Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

FAKULTÄT
FÜR WIRTSCHAFTS- UND
SOZIALWISSENSCHAFTEN

The Optimal Climate Policy of Mitigation and Adaptation: A Real Options Theory Perspective

Nicole Glanemann

WiSo-HH Working Paper Series
Working Paper No. 15
May 2014

WiSo-HH Working Paper Series
Working Paper No. 15
May 2014

The Optimal Climate Policy of Mitigation and Adaptation: A Real Options Theory Perspective

Nicole Glanemann Department of Economics, Hamburg University

ISSN 2196-8128

Font used: „TheSans UHH" / LucasFonts

Die Working Paper Series bieten Forscherinnen und Forschern, die an Projekten in Federführung oder mit der Beteiligung der Fakultät für Wirtschafts- und Sozialwissenschaften der Uni-versität Hamburg tätig sind, die Möglichkeit zur digitalen Publikation ihrer Forschungser-gebnisse. Die Reihe erscheint in unregelmäßiger Reihenfolge.

Jede Nummer erscheint in digitaler Version unter
https://www.wiso.uni-hamburg.de/de/forschung/working-paper-series/

Kontakt:
WiSo-Forschungslabor
Von-Melle-Park 5
20146 Hamburg
E-Mail: experiments@wiso.uni-hamburg.de
Web: http://www.wiso.uni-hamburg.de/forschung/forschungslabor/home/

The Optimal Climate Policy of Mitigation and Adaptation: A Real Options Theory Perspective[☆]

Nicole Glanemann

Department of Economics, Hamburg University, Von-Melle-Park 5, 20146 Hamburg
nicole.glanemann@wiso.uni-hamburg.de

Abstract

Climate policy can only be effective if it incorporates both mitigation and adaptation measures. However, the questions of how the two strategies can be optimally combined and how they affect each other are still far from being conclusive. To contribute to a better understanding of how uncertainty about future climate damage costs affects the climate policy design, this study analyses the decisions on adaptation and mitigation from a real options theory perspective. Real options quantify the opportunity costs of adopting policy now and making irreversible investments rather than waiting for new information to arrive, which could reduce the uncertainty. This paper develops a new framework in which the policy maker holds a portfolio of mitigation and adaptation options. Numerical simulations demonstrate that the dualistic approach to climate policy is impeded by the tension between uncertainty and economic irreversibility. They also disclose considerable asymmetry in the interaction and the magnitude of the real option values. This means that, compared with the common expected net present value approach, real options analysis places more emphasis on adaptation as the preferred measure.

Keywords: Climate policy, real options, mitigation, adaptation
JEL-Classification: C61, D81, Q51, Q54

[☆]This research paper is made possible through the advice and support of Michael Funke and Yu-Fu Chen. I wish to thank all the participants at the European Climate Change Adaptation Conference (ECCA) in Hamburg, March 2013, the 26th PhD Workshop on International Climate Policy in Paris, April 2013, and the EAERE FEEM VIU European Summer School in Resource and Environmental Economics in Venice, June 2013, for helpful comments. The research was supported by the International Max Planck Research School on Earth System Modelling (IMPRS-ESM).

1. Introduction

The negotiations at the past Conferences of the Parties to the UN Framework Convention on Climate Change have illustrated that the interests in and ideas about global cooperation on reducing emissions diverge considerably. At the same time, the global emission rates keep breaking new records every year and climate policy goals like the 2°C target become less likely to be achieved. Even if every country stopped emitting today, the warming trend would continue for several decades due to inertias in the climate system. Therefore, climate change is certain to happen and it will lead to changes in the environment and in the living conditions in more and more countries. Appropriately designed adaptation measures may help to gain from beneficial changes or to alleviate adverse impacts. Accordingly, climate policy can only be optimal if it factors in mitigation as well as adaptation. The best way to combine the two measures to fight climate change is, however, still far from being conclusive.

In the light of the urgency and relevance of this topic, the literature devoted to analysing the mix of the two measures is expanding rapidly . Kane and Shogren (2000) and Lecocq and Shalizi (2007) argue that mitigation and adaptation can be considered to be strategic complements and do not stand alone if policy is optimally designed. Mitigation prevents irreversible and potentially unmanageable ramifications, whereas adaptation is necessary to alleviate the impacts that are already locked in by climate change. Ingham et al. (2005) show that mitigation and adaptation are economic substitutes on the cost as well as on the benefit side. On the cost side, the investments in these measures compete for resources that are naturally scarce. On the benefit side, the usage of one option decreases the marginal benefit of the other. More precisely, mitigating emissions will successfully avoid damage and thus less adaptation is needed. Conversely, adapting effectively to global warming and the related consequences decreases the marginal benefit of emission reductions, as for example noted by Tol (2005a). As suggested by de Zeeuw and Zemel (2012), already the prospect of adapting in the future is increasing the current emission rate.[1] Quite recently, the existence of complementary and substitution effects was confirmed by Integrated Assessment Models such as AD-WITCH by Bosello et al. (2009, 2010, 2011) and Bosello and Chen (2010), AD-DICE by de Bruin et al. (2009), Ada-BaHaMa by Bahn et al. (2012) and AD-FAIR by Hof et al. (2009).[2] Interestingly, Bosello et al. (2010) and de Bruin et al. (2009) identify the trade-off between the two measures to be asymmetric. The two measures crowd each other out, but the effect of mitigation

[1]Aside from the outlined structure of the strategic complementarity and trade-offs, IPCC (2007) identifies specific examples of adaptation measures that can facilitate or exacerbate mitigation. If adaptation efforts involve an increased usage of energy, the total level of emissions that has to be mitigated increases. This is for example the case for air conditioning as a measure to adapt to heat or for seawater desalination as a measure to adapt to draughts. Other adaptation measures can facilitate mitigation, as they also decrease emissions. Buildings that are designed to reduce vulnerability to extreme weather events may also decrease the energy needs for heating and cooling.

[2]An extensive survey of this literature is provided by Agrawala et al. (2011b).

on adaptation is found to be weaker. In the short- and medium term, the benefits of mitigation are argued to be too small to reduce significantly the need to adapt. Moreover, both studies exhibit higher expenditures on adaptation, indicating that adaptation is the preferred measure. However, this result is very sensitive to the assumption concerning the discount rate: the more far-sighted the policy maker is assumed to be, the more attractive mitigation becomes. The reason is that the time gap between the occurrence of costs and the occurrence of benefits is much longer in the case of mitigation due to slow and lagged dynamics in the climate system. In contrast, adaptation can become effective as soon as it is fully implemented.

The understanding of how uncertainty affects the optimal mix is still at a "very early stage", as pointed out by Agrawala et al. (2011b). Felgenhauer and Bruin (2009) investigate the effects of uncertainty about climate sensitivity in a two-period model with learning. This kind of uncertainty is shown to reduce both mitigation and adaptation efforts. Furthermore, mitigation efforts are shown to be more sensitive to uncertainty than adaptation efforts. It is reasoned that uncertainty about climate sensitivity has long-run implications, affecting the decision about the long-run measure of mitigation more significantly. A multi-stage-decision under uncertainty about the benefits of both measures is qualitatively discussed by Felgenhauer and Webster (2013b), who suggest that the differences in the time lags between adopting a measure and learning about its benefits make adaptation and mitigation imperfect substitutes.

This paper aims to complement the research on the optimal policy mix of adaptation and mitigation under uncertainty by accounting for characteristics that cannot be fully captured by the normal net present value approach. It is generally agreed that the climate policy decision needs to take into account that (i) there is uncertainty about the future benefits of mitigation as well as of adaptation, (ii) waiting allows policy makers to gather new information about the uncertain future, (iii) the required investments in both policy measures are at least partially irreversible, which means that disinvesting cannot fully recover all the expenditures and (iv) the greenhouse gases accumulate and remain in the atmosphere long after they are emitted. On the one hand, the opportunity to wait for new information to arrive may induce the policy maker to delay costly and irreversible policy measures. On the other hand, a wait-and-see attitude may burden future generations with costs of an unknown size that are caused by irreversible climate damage. Hence, it may seem rational to adopt climate policy as soon as possible. These considerations show that the tension between uncertainty and these two types of irreversibility generates some value of delaying or accelerating investments. Differently from the above-mentioned studies, which apply a normal net present value approach, this paper explicitly accounts for this value.

