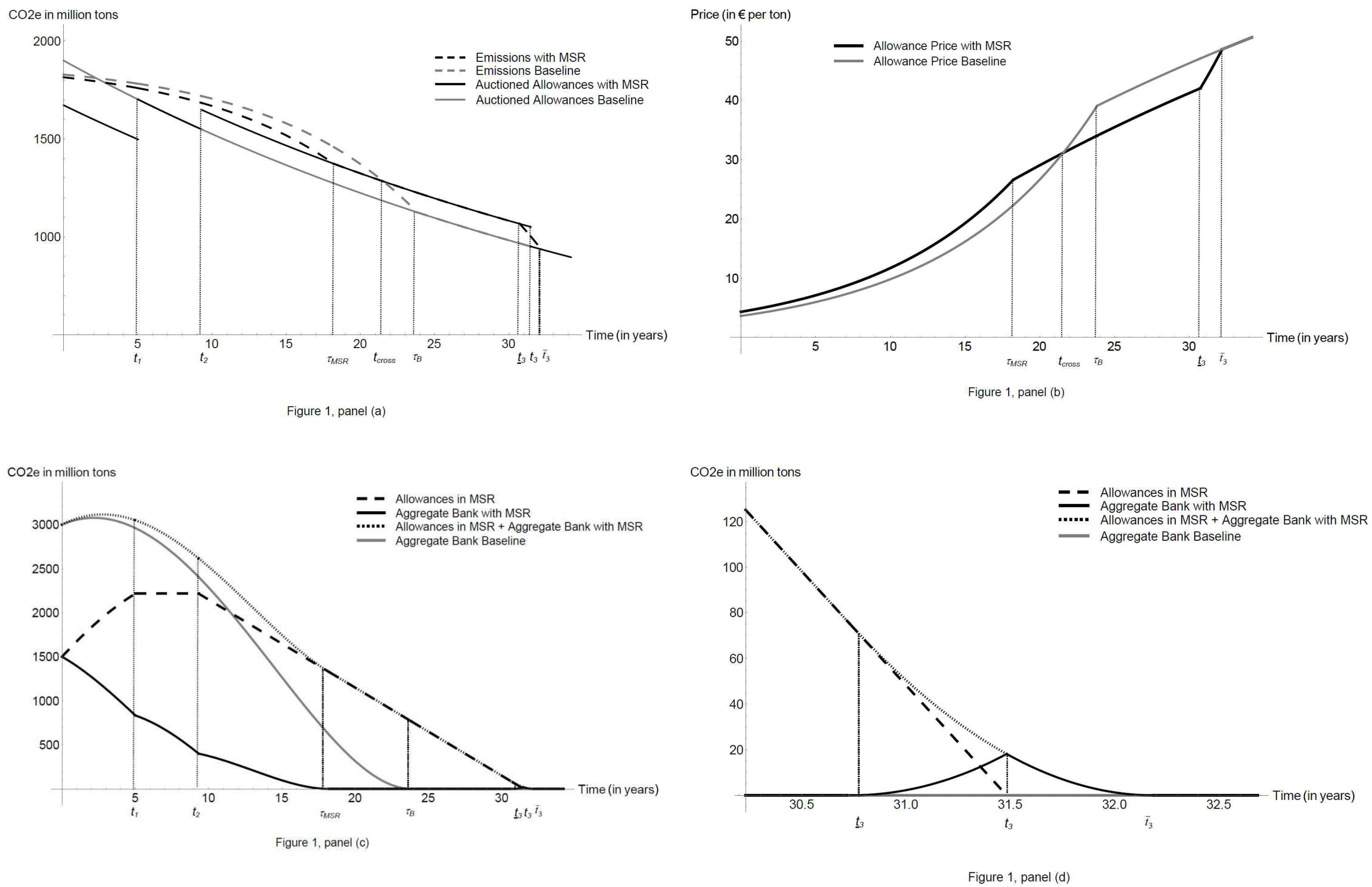

Figure 1: Comparison between baseline and MSR in deterministic setting. Panel (a) presents emission and allowance auction paths, panel (b) price paths, panel (c) aggregate banks and MSR and panel (d) zooms in on the evolution of the aggregate bank around t_3.