This value of waiting– also referred to as the value of managerial flexibility – is considered to be a real option. This concept has its roots in the evaluation of financial options as developed by Black and Scholes (1973) and Merton (1973). On financial markets, the investor pays a premium price to obtain the right, but not the obligation, to buy an asset for some time at a predetermined price. Profit is made when the price of the underlying asset rises above the predetermined price and the option is exercised. Even then, it can be profitable to wait to exercise the option and to speculate for a further price increase in the underlying asset. Hence, holding the option is still of value due to uncertainty about the future asset price. The concept soon turned out to grant considerable insights into capital investment decisions and is thus referred to as real options analysis (ROA). Similar to exercising a financial option, most capital investment decisions are (at least partially) irreversible due to sunk costs incurred by the investment. Furthermore, the investor often faces uncertainty about the profits the investment will generate, because the prices of inputs or outputs may vary over time. In such a situation, the flexibility to delay an investment may be of value, as more information about the involved uncertainties can be gained as time passes. ROA is designed to capture the value of waiting and thus exceeds the normal net present value approach. Early applications of ROA to investment decisions are for example given by McDonald and Siegel (1986) and Pindyck (1988, 1993). The studies by Kolstad (1996) and Ulph and Ulph (1997), published soon afterwards, focus their attention on the implications of irreversibility and uncertainty for climate policy. The ROA conception is that the policy maker has the "right" to adopt these climate policy measures in return for lower future damage costs. Accordingly, the real options value captures the opportunity costs of implementing such a policy now rather than waiting for new information to arrive. In almost all cases, ROA is conducted solely to examine either mitigation or adaptation and not both together. The mitigation option is investigated in the seminal work by Pindyck (2000, 2002) or later analyses by Anda et al. (2009), Baranzini et al. (2003), Chen et al. (2011a,b), Lin et al. (2007), Nishide and Ohyama (2009) and Wirl (2006).[3] The real option to undertake specific adaptation projects is explored by Dobes (2008, 2010), Hertzler (2007), Linquiti and Vonortas (2012), Nordvik and Lisø (2004) and Watkiss et al. (2013). In practice, however, more than one measure is available to fight climate change, and their optimal mix might be affected by uncertainty and irreversibility as well.[4] The first attempts to analyse the optimal balance of mitigation and adaptation by means of real options theory are presented by Maybee et al. (2012) and Strand (2011). As a result of a non-formal discussion, Maybee et al. (2012) anticipate that, due to the local nature of adaptation, the benefits of adaptation seem to be more guaranteed and thus greater priority is given to adaptation measures. Strand (2011) examines how the decision to mitigate is affected

[3]While the above-mentioned research deals with one global decision maker, the work by Barrieu and Chesney (2003) and Ohyama and Tsujimura (2006, 2008) analyses the strategic agents' decision on when to curb emissions.

[4]Evidence of adverse effects by uncertainty and irreversibility on climate policy is found in psychology. Gifford (2011) argues that the existence of sunk costs, uncertainty and risks belongs to the barriers or "dragons of inaction" that hinder mitigation and adaptation efforts.

by adaptation, but adaptation is not treated as a real option but as an exogeneously given process.

To provide a more realistic picture of the policy maker's portfolio to fight climate change, this paper develops a new modelling framework for a portfolio of mitigation and adaptation real options. The adaptation options allow the policy maker to postpone investment or to invest the optimal portion of the GDP in projects that alleviate climate change impacts. The mitigation option gives the opportunity to choose the optimal timing for curbing emissions. Incorporating both real options into the same framework implies that the values of the individual options are affected by each other's presence. This paper can thus investigate the interaction of the two values of waiting. How are the decisions to design the optimal mix of mitigation and adaptation affected by uncertainty and irreversibility?[5]

The remainder of this paper is organized as follows. Section 2 gives an overview of the most important properties of the modelling framework. Section 2.1 provides all the required equations to derive the optimal adaptation policy in Section 2.2 and the optimal mitigation timing in Section 2.3. The numerical simulations are presented in Section 3. Section 4 concludes the paper. More details are available in the technical Appendices A - D.

2. A Real Options Model of Adaptation and Mitigation

The decision regarding when to cut emissions is complicated. Firstly, the predicted benefits of mitigation involve huge uncertainties. Secondly, exercising the option to mitigate involves large sunk costs – for example induced by a switch to CO_2 neutral technologies in the energy sector. As soon as this option is exercised, the decision maker gives up the possibility to wait for new information to arrive. The combination of sunk costs and uncertainty generates opportunity costs of adopting the policy now. However, as the damage is largely irreversible, exercising this option can also create sunk benefits. The net of these opportunity costs and benefits is reflected by the value of the real option and must be included in the decision model. In contrast to normal cost-benefit analysis, this approach can therefore explain how optimal policy and its timing is influenced by uncertainty and irreversibility.[6]

In practice, a decision maker also holds the option to invest in a better adjustment to the future impacts of climate change. The possibilities to adapt are manifold. The category

[5] Alternatively, one may consider only one real option that offers the opportunity to switch between different "modes" of climate policy, e.g. to do nothing, to mitigate only, to invest a certain, albeit not optimised, portion of the GDP in adaptation or to do both (in this case switching between all the modes might not be allowed). These kinds of real options models are for example applied to assess decisions to invest in the electricity sector, e.g. see Fuss et al. (2009, 2011). Obviously, this simplification cannot adequately encapsulate the interaction of the respective values of waiting, as they are not individually modelled.

[6] A more detailed introduction to the real option modelling framework is provided by Dixit and Pindyck (1994) and Stokey (2009).

of adaptation measures that is most relevant to the context of climate policy design is referred to as anticipatory or proactive adaptation, because it can be planned and taken in advance.[7] Additionally, these measures may be classified on the basis of the type of damage they reduce. Dykes and early warning systems are meant to lessen the impacts of occasionally occuring climate catastrophes. Other measures, like sea water desalination, land-use zoning, air conditioning, thermal insulation, vaccination programmes or the breeding of more resilient crops, help to alleviate everyday life that has been made difficult by gradually evolving climate change. Although differing in the purpose they address, almost all adaptation efforts require investments that are largely sunk. Furthermore, it is not clear in advance whether their design is both perfectly suitable for and effective in decreasing the future damage costs. The combination of irreversible investments and the uncertainty of the resulting benefits implies that adaptation projects can be modelled as real options. Consequently, the policy maker holds a portfolio of different option types: one option to mitigate and options to invest in adaptation.

In this paper, the options are modelled to reflect certain characteristics of adaptation and mitigation. Mitigation addresses the source of the climate change problem by reducing the amount of emitted greenhouse gases (GHGs). Once abated, these emissions cannot cause future damage. Therefore, early mitigation efforts can be considered to be the best insurance against climate change damages. Adaptation addresses the outcome of the climate change problem by alleviating the present or expected damages.

The decision to mitigate is modelled as a commitment to a certain emission reduction target, but it is not meant to be a continuous investment decision that can be immediately adjusted if necessary. In this context, making this distinction is important, as the first mentioned specification resembles a one-off decision and implies less flexibility to react to shocks or to new information pouring in. This idea of modelling better reflects reality, as mitigation efforts are negotiated in terms of emission reduction targets and are stipulated by an international treaty for longer periods of time. In contrast, adaptation is not about committing to a particular target but about investing in suitable projects wherever and whenever required. Accordingly, in the model, the decision maker can switch between waiting to invest and investing the optimal amount of money. However, the implementation of adaptation projects is assumed to take time.

The model also incorporates a stylized notion of adaptation capacity. The capacity is here understood to comprise all the means that enable the adoption of adaptation measures rather quickly.[8] Climate damage is assumed to compromise these means. This is consistent with the observation, as for example indicated by Smith et al. (2001), that countries already suffering from climate damage lack the capacity for quick adaptation. Finally, adaptation provides a local public good in most cases. Hence, economic theory

[7]The measures falling into the opposite category are implemented as soon as the damage occurs (reactive adaptation); see Smit et al. (2000) for this and other categorizations.

[8]The capacity to adapt depends on many factors, e.g. on the institutional system, economic and technological development, knowledge, values, ethics and cultures; see for example Adger et al. (2009).

suggests that adaptation should be supplied by the countries or local communities that benefit from these measures in the first place. However, as outlined by Lecocq and Shalizi (2007), several reasons corroborate the idea of modelling adaptation as a strategy that requires international collective action. A large number of countries lack the institutional, technological and financial capacities to meet their adaptation needs, a fact that calls for international aid and cooperation. Moreover, while mitigation forces the polluter to pay, adaptation is required where the damage occurs and not necessarily where it is primarily caused. Hence, equity justifies the international funding of adaptation projects. Furthermore, planning adaptation internationally could be effective. For example, it is beneficial to internalize externalities that may be caused by adaptation measures. Some projects may be operated in a more cost-effective fashion if they are carried out transnationally. In fact, the United Nations negotiates on adaptation and mitigation in the same breath.[9] Accordingly, in this paper, both mitigation and adaptation are considered to concern global policy.

For simplicity, technological progress is not incorporated into the modelling framework. Accounting for further real options that allow investment in R&D of one or the other climate policy measure would be a valuable next research step. Alternatively, the technological progress in these measures could be modelled as additional sources of uncertainty. However, the implementation of exogenously defined technological progress based on some ad hoc assumptions about how the technologies to mitigate and or to adapt may develop is not considered to be a worthwhile improvement of this analysis.

The procedure for incorporating both real options into one framework is as follows. The policy maker has the choice of when to switch from the high- to the low-emission scenario. In both scenarios, adaptation efforts are undertaken optimally. The optimal timing of mitigation is then inserted back into the adaptation model to obtain the optimal adaptation policy given that the emissions are optimally reduced.

2.1. The Model

In the following model, it is assumed that a forward-looking and risk-neutral policy maker strives to find the optimal policy for adaptation and mitigation by weighing the flow of consumption against the policy costs. More precisely, the policy decision is based on maximizing welfare, which can be expressed by

$$W = \mathbb{E}_0 \left[\int_0^\infty [\ Y(t)\,(1 - D(t)) - C_a(t) - C_m(t)\ ]\, e^{-rt} dt \right], \tag{1}$$

where $\mathbb{E}_0$ describes the expectations operator conditioned on the information given in the present period $t = 0$. Here, the level of consumption is assumed to be equivalent to

[9] Accordingly, the Kyoto Protocol has not only stipulated emission reductions, but also established a fund that finances adaptation projects and programmes in needy member states; see `http://www.adaptation-fund.org/`.

the level of the GDP $Y(t)$. Climate change causes damage costs $D(t)$, which reduce the level of the GDP. The costs of adaptation and mitigation are given by $C_a(t)$ and $C_m(t)$, respectively. The discount rate is described by r.
In the following, $Y(t) \equiv Y$ is assumed to be constant. Hence, all the processes that drive economic growth are ignored, in particular technological change.[10]
The proportion of climate damage costs D in equation (1) can be expressed by an exponential function:

$$D(t) = 1 - e^{-\frac{\rho\theta(t)M(t)^{\psi}}{(1+\alpha A(t))^{\phi}}}, \tag{2}$$

where $\rho \in [0, 1)$, $\alpha, \phi, \psi \in \mathbb{R}_+$. The exponent ϕ determines how quickly the effectiveness of adaptation decreases. This exponential function depends on the functions $M(t)$, which describes the accumulation of GHGs in the atmosphere, $A(t)$, which reflects the adaptation efforts, and $\theta(t)$, which causes stochasticity in the social costs of climate change. For notational ease, the exponential function is referred to:

$$\Upsilon(\theta(t), M(t), A(t)) = e^{-\frac{\rho\theta(t)M(t)^{\psi}}{(1+\alpha A(t))^{\phi}}}. \tag{3}$$

The uncertainty regarding the gravity of the losses inflicted by pollution is either caused by a lack of knowledge about the values of certain key parameters or intrinsically given. Economic models exhibit a substantial degree of instrinsic uncertainty. Even if all the parameters were known, there would still be uncertainty due to random exogenous events and fluctuations in the system. This kind of uncertainty is immense over long time horizons, which need to be considered to assess climate policies. Therefore, it is important to analyse the effects caused by intrinsic uncertainty. Pindyck (2000) suggests modelling the intrinsic uncertainty in the damage costs by utilizing a geometric Brownian motion with drift μ, variance σ and Wiener process z:

$$d\theta = \mu\theta dt + \sigma\theta dz. \tag{4}$$

Let θ capture all the processes that cannot be controlled by the policy maker, e.g. tastes or population growth. This process reflects the above-described characteristics of intrinsic

[10] How the GDP growth affects the optimal policy mix is not the pivotal question in this paper and it is thus ignored in the following for the sake of limiting the computational effort. It is certainly worthwhile addressing this question as well, as the implementation of these policy measures and economic growth may exhibit interesting interaction effects. Some adaptation projects are thought necessary to allow for / facilitate economic growth, especially in developing countries. Conversely, as for example pointed out by Jensen and Traeger (2013), economic growth increases the expected future wealth, which may delay mitigation, as present generations are less willing to forego consumption today. Tsur and Withagen (2013) argue that these future, richer generations could more easily afford to invest in adaptation. However, it should not be forgotten that economic growth is the main driver of emissions and thus of the climate problem. A worsening of the climate conditions limits the possibilities to adapt and requires ever more refined technologies to alleviate the impacts. How economic growth affects both policy measures is thus a question of whether these technologies will be available and how costly they will be. The ambiguity in the relationship between the GDP and the adaptation costs is emphasized by Agrawala et al. (2011a). They find that in AD-WITCH and AD-DICE contrary but valid assumptions are made about this relationship.

uncertainty. First of all, the present level of social costs can be observed, whereas the future costs remain uncertain. Secondly, the longer the time horizon considered, the more uncertainty increases, which makes a reasonable decision on climate policy strategy difficult.[11]

For simplicity, I assume that there is no ecological uncertainty in the accumulation of GHGs in the atmosphere. As in Nordhaus (1994), it evolves according to:

$$\frac{dM}{dt} = \beta E(t) - \delta M(t), \tag{5}$$

where β is the marginal atmospheric retention of emissions E. The natural rate of depletion is given by δ, $0 < \delta < 1$. Once emitted, a certain percentage of the GHGs will stay in the atmosphere for a long time, as described by equation (5). For simplicity, the emissions are assumed to be proportional to the GDP without losses:

$$E(t) = \epsilon(1 - m(t))Y, \tag{6}$$

which can be curbed according to an emission reduction target m, $0 \leq m \leq 1$. Mitigation, however, incurs costs of:

$$C_m(t) = \kappa_1 m(t)^{\kappa_2} Y, \tag{7}$$

with $\kappa_1 \geq 0$ and $\kappa_2 > 1$ so that $C_m(t) < Y$ for all t holds.[12] The convexity of this function relates to the increased costs and efforts required when choosing a higher emission reduction rate m.

Proactive adaptation can be considered to be a capital stock that lowers the harm inflicted by climate change; see Bosello et al. (2009, 2010, 2011) and Bosello and Chen (2010). The evolution of this capital stock is given by:

$$\frac{dA}{dt} = a(t)Y - \xi A(t). \tag{8}$$

This stock depreciates at a rate of $\xi \in (0, 1)$. The decision maker can allocate a share $a(t)$, $a(t) < 1$ for all $t \geq 0$, of the GDP to investments in adaptation capital. These investments are assumed to be irreversible, i.e. $a(t) \geq 0$ for all $t \geq 0$. The investment costs are assumed to be convex, i.e. adaptation efforts take time. To account for the adaptive capacity, I also assume that the time to adapt increases with unabated damage. In other words, the (financial, institutional, technological, etc.) means that facilitate

[11] Alternatively, stochasticity could be modelled by a mean reverting process. This approach would imply that the policy maker has a good idea, albeit not perfect knowledge, of how the social costs will develop over long time horizons. The uncertainty about the costs in the very distant future is thus not significantly greater than the uncertainty about the costs in the near future. This would certainly be a feasible assumption if the climate damage cost function only depends on the atmospheric pollution, which is perfectly known in this model set-up. Here, I argue that there are many more factors that influence the damage costs, in particular economic factors, which are difficult to anticipate over long time horizons.

[12] Equations (6) and (7) are versions of the corresponding functions in Nordhaus (2010) without any technological progress.

the quick conducting of adaptation measures deteriorate due to unabated climate damage. Accordingly, the "unit costs" of adaptation are the same, but the adjustment costs increase if the climate damage worsens.[13] These cost effects are disentangled by:

$$C_a(t)) = \gamma_1 a(t)Y + \frac{1}{2}\gamma_2 \frac{(a(t)Y)^2}{\Upsilon(\theta(t), M(t), A(t))}, \tag{9}$$

where the parameters γ_1 and γ_2 are positive. Additionally, the calibration of these two parameters must rule out $a \geq 1$.

Accounting for all the above-mentioned equations, the model is solved by first determining the optimal flow of investments $(a(t)Y)_{t\geq 0}$ for the high- ($m = 0$) and low-emission ($m > 0$) scenarios, as outlined in Section 2.2.

2.2. Adaptation Policy

The decision maker strives to find the optimal strategy for investing in adaptation given emission policy E or m. Welfare is thus rephrased as:

$$\begin{aligned} W(\theta(t), M(t), A(t); m(t) \equiv m) = \max_{0\leq a(t)\leq 1} \mathbb{E}_0 \Bigg[\int_0^\infty \Bigg(Y \Big(\Upsilon(\theta(t), M(t), A(t)) \\ - \gamma_1 a(t) - \frac{1}{2}\frac{\gamma_2}{\Upsilon(\theta(t), M(t), A(t))} a(t)^2 Y \Big) - C_m \Bigg) e^{-rt} dt \Bigg]. \end{aligned} \tag{10}$$

By applying Ito's Lemma, this optimization can be expressed by a Hamilton-Jacobi-Bellman equation:

$$\begin{aligned} rW = Y\Upsilon - C_m + (\beta\epsilon(1-m)Y - \delta M)\frac{\partial W}{\partial M} + \mu\theta\frac{\partial W}{\partial \theta} + \frac{1}{2}\sigma^2\theta^2\frac{\partial^2 W}{\partial \theta^2} \\ - \xi A\frac{\partial W}{\partial A} + Y \max_{0\leq a(t)\leq 1}\left\{ a\frac{\partial W}{\partial A} - \gamma_1 a - \frac{1}{2}\frac{\gamma_2}{\Upsilon}a^2 Y \right\}, \end{aligned} \tag{11}$$

where the functional arguments are dropped to simplify the notation. Equation (11) implies the first-order condition for the optimal investment:

$$a^* = \frac{\Upsilon}{\gamma_2 Y}\left(\frac{\partial W}{\partial A} - \gamma_1\right). \tag{12}$$

The optimality condition clarifies whether and how much to invest. The marginal welfare of adaptation increases with higher pollution M and a higher θ. Therefore, the investment efforts increase in a situation of worse climate impacts. However, these efforts are slowed down by a decrease in $\Upsilon(\theta, M, A)$, reflecting a reduced adaptive capacity. Accordingly, the optimal policy design needs to incorporate considerations about maintaining sufficient adaptive capacity so that future generations are not limited in their options to adapt to