The effect captured by proposition 3, that the introduction of a MSR increases allowance prices in the short run, is the main motivation for introducing a reserve to the EU ETS. Propositions 1 to 3 reveal that in a deterministic setting and in the absence of market failures, this effect is possible but by no means guaranteed. Proposition 3 reveals that even if the MSR increases prices in the short run, this has to be traded off against a drop in prices in the medium run. That the MSR creates incentives to invest in low carbon technologies, especially those with long investment cycles and life spans such as power plants can therefore not be taken for granted.

Real-world emissions trading schemes of course do not operate under such stylized conditions. We therefore now consider economic uncertainty.

5 The Effect of Economic Uncertainty

The polluting industry can be subject to different forms of uncertainty. Here, we focus on economic uncertainty affecting abatement cost functions but abstract from long-term political or regulatory uncertainty about the design and stringency of a cap-and-trade scheme.[9] Changes in marginal abatement costs can be caused by economic fluctuations, introduction or removal of overlapping climate policies such as support for renewables, a phase-out of nuclear technology or technological progress.

We start out with a more general representation of uncertainty before we focus on a more specific yet illustrative case. In concreto, we assume that there is a shock $\varepsilon(t)$ to the level of unregulated emissions u of the representative firm. The

[9] Salant (2015) investigates the impact of regulatory uncertainty on allowance prices in the EU ETS.

abatement cost function under uncertainty is therefore

$$\frac{c}{2}\left(u+\varepsilon(t)-e_i(t)\right)^2, \tag{29}$$

where the distribution of $\varepsilon(t)$ is characterized by the density function $\phi(\varepsilon(t),t)$. Shocks might be persistent such that once they have occurred, the mean is (temporarily or permanently) adjusted.

A risk-neutral firm's optimization problem therefore becomes

$$\begin{aligned} min_{e_i(t),x_i(t)} \quad & E_0\left[\int_{t=0}^{\infty}\left[e^{-\rho t}C\left(u+\varepsilon(t)-e_i(t)\right)+p(t)x_i(t)\right]dt\right] \\ s.t.: \quad & \dot{b}_i(t)=x_i(t)-e_i(t) \\ & b_i(t)\geq 0 \qquad \forall t \end{aligned} \tag{30}$$

where $E_t[\cdot]$ is the expected value given all information available at time t and ρ is the interest rate inclusive of the asset-specific risk premium. The expected price path, given what is known at time t, satisfies (see Schennach, 2000)

$$E_{\underline{t}}[\dot{p}(t)] \;=\; E_{\underline{t}}\left[\rho p(t)-\mu_i(t)e^{\rho t}\right], \quad \forall t\geq \underline{t}, \tag{31}$$

which is the equivalent of condition (12) under uncertainty. In the context of an emissions trading scheme, condition (31) implies the formation of a futures market for allowances. If the borrowing constraint is not binding with certainty in an interval $[\underline{t},\bar{t}]$, i.e. $E_{\underline{t}}[\mu_i(t)]=0$ for all $t\in[\underline{t},\bar{t}]$, then the allowance price rises at rate ρ within this interval. If there is a positive probability that the borrowing constraint is binding, i.e. $E_{\underline{t}}[\mu_i(t)]>0$, the expected price will rise at a rate less than ρ over the respective interval.