[13] Please note, that the "unit costs" of adaptation only stay the same in the absence of technological progress.

climate change. This emphasizes the importance of the assumption of $\frac{\gamma_2}{\Upsilon(\theta(t),M(t),A(t)}$ being the adjustment cost parameter in equation (9).
Depending on the marginal value of adaptation, the investment strategy can then be summarized as:

$$a^* = \begin{cases} 0 & \text{for} \quad 0 \leq \frac{\partial W}{\partial A} \leq \gamma_1 \\ \frac{\Upsilon}{\gamma_2 Y}\left(\frac{\partial W}{\partial A} - \gamma_1\right) & \text{for} \quad \gamma_1 < \frac{\partial W}{\partial A} \leq \gamma_1 + \frac{\gamma_2 Y}{\Upsilon} \\ 1 & \text{for} \quad \frac{\partial W}{\partial A} > \gamma_1 + \frac{\gamma_2 Y}{\Upsilon} \end{cases}. \tag{13}$$

It is optimal to start investing in adaptation as soon as the marginal welfare of adaptation is higher than γ_1. Please note that $a^* = 1$ is ruled out and only serves as a upper boundary. When reinserting the optimal investment policy (13) into equation (11), the resulting Hamilton-Jacobi-Bellman equation is defined differently in the range of possible values $\mathbb{R}^3_+ = \{(\theta, M, A) : \theta, M, A \geq 0\}$. In the region $S_1 = \{(\theta, M, A) : 0 \leq \frac{\partial W}{\partial A} \leq \gamma_1\} \subset \mathbb{R}^3_+$, welfare can be expressed by:

$$rW = Y\Upsilon - C_m + (\beta\epsilon(1-m)Y - \delta M)\frac{\partial W}{\partial M} + \mu\theta\frac{\partial W}{\partial \theta} + \frac{1}{2}\sigma^2\theta^2\frac{\partial^2 W}{\partial \theta^2} - \xi A\frac{\partial W}{\partial A}. \tag{14}$$

If the marginal welfare of adaptation is sufficiently low, it is optimal not to invest. Then, the decision maker receives the expected present welfare given for the scenario of never investing in adaptation. However, the stochastic fluctuations of θ may cause less favourable conditions and increase the marginal welfare of adaptation in the future. The value of the opportunity to invest in the future is clearly influenced by these stochastic fluctuations and by the fact that the investment costs are sunk. Accordingly, this opportunity is quantified by a real options value. The welfare in the region S_1 is therefore given by the sum of the expected present welfare of never investing and the real options value to expand the existing adaptation capital stock in the future.
In the region $S_2 = \{(\theta, M, A) : \gamma_1 < \frac{\partial W}{\partial A}\} \subset \mathbb{R}^3_+$, welfare can be expressed by:

$$\begin{aligned} rW = Y\Upsilon - C_m + (\beta\epsilon(1-m)Y - \delta M)\frac{\partial W}{\partial M} + \mu\theta\frac{\partial W}{\partial \theta} \\ + \frac{1}{2}\sigma^2\theta^2\frac{\partial^2 W}{\partial \theta^2} - \xi A\frac{\partial W}{\partial A} + \frac{\Upsilon}{2\gamma_2}\left(\frac{\partial W}{\partial A} - \gamma_1\right)^2. \end{aligned} \tag{15}$$

As soon as the marginal welfare trespasses on the value γ_1, the policy maker starts to invest at the optimal rate given by equation (12). However, it is possible that the stochastic fluctuations of θ may decrease the marginal welfare of adaptation in the future. Such a decrease in the social costs may render investments in adaptation unnecessary and the policy maker can stop investing without costs. Therefore, the solution to the welfare in the region S_2 is only given by the expected present value of investing $a^* = \frac{\Upsilon}{\gamma_2 Y}\left(\frac{\partial W}{\partial A} - \gamma_1\right)$.
As the threshold at which the decision maker optimally switches from one investment regime to the other as well as the rate of optimal investment are given in terms of the

marginal welfare of adaptation, the system is solved by deriving the partial derivatives of equations (14) and (15). More precisely, with the abbreviations $w = \frac{\partial W}{\partial A}$, $w_\theta = \frac{\partial^2 W}{\partial \theta A}$, $w_{\theta\theta} = \frac{\partial^3 W}{\partial \theta\theta A}$, $w_A = \frac{\partial^2 W}{\partial A^2}$ and $w_M = \frac{\partial^2 W}{\partial MA}$ the marginal welfare of adaptation for S_1 can be expressed as:

$$\begin{aligned}(r+\xi)w =& Y\Upsilon\frac{\alpha\phi\rho\theta M^\psi}{(1+\alpha A)^{\phi+1}} + \left(\beta\epsilon(1-m)Y - \delta M\right)w_M + \mu\theta w_\theta + \frac{1}{2}\sigma^2\theta^2 w_{\theta\theta} \\ &- \xi A w_A \quad \forall(\theta, M, A) \in S_1; \end{aligned} \tag{16}$$

its equivalent for S_2 is given by:

$$\begin{aligned}(r+\xi)w = & Y\Upsilon\frac{\alpha\phi\rho\theta M^\psi}{(1+\alpha A)^{\phi+1}} + \left(\beta\epsilon(1-m)Y - \delta M\right)w_M + \mu\theta w_\theta + \frac{1}{2}\sigma^2\theta^2 w_{\theta\theta} \\ &+ \left(\frac{\Upsilon}{\gamma_2}(w-\gamma_1) - \xi A\right) w_A + \Upsilon\frac{\alpha\phi\rho\theta M^\psi}{(1+\alpha A)^{\phi+1}}\frac{1}{2\gamma_2}(w-\gamma_1)^2 \quad \forall(\theta, M, A) \in S_2; \end{aligned} \tag{17}$$

By equations (14) - (17) as well as equation (13) describing the threshold between S_1 and S_2 in terms of the marginal welfare of adaptation, the system is fully described. However, due to the complexity, the system cannot be solved analytically but requires numerical treatment. The applied numerical routine is a fully implicit finite difference method, as explained in Appendix A.

2.3. Mitigation Policy

The timing of undertaking mitigation efforts, i.e. increasing $m = 0$ to some $m > 0$, depends on the optimal adaptation policy that is conducted in these emission scenarios. Hence, the recipe in Section 2.2 needs to be applied to derive the welfare of adaptation for $m = 0$ and for $m > 0$, respectively. The difference in the respective welfare values $W(\theta, M, A; m > 0) - W(\theta, M, A; m = 0)$ would describe the benefits of reducing emissions, if the decision to mitigate were a now-or-never decision. This net present value consists of the direct benefits that are given by less pollution and of the indirect benefits from prescribing a different adaptation strategy. These indirect benefits can be understood as the value of the additional flexibility in adaptation investments. As opposed to a now-or-never decision, the decision on when to cut emissions involves uncertainty and irreversibility, which gives waiting to mitigate a value that is expressed by its real option $W^M(\theta, M, A; m = 0)$. Depending on the optimal adaptation activities in the no-mitigation scenario $m = 0$, the real option to mitigate is expressed as follows:

$$\begin{aligned} rW^M =& \left(\beta\epsilon Y - \delta M\right)\frac{\partial W^M}{\partial M} + \mu\theta\frac{\partial W^M}{\partial\theta} + \frac{1}{2}\sigma^2\theta^2\frac{\partial^2 W^M}{\partial\theta^2} \\ &- \xi A\frac{\partial W^M}{\partial A} + \frac{\Upsilon}{2\gamma_2}\left(\frac{\partial W^M}{\partial A} - \gamma_1\right)^2 \mathbb{1}_{\{(\theta,M,A)\in S_2\}}, \end{aligned} \tag{18}$$

where $\mathbb{1}_{\{(\theta,M,A)\in S_2\}}$ is one in the region S_2 and zero in the region S_1.

The threshold of mitigation is derived by comparing the real options value (18) with the benefits of switching from the high- to the low-emission scenario, $W(\theta, M, A; m > 0) - W(\theta, M, A; m = 0)$. Again, the solution cannot be found analytically but requires numerical treatment, as described in Appendix B.

To obtain the optimal policy thresholds, the mitigation threshold is computed by taking the optimal adaptation policy into account and the optimal adaptation policy needs to incorporate the optimal timing of the emission reduction efforts.

3. Numerical Simulation

To achieve a better understanding of the interaction of mitigation and adaptation, a numerical analysis of the model needs to be conducted. This analysis consists of four parts. First, the optimal policy mix is investigated. Then, the interaction of the two measures is explored. Afterwards, the contribution of the ROA to the analysis of the climate policy decision is demonstrated. It must be noted that calibrating the model is particularly challenged by the lack of estimates concerning adaptation. As emphasized by Agrawala and Fankhauser (2008) and Bosello et al. (2009), studies of adaptation have been limited to a few economic sectors, countries and measures and are thus insufficent to provide reliable estimates. Accordingly, some caution is required when interpreting the quantitative insights of modelling exercises based on these estimates. The last part of this numerical analysis is thus devoted to a sensitivity analysis of these parameters and other parameters that are controversially discussed. Studying the model as proposed may be comprehensive enough to provide some meaningful insights into the pivotal effects.

The base calibration is as follows. Emissions E assume the value of 0.033 trillion of CO_2 metric tonnes, as estimated by EDGAR (the Emission Database from Global Atmospheric Research) for 2011.[14] For the same year, the IMF reports the global GDP to amount to $Y = 78.97$ trillion US dollars (PPP).[15] The present concentration of almost 400 ppm translates into $M = 40$ ($\times$ 10 ppm).[16] A considerable simplification of the numerical routine is offered by assuming that $\delta = 0$. This means that the parameter β has to be adjusted to reflect the average increase in the atmospheric CO_2 concentration over the time horizon of interest. According to the latest measurements from the Mauna Loa Observatory, Hawaii, the current atmospheric CO_2 concentration can be assumed to increase by about 3 % per year. The parameter β is thus computed to be 9.09 ppm per trillion CO_2 metric tonnes.

In most integrated assessment models, as in DICE, the damage function implictly factors in optimal adaptation efforts. Therefore, de Bruin et al. (2009) recompute the damage

[14] This database is created by European Commission and the Joint Centre (JRC)/PBL Netherlands Environmental Assessment Agency; see `http://edgar.jrc.ec.europa.eu`.

[15] The data originate from the World Economic Outlook Database, October 2012 edition.

[16] Information about the measurements can be retrieved from `http://co2now.org`.

function in DICE by disentangling the adaptation costs from the damage costs. I adapt the damage function (3) to their calculations for the doubling of CO_2 and to the rather arbitrary assumption, which is needed for the numerical routine, that an extremely high concentration of 4200 ppm CO_2 in the atmosphere would lead to a total loss of GDP. For the concentrations that are likely to be reached in the near future and are thus of relevance to this study, the resulting damage function is very similar to the damage function presented by de Bruin et al. (2009). The current value of the cost parameter is set to be $\theta = 10$. This choice implies an approximate value for the social cost of CO_2 of about 16 US dollars, which is in the range of the estimates surveyed by Tol (2005b).[17] Just as controversial and crucial as the calibration of the damage function is the assignment of a value to the discount rate r. Here, I settle for a 2.5% discount rate. The mitigation costs are chosen to be slightly lower than the function estimated by Cline (2011), which is based on a large set of model results compiled by the Stanford Energy Modeling Forum study EMF 22.[18] The mitigation target m is premised on the emission reduction targets that countries would have to adhere to in order to satisfy the Copenhagen Accord by 2020. As reported by Cline (2011), these efforts would mean a 9% reduction in global emissions.

As many diverse forms of capital can be considered to be adaptation capital, it is also controversial to determine the depreciation rate ξ in equation (8). For example, Bosello et al. (2011) and de Bruin et al. (2009) choose a value of 10%, while Agrawala et al. (2011a) and Felgenhauer and Webster (2013a) settle for a depreciation rate of 5%. I choose to compromise with $\xi = 0.075$.

As pointed out by Nishide and Ohyama (2009), the stochastic path of θ should be chosen somewhat arbitrarily, since associated data are lacking. A plausible calibration is represented by $\sigma = 0.07$ and $\mu = 0$.

The other parameters, such as those describing the costs and the effectiveness of adaptation, are chosen to comply with the rather broad estimates that are also used to calibrate the AD-DICE model by de Bruin et al. (2009) and the AD-WITCH model by Bosello et al. (2009, 2010, 2011). The reference point of calibration is the doubling of atmospheric CO_2. Concerning this point, an extensive review of the impact assessment literature by Tol et al. (1998) values the adaptation costs at about 7% - 25% of the total damage costs. Further studies, for example by Mendelsohn (2000) and Reilly et al. (1994), give the impression that the amount of damage that is reduced by adaptation in the calibration point could lie between 30% and 80%. Consistent with these ranges, the calibration of γ_1 and γ_2 determines the adaptation costs that are incurred by reacting to a doubling

[17] The social cost is derived by taking the net present value of the future damages caused by an additional ton of CO_2. The review by Tol (2005b) illustrates the diversity of the social cost assessments. In order to obtain a vague idea about whether the calibration of θ is feasible, i.e. the implied social cost is within the range of assessments, the exponential function in equation (3) can be roughly approximated by its first-order Taylor expansion.

[18] An overview of the EMF scenarios caan be found in Clarke et al. (2009).

of CO_2 to be at least 0.18% of the GDP.[19] This number is thus of the same order of magnitude as the estimates produced by the AD-WITCH and AD-DICE models.[20] The full listing of the parametrization is given in Appendix C.

In the following, the simulation results are demonstrated by three-dimensional graphs of the state variables that are assumed to be given at the point in time when the decision has to be made. For each combination of already installed adaptation capital A and level of atmospheric CO_2 concentration M, the threshold of taking action is derived in terms of the observed value of θ. The resulting threshold curves thus divide the space of (θ, M, A) values into regions of optimal policy. The lower region spans all the values in which it is optimal to postpone policy adoption. In all the values above the threshold, the policy maker implements the policy immediately. In the case of adaptation, it additionally holds true that the intensity of investment efforts is higher the greater the distance to the threshold. The purpose of this representation is to investigate how the curves shift under alternative assumptions and to draw conclusions concerning the implied policy decisions.

Figure 1 illustrates the optimal policy of adaptation and mitigation. In this simulation, the two climate policy measures interact with each other. The adaptation threshold shown by Figure 1a takes into consideration the optimal timing of mitigation. The mitigation threshold given by Figure 1b is obtained by incorporating the information about the optimal investment into adaptation. The optimal policy mix unfolds by considering Figure 1c, which displays both thresholds together.

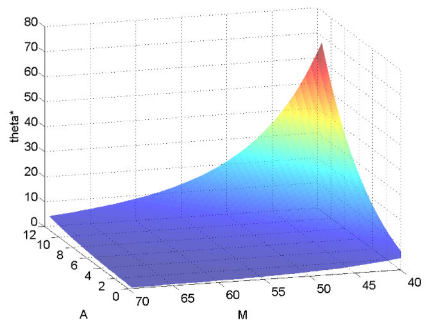
Figure 1a shows that the threshold of adaptation shifts upwards for more installed adaptation capital, i.e. investment becomes less necessary. This effect is more pronounced for lower values of M. Moreover, it is clear that the region of inaction shrinks for higher pollution M. Therefore, the results confirm what intuition tells us: investment in adaptation needs to be undertaken sooner the more the economy is exposed to climate change damage.

The mitigation threshold in Figure 1b reveals some familiar features, which have already been observed in the mitigation real options literature as well as some new characteristics. As is generally known, the mitigation threshold shifts downwards for higher pollution levels. That means that a higher atmospheric CO_2 concentration increases the urgency to cut emissions soon. In contrast to the hitherto existing research that focuses on mitigation as the only real option, the mitigation threshold in this paper features discontinuities or sudden jumps, which appear to be located on a curve. Figure 1c explains that the source of these discontinuities is the intersection of the two threshold curves. Indeed, at these points, the description of the mitigation real options (18) switches to a different functional

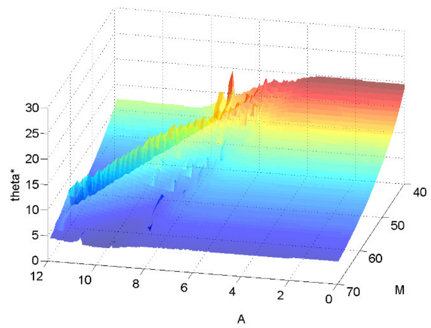
[19]This value is calculated assuming that $\theta \equiv \theta_0$.

[20]These models estimate the costs to be 0.19% (AD-WITCH) and 0.28% (AD-DICE) of the GDP. For a comparison of the two models, see Agrawala et al. (2011a).

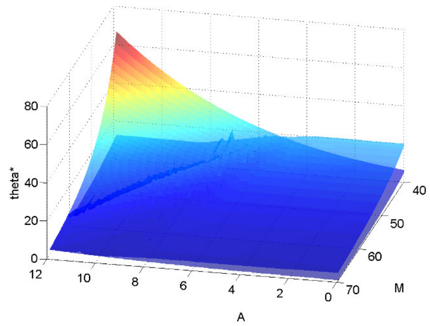
form, which causes the associated threshold to drop (seen from low to high A-levels).[21] In other words, this drop is attributed to the phasing out of adaptation investments. Hence, adding adaptation to the model grants a new perspective on optimal mitigation, which can be discussed in more detail by considering Figure 1c.



(a) The Optimal Adaptation Threshold



(b) The Optimal Mitigation Threshold



(c) The Optimal Policy Mix Given by Both Thresholds

Figure 1: The Optimal Policy

Figure 1c discloses two different regimes of optimal policy. For low levels of adaptation capital A or high pollution M, the mitigation threshold hovers above the adaptation threshold. In this area, the optimal policy action can be described as follows. Below the adaptation threshold, the policy maker will neither invest in adaptation nor undertake any emission reduction efforts. In between the two thresholds, the optimal strategy is to expand only the adaptation capital stock. As soon as the upper threshold has been reached for the first time, mitigation complements adaptation. Accordingly, the policy maker is advised to invest first in adaptation before curbing emissions. The question of why adaptation is the preferred alternative is answered by the acute exposure to

[21] As the numerical solution procedure approximates the partial derivatives, these discontinuities cause errors in their neighbourhood, which materialize as single-point peaks. The induced errors vanish at more distant points to the intersection. For illustrational purposes, some of the single-point peaks are not displayed in Figure 1. The corresponding graphs with all the single-point peaks are available on request.

climate damage with low A and/or high M values.[22] If properly planned and managed, the adaptation projects that are undertaken first are relatively inexpensive, completed quickly and effective. Emission reduction is of less importance, because it does not help to cure the present vulnerability.