Trivially, proposition 2 still holds as unrestricted borrowing by design neutralizes the effect of a MSR irrespective of uncertainties present. However, proposition 1 needs to be adjusted as follows:

Proposition 4 *Under economic uncertainty represented by the random variable $\varepsilon(t)$, characterized by the density function $\phi(\varepsilon(t))$, it holds that, if all equilibrium emission paths of the baseline case which feature a non-zero value of the density function ϕ are still feasible under the market stability reserve in case borrowing is not permitted, i.e.*

$$\int_{s=0}^{t} e_B^*(s,\varepsilon(s))ds \quad \leq \quad b^0 + \int_{s=0}^{t} S_{MSR}(s,\varepsilon(s))ds \qquad (32)$$
$$\forall t \leq \tau_B \quad and \quad \varepsilon(t) : \phi(\varepsilon(t)) > 0,$$

then both the equilibrium price and emission paths are unaffected by the market stability reserve, i.e.

$$e_{MSR}^*(t,\varepsilon(t)) = e_B^*(t,\varepsilon(t)) \quad \forall t, \qquad (33)$$

$$p_{MSR}^*(t,\varepsilon(t)) = p_B^*(t,\varepsilon(t)) \quad \forall t. \qquad (34)$$

Propositions 2 and 4 establish when a MSR has no effect on equilibrium emission and price paths under economic uncertainty. To identify the impact of a binding MSR, we turn to a more specific shock. Let us assume that it is common knowledge that at a given point in time t_{shock} (with $b(t_{shock}) > 0$), the random variable ε takes one of two values $\{\varepsilon^L, \varepsilon^H\}$ with probabilities $1 - w^H$ and w^H, respectively. Furthermore, assume that the MSR binds if $\varepsilon = \varepsilon^H$ but not if $\varepsilon = \varepsilon^L$. The cap-and-trade scheme still induces scarcity in the market for allowances in the long run in both cases.

While this setting is restrictive, it serves to illustrate some interesting properties of a MSR in the context of economic uncertainty. It allows to isolate how a MSR changes the price response of a cap-and-trade scheme when uncertainty over the future demand for allowances is resolved. This is of particular interest, because the EU explicitly quotes an expected increase in the resilience of the EU ETS toward supply-demand imbalances as a reason for introducing the MSR (EP, 2015).

Before we turn to the effect of the MSR on equilibrium paths, we first study how the shock on unregulated emissions, i.e. the demand function for emission allowances, affects prices.

Lemma 1 *Ceteris paribus, an unexpected, persistent shift of the demand function to the left (right), i.e. a reduction (increase) in u, reduces (increases) the level of the equilibrium price path ($\partial p_B^0/\partial u > 0$) for all t. An increase (decrease) in the initial bank b^0, i.e. an increase (decrease) in the supply of allowances, reduces (increases) the level of the equilibrium price path for all $t < \tau_B$.*

Adjustments in the supply and demand for allowances thus have an intuitive effect on equilibrium prices. An unexpected recession would cause the allowance price to drop.

Consider the point in time when the shock materializes (t_{shock}) and assume that $b_B(t_{shock}) = b_{MSR}(t_{shock}) + R(t_{shock})$, i.e. that initial conditions are comparable when the shock occurs (which will not be the case in equilibrium). If the demand for allowances is lower than expected ($\varepsilon = \varepsilon^L$), the MSR is not binding by assumption. The equilibrium emission path of the baseline case is therefore feasible and remains optimal (proposition 1).

If the demand for allowances is higher than expected ($\varepsilon = \varepsilon^H$), the MSR binds. Given initial conditions at t_{shock}, the impact of the MSR on price and emission paths are captured by proposition 3. Hence, prices rise above the baseline case in the short run, then drop below the baseline in the medium run and are unaffected in the long run. Note that the positive demand shock increases the price at t_{shock} in baseline case already (lemma 1). At least in the short run and with comparable conditions at t_{shock}, the MSR adds to this effect.