For bigger adaptation capital stocks, the adaptation threshold moves above the mitigation threshold. Below the mitigation threshold, climate policy efforts are dispensable, because the climate damage costs are very low. As soon as the mitigation threshold has been crossed for the first time, emissions need to be reduced. Only if the process moves above the adaptation threshold is investment in adaptation optimal. This area describes the optimal policy of well-adapted economies, which are less exposed to climate damage. Investing more in adaptation becomes inefficient, while mitigation becomes the preferred measure. All in all, Figure 1c thus points out the key role of being well adapted: it is optimal to reduce the current vulnerability to climate change first and then to cut emissions to reduce the future impacts.

To understand the curvature of the mitigation threshold, we have to dissect the components of adaptation, which are added to the mitigation model, and examine their effects on the timing of mitigation. Adaptation means (i) to enjoy the benefits of the already existing capital stock and (ii) to have the opportunity to expand this stock. If the existing stock is responsible for the curvature, we may speak of a complementarity effect: a sufficient build-up of adaptation capital would ensure the availability of the (financial) means to take care of the future generations' fate by curbing emissions. The better the economy is adapted, the sooner emissions are to be curbed. However, for very low and very high stocks, the mitigation threshold in Figure 1c appears to be insensitive to the A-levels. To clarify this issue ultimately, Figure 2 illustrates the optimal timing of mitigation under the assumption that adaptation capital exists but the opportunity to expand it is not given. First of all, it is confirmed that the timing of mitigation is rather insensitive to the existing adaptation capital stock size. The reason is that adaptation capital only grants short- or medium-term benefits, as it depreciates over time. In contrast, the benefits of mitigation are rather small in the near future and are expected to accumulate over longer time horizons. Accordingly, the current level of adaptation capital cannot have a significant effect on the decision regarding whether to adopt a measure that pays in the distant future. Put differently, a high adaptation capital stock does not accelerate mitigation. As Figure 2 suggests, the current A-levels slightly decrease the benefits of mitigation. This small effect of substitution on the benefit side is, however, hardly visible in Figure 1b. If the adaptation capital stock is not responsible for the mitigation threshold curvature, the opportunity to expand it is. It is recognizable that for lower adaptation capital stocks A the threshold in Figure 1b is much higher than its equivalent in Figure

[22] Here, I use the terms "exposure to climate change damage" and "vulnerability to climate change" interchangeably.

2. This means that taking the opportunity to invest in adaptation delays the mitigation efforts, presumably due to substitution effects on the cost and benefit side. Indeed, the opportunity to invest in adaptation decreases the benefits of mitigation. In other words, the benefits of mitigation would be very high if the economy continues to be so highly exposed to climate damage. However, investing in adaptation reduces the vulnerability to climate change and thus decreases the future benefits of mitigation. In addition, investing in adaptation leave less financial means for adopting emission cuts. Consequently, the mitigation threshold shifts upwards. With higher A-values, the threshold in Figure 1b converges to the one displayed in Figure 2. The opportunity to expand the adaptation capital loses its value due to the decreasing effectiveness of the capital. Therefore, the effect of the opportunity to invest in adaptation on the mitigation threshold vanishes. All in all, the curvature of the mitigation threshold arises from the decreasing value of expanding the adaptation capital stock.

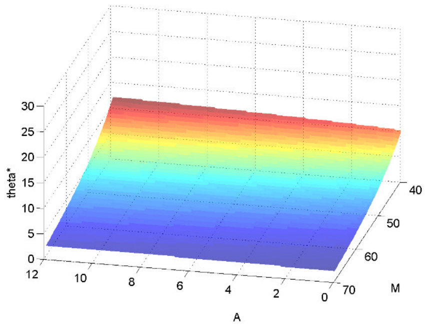
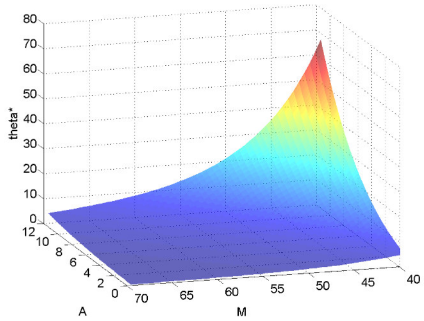
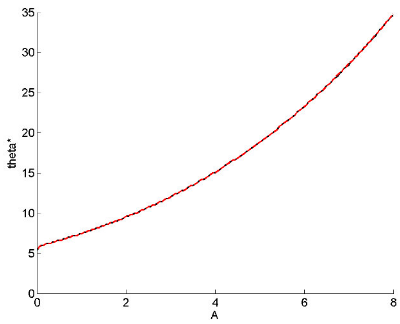


Figure 2: The Mitigation Threshold under the Assumption that Investing in Adaptation Is Not Possible

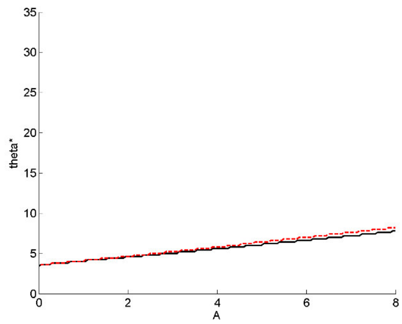
Next, the effects of mitigation on the adaptation option are examined. Considering Figure 1a once again, we can see that the decision to cut emissions does not lead to any noticeable jumps in the adaptation threshold. Therefore, the interaction between the two measures is obviously not of a symmetric nature. By analogy with Figure 2, Figure 3a demonstrates the threshold of adaptation for the scenario in which emissions cannot be curbed. The comparison of the thresholds in the optimal-emission scenario (Figure 1a) and the high-emission scenario shows no visible difference. Indeed, Figure 3b proves that the two thresholds are even identical at the present pollution level. The timing the investment in adaptation is thus determined by the present magnitude of atmospheric pollution M but not by the future development of M or the opportunity to slow down its growth. The reason is that for the decision on whether to adopt a short-term measure, such as adaptation, the present impacts matter more than the future threats. Figure 3c illustrates the same thresholds for the deterministic case, in which σ and the real options value of adaptation are zero. Obviously, the thresholds are much lower, which means that the policy maker is more willing to shoulder the sunk costs caused by adaptation when certain about the resulting benefits. Figure 3c also shows that there is a difference, albeit marginal, between the two thresholds in the deterministic case. This simulation thus confirms earlier findings in the literature, which describe the crowding out effect of mitigation on adaptation as rather small. It is reasoned that in the short- and medium-term the benefits of mitigation are too small to reduce significantly the current need to adapt. Comparing Figure 3b with Figure 3c leads to the conclusion that this effect of substitution with respect to timing vanishes when taking a real options perspective. The benefits of mitigation are not only too small but also too uncertain to influence the timing of the adoption of a measure that promises to improve the situation soon.



(a) The Adaptation Threshold in the High-Emission Scenario



(b) The Adaptation Threshold in the High-Emission Scenario (Black) and the Adaptation Threshold in the Optimal Mitigation Scenario (Red) Displayed in $M = 40$



(c) Both Adaptation Thresholds in the Absence of Uncertainty in M =40

Figure 3: The Adaptation Thresholds

In order to conduct a more comprehensive analysis of the interaction effects, the adaptation investment levels need to be examined as well. To this end, Figure 4a provides information about the optimal adaptation efforts - specified as a percentage of the GDP - in the high-emission scenario for $M = 40$. For the present level of $\theta = 10$ the investment efforts are rather small for low levels of adaptation capital A and zero for higher levels of adaptation capital.[23] Figure 4b comprises the cuts of investment efforts when emissions are curbed optimally. These cuts range from 0% up to almost 0.03% of the GDP, if all the value combinations of A and θ are considered. For the θ values that can be assumed in the near future, the reduction of efforts is significantly lower than 0.01%. Figure 4c and Figure 4d illustrate the investment efforts for the deterministic version of the model. Comparing Figure 4a with Figure 4c, we can see that uncertainty makes the policy maker less willing to invest in adaptation, as shown before in Figure 3. Figure 4b and Figure 4d demonstrate that cutting emissions optimally allows the policy maker to invest less in the deterministic case than under uncertainty. In other words, if the benefits of mitigation cannot be counted on with absolute certainty, the adaptation investment efforts must not be too severly cut back.

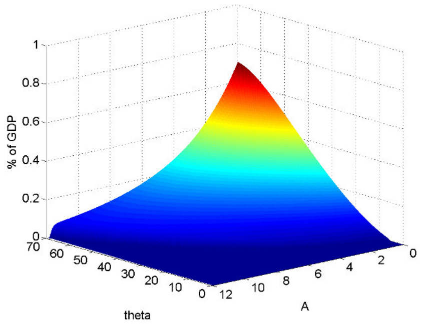
Having considered the effects of interaction, we may conclude that there is considerable asymmetry in the interaction of the two real options. The timing of mitigation is not sensitive to the currently installed adaptation capital stock. However, the opportunity to expand the adaptation capital stock affects the benefits of mitigation greatly. Contrariwise, adaptation activites are only slightly influenced by the real option to mitigate.

Next, the contribution of taking the real options perspective when analysing the climate policy decision is addressed. For this, Figure 5 presents the optimal policy threshold curves under alternative assumptions. Figure 5a illustrates the case in which the uncertainty parameter σ and the real options values are zero. A deterministic view on the optimal policy decision is for example taken by Bosello et al. (2009, 2010, 2011) and de Bruin et al. (2009). Figure 5b takes a step further by prescribing $\sigma = 0.07$ as in the base calibration, but it postulates that only the expected net present value matters to the policy decision. The existence of any effects generated by the interaction of uncertainty and the irreversibilities are neglected. The strand of literature that accounts for uncertainty but exclusively follows the expected net present value approach to determine the optimal policy mix is represented by Felgenhauer and Bruin (2009) and Felgenhauer and Webster (2013a,b).[24]

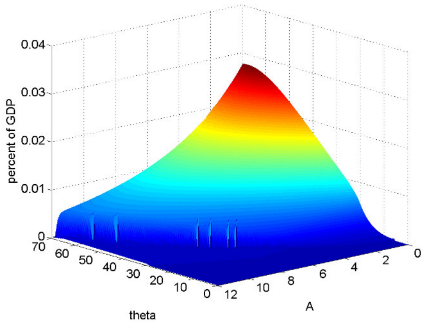
Comparing the graphs in Figure 5 with each other and with Figure 1c, we can see that neglecting uncertainty and the real options values shifts the thresholds downwards to a great extent. Accounting for uncertainty but ignoring the real options approach alters the threshold curves less. For low A values, the area in which emissions are not curbed is enlarged. The timing of adaptation is only slightly affected by accounting for uncertainty. What really has a big impact on the decision is the incorporation of the

[23]The graph also indicates that with extremely high values of θ and extremely low values of A, the investment efforts may rocket upwards to approximately 0.7% of the GDP. This static analysis, however, hides the fact that this combination of very high values of θ and extremely low values of A will not occur, as the policy maker expands the capital stock long before the stochastic process can fluctuate to this level. Therefore, it is not deemed necessary to implement an explicit investment budget constraint, which would add just another parameter posing calibration difficulties.

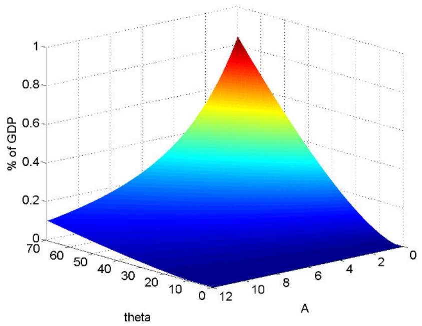
[24]It must be noted that these studies explore the effects of uncertainty attached to different components of the model. A direct comparison with these studies is thus not possible.



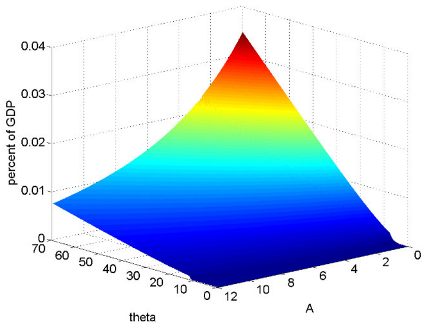
(a) Investment Efforts in the High-Emission Scenario under Uncertainty



(b) A Reduction in the Investment Efforts due to Optimal Mitigation in the Uncertainty Scenario



(c) The Investment Costs, Assuming No Uncertainty and No Mitigation

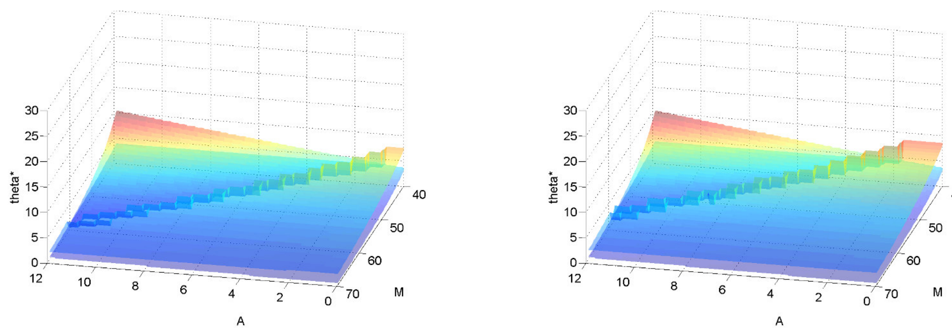


(d) A Reduction in the Investment Efforts due to Optimal Mitigation in the Deterministic Scenario

Figure 4: The Adaptation Investment Costs in M =40

values of waiting or the real options values generated by the tension between uncertainty and the irreversibilities, as emphasized by Figure 1c. The area of inaction, in which neither of the climate policy measures is adopted, is shown to be significantly larger in this graph. Accordingly, this result given by the real options approach is in accordance with the existing global climate policy inaction. In contrast, Figure 5b indicates that a global climate policy of adaptation and mitigation would already have been adopted if the policy makers had not incorporated any considerations of delaying policy adoptions and waiting for more information to arrive.

Taking a closer look at the graphs, we can see that the area in which adaptation is the preferred measure is widened by the real options approach. ROA thus gives more weight to adaptation to fight acute exposure to climate change damage than the ordinary expected present value approach. As uncertainty is also accounted for in the expected net present value approach, this observation can only be explained by the interaction of uncertainty and the economic irreversibilities. Investments in adaptation are allowed to be of a small scale, which makes it possible to limit the magnitude of the sunk costs. In contrast, mitigation imposes relatively high sunk costs. The combination of comparatively low sunk costs and being less affected by uncertainty restricts the real options value of adaptation, which gives adaptation greater priority in a more vulnerable economy. On the contrary, mitigation is delayed due to its rather high sunk costs and its rather uncertain benefits. However, the marginal real options values cause the marginal benefits of adaptation to decrease much faster for high A values. Accordingly, for a better-adapted economy, this approach favours the stand-alone policy of curbing emissions more than the expected net present value approach does. ROA widens the areas in which only one measure is adopted, i.e. the associated values of waiting delay the implementation of the measure that is least favoured. Consequently, the benefits of taking a real options perspective are not trivial. This perspective helps us to understand the existing reservations regarding early climate policy activities. In addition, it points out that the policy maker is rather reluctant to adopt two measures that cause sunk costs and generate more or less uncertain benefits.



(a) The Optimal Climate Policy Thresholds in the Deterministic Framework

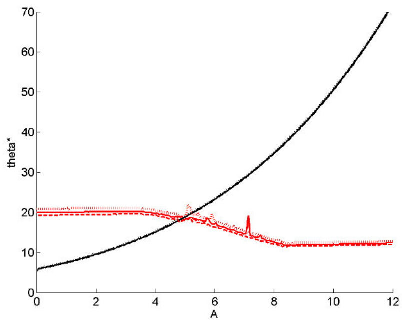
(b) The Optimal Climate Policy Thresholds in the Expected Net Present Value Framework

Figure 5: The Optimal Climate Policy Thresholds under Alternative Methodological Assumptions

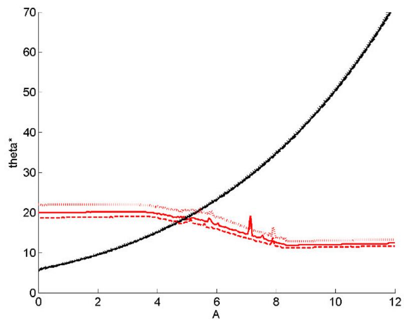
The optimal policy mix certainly depends on the above choices of the parameter values. Clarification of the involved sensitivity of the results is provided by Figure 6 - Figure 9. For the purpose of a clear visual representation, the response of the mitigation threshold (red) and the adaptation threshold (black) to alternative assumptions on the parameter values is only given for the value $M = 40$. In each case, the base calibration of the investigated parameter is varied by $\pm$ 10%. If clarity requires it, an additional graph for a $\pm$ 20% parameter variation is presented.[25] The thresholds resulting from the new simulations are then compared with the thresholds of the base calibration.
Figure 6 indicates that mitigation is more sensitive to changes in uncertainty σ. A $\pm$ 10% variation as shown by Figure 6a causes only small shifts in the mitigation threshold

[25]The other graphs for a $\pm$20% variation are listed in Appendix D.

and no visible changes in the adaptation threshold. More pronounced is the result for a ± 20% variation given by Figure 6b. The adaptation threshold appears to be almost insensitive. The benefits of mitigation, which evolve slowly over the considered time horizon, are crucially affected by intrinsic uncertainty, as it grows over time as well. In contrast, the adaptation decision is based on the benefits that this capital will grant in its rather short life-time. These benefits are thus more guaranteed and less affected by variations in σ. This result, however, does not imply that uncertainty is not important for the adaptation decision at all, as proven by the comparison of Figure 1c with Figure 5.