How does the anticipation of the response to the shock affect firm behavior for $t < t_{shock}$? At the instant t_{shock}, the co-state variable $\lambda(t)$ has to meet the following condition (Goeschl & Perino, 2009):

$$\lim_{t \to t_{shock}} \lambda(t_{shock}) = E[\lambda(t_{shock})] = w^H \lambda^H(t_{shock}) + (1 - w^H)\lambda^L(t_{shock}), \quad (35)$$

where superscript H represents the value of the co-state if $\varepsilon = \varepsilon^H$ and L if $\varepsilon = \varepsilon^L$. The intuition for condition (35) is as follows: If the (expected) value of the co-state jumped, firms would have an incentive to shift purchases of allowances either forward or backward in time. However, by assumption the shock occurs when the bank is strictly positive, i.e. the expected allowance price rises at rate r and hence firms are ex-ante indifferent between purchasing an additional allowance just before or just after the shock occurs. Expected jumps in λ are therefore ruled out by the no-arbitrage condition.

Due to conditions (11) and (35) the allowance price satisfies $\lim_{t \to t_{shock}} p(t_{shock}) = E[p(t_{shock})]$. Above we have established that if $b_B(t_{shock}) = b_{MSR}(t_{shock}) + R(t_{shock})$, then $p^H_{MSR}(t_{shock}) > p^H_B(t_{shock})$ and $p^L_{MSR}(t_{shock}) = p^L_B(t_{shock})$. Given the strictly positive bank, the price rises at rate ρ before the shock occurs in both institutional settings. If expected prices at t_{shock} differ, so do the paths for $t < t_{shock}$. Because price paths fully determine emission paths and subsequently the total amount of unused allowances stored in the bank and the MSR, $b_B(t_{shock}) = b_{MSR}(t_{shock}) + R(t_{shock})$ cannot occur in equilibrium, as the expected price level at t_{shock} differ.

In equilibrium, the price path before the demand shock with the MSR will be above the path without a MSR, but less so than would be implied by $b_{MSR}(t_{shock}) + R(t_{shock}) = b^*_B(t_{shock})$. The (relatively small) increase in the allowance price under the MSR prior to the shock reduces emissions and increases the sum of allowances stored in the bank and the MSR. This in turn reduces the price level after the shock has occurred (lemma 1) and therefore the expected price level at t_{shock}. In this setting the following proposition holds:

Proposition 5 *In case of a simple demand shock $\varepsilon \in \{\varepsilon^L, \varepsilon^H\}$ at t_{shock}, where the MSR is binding only after a positive demand shock and $b^*_{MSR}(t_{shock}) > 0$, it holds* *that*

$$
\begin{aligned}
p^*_{MSR}(t) &> p^*_B(t) \quad \forall t < t_{shock}, && (36)\\
p^{H*}_{MSR}(t_{shock}) &> p^{H*}_B(t_{shock}), && (37)\\
p^{L*}_{MSR}(t_{shock}) &< p^{L*}_B(t_{shock}), && (38)\\
b^*_{MSR}(t_{shock}) + R^*(t_{shock}) &> b^*_B(t_{shock}) && (39)
\end{aligned}
$$

i.e. the MSR amplifies the response of the allowance price to positive and negative demand shocks.

Figure 2 illustrates proposition 5 by presenting the price paths with (black) and without a MSR (gray).[10] If a MSR is in place, initially ($t < t_{shock}$) prices are higher as firms want to bank additional allowances in order to reduce scarcity in case of a positive demand shock that would make the MSR binding. Once uncertainty is resolved, prices in both institutional settings jump to their new equilibrium levels. Directly after a positive demand shock, allowance prices are significantly higher if a MSR is in place than if not. Note that at t_{shock} initial conditions in the two settings are no longer the same as firms have abated more under the MSR and hence the sum of allowances stored in the aggregate bank and the MSR is larger than without the MSR. Price paths intersect at t^H_{cross}. When the MSR eventually nears depletion, a second banking phase makes for a smooth transition and price paths with and without the MSR merge at $\bar{t}_3$ and are identical thereafter. After a negative demand shock when the MSR by assumption is not binding, prices in both settings drop, but slightly more so in case where a MSR is in place, because scarcity of allowances is somewhat lower due to the higher level of abatement before the shock. In the long run when banking has ceased, the price paths with

[10]Note that parameter values differ from those used in figure 1 and are chosen to meet the assumptions made above and to produce a clear graph rather than to closely represent the EU ETS.

and without the MSR are identical.

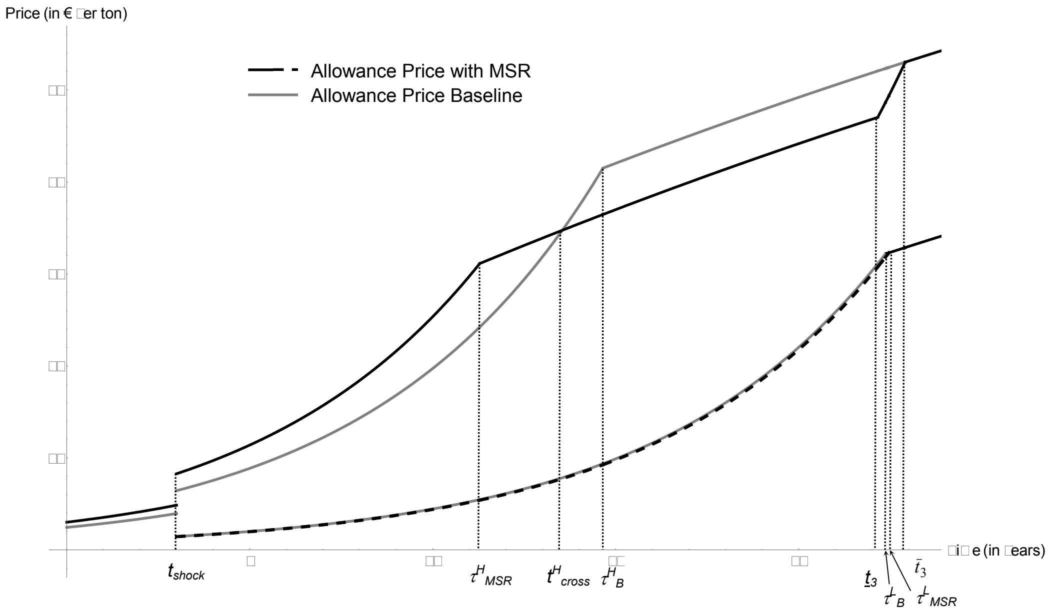

Figure 2: Comparison of allowance price paths with and without MSR in stochastic setting. The price path with the MSR following a negative demand shock (black dashed) is strictly below the path without the MSR for all $t \in [t_{shock}, \tau^L_{MSR}[$.

Compared to the deterministic case, economic uncertainty substantially increases the likelihood that the introduction of a MSR raises the equilibrium price path in the short term. However, we present a simple example where the price response to a demand shock is amplified and not mediated by the MSR. Hence, the effect of the MSR is contrary to what has been intended by policy makers who introduced the MSR, namely to increase the resilience of the EU ETS to shocks in demand. With the MSR, allowance prices can be expected to become in fact more, and not less volatile.

6 Conclusion

Burdened by an excess supply of allowances in form of a systemic 'surplus' (aggregate bank), the EU ETS is currently thought to not produce a sufficient price signal to incentivize investment towards low-carbon technologies and to adequately reflect the social cost of carbon. At the heart of a reform proposal to address these findings, the European Commission has devised a reserve mechanism that systematically postpones the issue date of allowances in times of high surpluses. One objective of the implementation of such a MSR is to increase scarcity in the market for allowances to reach higher price levels at an earlier date. This should nudge firms to undertake more abatement and low carbon investments. Furthermore, the reserve should guard the system against demand shocks, i.e. reduce price volatility by acting as a responsiveness mechanism when the aggregate bank moves outside a pre-defined trigger bandwidth. In this paper, we studied the effects of such an allowance-preserving reserve mechanism on optimal emission and price paths of market participants in a perfectly competitive market using a dynamic optimization framework. In a baseline scenario without a MSR, a representative firm depletes its bank to smooth the rise in abatement costs as the allocation schedule is tightened over time. Once the aggregate bank is empty, the non-borrowing constraint binds and prices rise at a rate less than the market interest rate. Including a MSR impacts the allocation schedule. Even so, as long as the optimal baseline emission path is still feasible under the changed allocation schedule, the MSR has no effect in absence of market failures. This is always the case if unrestricted borrowing is allowed.

However, when the MSR keeps firms from following their optimal baseline emission path, firms counteract the drainage of liquidity with increased abatement to counter the depletion of the aggregate bank, which in turn leads to a higher price path in the short run. However, as the MSR is allowance-preserving, it will successively re-inject all stored allowances once the aggregate bank drops below a lower trigger level. Because the aggregate bank is depleted at an earlier point in time, the price path with MSR is affected by the lower slope due to the non-borrowing constraint. In the medium run, the equilibrium emission path is then higher and the corresponding price path lower than in the baseline case. Furthermore, the banking phase is no longer unique. To smooth the jump in the allocation schedule once the reserve is empty, the firm banks for a second time, leading again to increased abatement and the price rising at the market interest rate. This creates an additional kink in both the emission and price path until the renewed aggregate bank is once more depleted. In the long run, this leads to identical emission and price paths, as the reserve's impact completely vanishes from the system.

We find that the introduction of a binding MSR, i.e. a mechanism that creates real scarcity, does not lead to a sustainable increase in prices over a baseline scenario but may in fact lower medium-term price expectations and thus obstruct investment in low-carbon technologies with long investment cycles. In our stylized model, this would run reverse to what the regulator intends to accomplish with its reform. In an ensuing investigation, we introduce uncertainty by modeling a simple demand shock. Such a shock makes it more likely that the MSR binds and thus short-term prices rise as the reserve reduces the probability to realize the optimal baseline emission path. We show that a demand shock arriving while there are still allowances held in the aggregate bank can lead to an amplification of the response of the allowance price. Instead of reducing price volatility, the MSR increases the impact of the shock. Again, this simple example shows that the MSR may indeed not accomplish what it is designed for, but further add to the EU ETS's struggles to create a desirable and reliable price signal.

Combining our findings, it is far from clear that the MSR will live up to its expectations. Further research needs to be devoted to the investment decisions of firms under the influence of the contrastive effects on prices by the MSR and on sources of market failures with respect to the inter-temporal optimization of market participants.

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A Proofs

A.1 Proof of Proposition 1

Note that in (20)-(22) the exact time profile of $S(t)$ for all $t < \tau_B$ is irrelevant. Requiring feasibility, (25) ensures that (23) is met. Hence, given feasibility, the baseline paths are still an equilibrium under the market stability reserve. Condition (24) together with only one banking period in the baseline equilibrium implies that proposition 1 holds if the MSR has no impact on the number of allowances auctioned at and after τ_B.

A.2 Proof of Proposition 2

Borrowing eliminates the constraint on the state variable (4) and therefore the feasibility condition (16) can be dropped.

A.3 Proof of Proposition 3

By definition, the bank is strictly positive for all $t \leq t_2$. Hence, it holds that $\tau_{MSR} > t_2$. If $\tau_{MSR} > t_3$, conditions (13)-(15) and (17) perfectly coincide with conditions (20)-(22) and (24), since $R(\tau_{MSR}) = \dot{R}(\tau_{MSR}) = 0$ in this case. Hence, $\tau_{MSR} > t_3$ would imply that $\tau_{MSR} = \tau_B$ and $e^*_{MSR}(t) = e^*_B(t)$ for all t. However, proposition 3 considers the case where $e^*_B(t)$ is not feasible due to the MSR. It follows that $\tau_{MSR} \leq t_3$ and hence that $e^*_{MSR}(\tau_{MSR}) = S_0 e^{-a\tau_{MSR}} + I$. The interval $[t_2, t_3]$ also marks where the emission path of the baseline case violates the feasibility constraint under the MSR. Hence, all emission paths at or above $e^*_B(t)$ are not feasible under the MSR. It therefore holds that $e^*_{MSR}(t) < e^*_B(t)$ and $p^*_{MSR}(t) > p^*_B(t)$ for all $t \leq \tau_{MSR}$.

Once the aggregate bank is depleted, the equilibrium price of allowances under the MSR does no longer rise at the rate of interest r, but at a lower rate. The non-borrowing constraint becomes binding and aggregate emissions are equal to allowances auctioned at t. However, this holds only temporarily. At t_3, the auctioning schedule $S_{MSR}(t)$ is discontinuous (it drops by I) as the MSR stops issuing allowances. If the bank was zero and emissions equaled the amount auctioned, the allowance price would make a discontinuous upward jump at t_3. Since the depletion of the MSR is perfectly foreseen, in equilibrium firms will bank allowances and the equilibrium price will rise at the rate r during a second banking interval $[\underline{t}_3, \bar{t}_3]$. Since $\bar{t}_3 > t_3$, i.e. the MSR is empty at the end of the second banking phase, it holds that $e^*_{MSR}(\bar{t}_3) = e^*_B(\bar{t}_3)$ and $p^*_{MSR}(\bar{t}_3) = p^*_B(\bar{t}_3)$.

As the aggregate bank and the MSR are empty in both scenarios, and the total number of allowances issued up to $\bar{t}_3$ is identical as well (the MSR is allowance preserving), the result that $e^*_{MSR}(t) < e^*_B(t)$ and $p^*_{MSR}(t) > p^*_B(t)$ for all $t \leq \tau_{MSR}$ implies that for at least some t within $[\tau_{MSR}, \bar{t}_3]$ the opposite must hold. The point in time when emission and price paths with and without a MSR intersect, t_{cross}, has to be to the left of τ_B because the slope of the price path in the baseline case is constant and lower than r for all $t > \tau_B$ and the price path with the MSR approaches from below at $\bar{t}_3$. The two banking phases are strictly separate, i.e. $\tau_{MSR} < \underline{t}_3$, because otherwise $\tau_{MSR} = \bar{t}_3 > t_3$. As has been shown above, this can only be an equilibrium if the MSR is non-binding.

For all $t > \bar{t}_3$ the non-borrowing constraint is always binding since the price of allowances and hence marginal abatement costs for $e(t) = S_0 e^{-at}$ increase at a rate less than r, i.e. it holds that $\bar{t}_3 > 1/a \ln[(r+a)S_0/(ru)]$.

A.4 Proof of Lemma 1

Using conditions (13)-(17), the following system of equations can be obtained:

$$u\tau_B - \frac{p_B^0}{rc}\left(e^{r\tau_B} - 1\right) = b^0 + \frac{S_0}{a}\left(1 - e^{-a\tau_B}\right) \tag{40}$$

$$u - \frac{p_B^0}{c}e^{r\tau_B} = S_0 e^{-a\tau_B}, \tag{41}$$

which determines the point in time when the bank is depleted (τ_B) and the initial price level, p_B^0. Using Cramer's rule, the effect of a change in u is identified as $\frac{\partial p_B^0}{\partial u} = \frac{\tau_B rc}{e^{r\tau_B}-1} > 0$ and the effect of a change in b^0 as $\frac{\partial p_B^0}{\partial b^0} = -\frac{rc}{e^{r\tau_B}-1} < 0$. This holds for all $t < \tau_B$.

For all $t \geq \tau_B$, prices rise at a rate below the rate of interest and emissions are equal to the number of allowances auctioned at time t (condition (17)). Hence, $p(t) = c\left(u - S_0 e^{-at}\right)$ and therefore $\frac{\partial p(t)}{\partial u} = c > 0$. The size of the initial bank is irrelevant for the price path once the bank has been depleted. The proof with a MSR is analogous.