(a) Sensitivity to Alternative σ Values Generated by a ± 10% Variation: $\sigma = 0.07$ (Solid Line, Base Calibration), $\sigma = 0.063$ (Dashed Line), $\sigma = 0.077$ (Dotted Line)

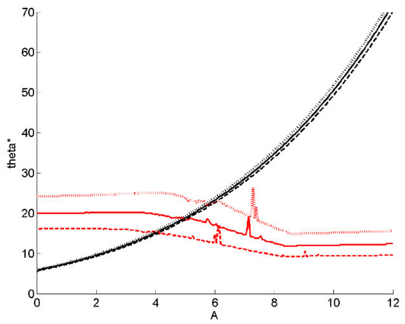


(b) Sensitivity to Alternative σ Values Generated by a ± 20% Variation: $\sigma = 0.07$ (Solid Line, Base Calibration), $\sigma = 0.056$ (Dashed Line), $\sigma = 0.084$ (Dotted Line)

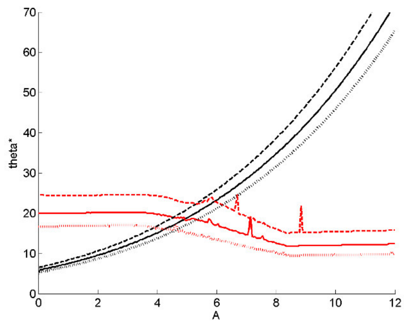
Figure 6: Sensitivity of the Optimal Policy Mix to Uncertainty Depicted by the Threshold of Mitigation (Red) and the Threshold of Adaptation (Black) in $M = 40$

The mix of short- and long-term policy measures may depend on the policy maker's weighting of future welfare. To this end, the effects of alternative assumptions on the discount rate value r are examined. The results in Figure 7a emphasize the importance of the appropriate discount rate choice. As the lifetime of adaptation capital is relatively short compared with the effects of mitigation, the adaptation threshold is only slightly influenced by the choice of the discount rate. In contrast, small variations in the discount rate can generate huge differences in the timing of mitigation. Mitigation becomes more attractive for lower discount rates. A far-sighted policy maker cares more about the future damage costs and thus finds the long-term solution to the climate problem, mitigation, more appealing.

As mentioned in Section 2.1, it is debatable how GDP growth affects the measures and their technologies and therefore this is not explicitly modelled in this paper. Nonetheless, the role of alternative GDP values will be examined. Assuming a higher (lower) GDP value is tantamount to having greater (fewer) financial resources available to spend on climate policy efforts, but also to having higher (lower) emissions, higher (lower) climate damage costs in the future and a higher (lower) total amount of emissions to reduce. The question arises of whether a higher GDP gives more priority to adaptation or to mitigation in this modelling framework. Figure 7b reports that a higher GDP level means that the adoption of both measures is accelerated. Whether one or the other option is preferred cannot be answered in general, but depends on how exposed the economy is to the climate impacts. In a well-adapted economy, the two thresholds appear to be equally sensitive to variations in the GDP level. In a badly adapted economy, adaptation is not very sensitive to the GDP, because early investment is mandatory irrespective of having a 10% higher or lower GDP level. However, a richer world can cut emissions sooner, because more financial means are left after undertaking adaptation efforts. The sensitivity of mitigation is thus higher in the area of low A values.



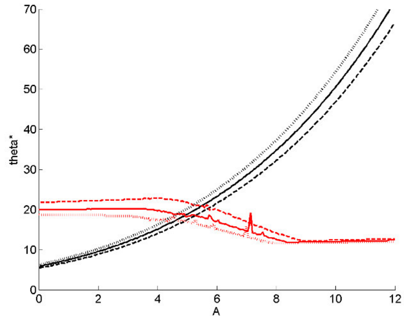
(a) Sensitivity to Alternative Discount Rates Generated by a $\pm$ 10% Variation: $r = 0.025$ (Solid Line, Base Calibration), $r = 0.0225$ (Dashed Line), $r = 0.0275$ (Dotted Line)



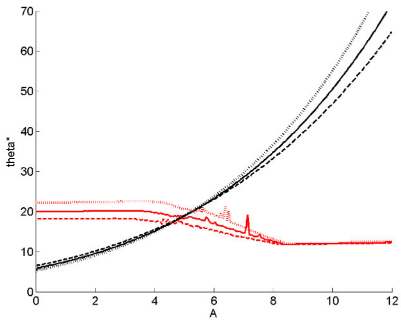
(b) Sensitivity to Alternative GDP Values Generated by a $\pm$ 10% Variation: $Y = 78.97$ (Solid Line, Base Calibration), $Y = 71.07$ (Dashed Line), $Y = 86.87$ (Dotted Line)

Figure 7: Sensitivity of the Optimal Policy Mix to Discounting and GDP

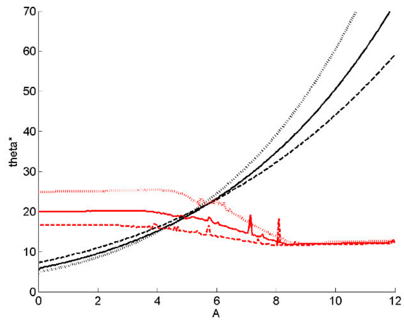
As already mentioned, it is necessary to examine the results concerning the rather vague calibration of the adaptation model. Figure 8 delivers insights into the sensitivity to the depreciation rate and the effectiveness of adaptation capital. Intuition suggests that the optimal policy mix may depend on the depreciation rate ξ of adaptation. A high depreciation rate implies that the involved investments bring only short-term effects, while a lower rate makes adaptation compete with mitigation as a long-term policy. More precisely, a lower depreciation rate makes adaptation a measure that not only helps to alleviate the impacts of the current damage but that also reduces the impacts in the more distant future. Consequently, adaptation partially crowds out mitigation in the optimal policy portfolio. This is confirmed by Figure 8a, as the shifts in the thresholds imply that the policy maker invests in adaptation sooner (later) and curbs emissions later (sooner) if the capital stock depreciates slower (faster). Thus, the effects of substitution between both policy measures is decisively affected by the durability of the adaptation projects undertaken.



(a) Sensitivity to Alternative Depreciation Rates Generated by a $\pm$ 10% Variation: $\xi = 0.075$ (Solid Line, Base Calibration), $\xi = 0.0675$ (Dashed Line), $\xi = 0.0825$ (Dotted Line)



(b) Sensitivity to Alternative Adaptation Effectiveness Parameters Generated by a $\pm$ 10% Variation: $\phi = 4.5$ (Solid Line, Base Calibration), $\phi = 4.05$ (Dashed Line), $\phi = 4.95$ (Dotted Line)

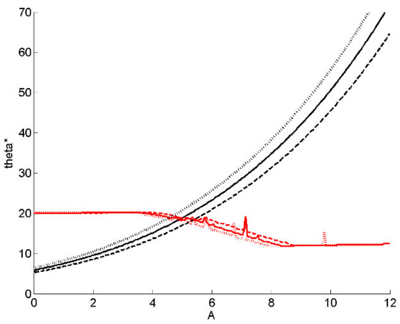


(c) Sensitivity to Alternative Adaptation Effectiveness Parameters Generated by a $\pm$ 20% Variation: $\phi = 4.5$ (Solid Line, Base Calibration), $\phi = 3.6$ (Dashed Line), $\phi = 5.4$ (Dotted Line)

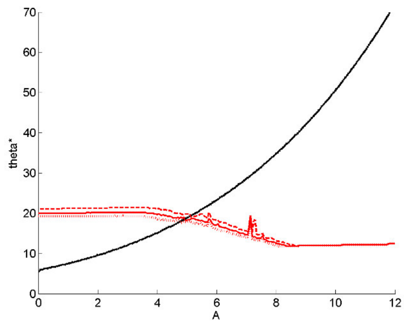
Figure 8: Sensitivity of the Optimal Policy Mix to the Calibration of the Adaptation Parameters

The sensitivity to alternative assumptions on the effectiveness of adaptation is examined in Figure 8b and Figure 8c. How the effectiveness parameter ϕ affects adaptation depends on the size of the currently operating adaptation capital stock. For low values of A, a higher level of effectiveness of adaptation clearly incentivizes early investment to build up a sufficient stock size. The meaning of “sufficient” also relies on the effectiveness parameter. Consequently, a higher ϕ-value implies that investment in adaptation can be cut back sooner. The effectiveness parameter affects mitgation significantly as well. If adaptation works well to fight climate change impacts, it becomes less of an imperative to fight the root of the climate change problem. Accordingly, mitigation can be delayed. The lack of empirical evidence requires us to test alternative values for the “unit costs” γ_1. Figure 9a indicates that higher investment costs make adaptation efforts less attractive and the threshold shifts upwards, the greater the installed adaptation capital stock is. Although adaptation efforts are more costly, the timing of mitigation is not affected. On the one hand, one may suspect that mitigation is delayed if it is undertaken after investing in adaptation, because the investment claims a bigger share of the financial resources. On the other hand, the adoption of mitigation could be accelerated in order to make the future generations less dependent on expensive investments in adaptation. At this point of our analysis, Figure 9a offers no other choice than to conjecture that the two effects balance each other out, leading to the insensitivity of the mitigation threshold.

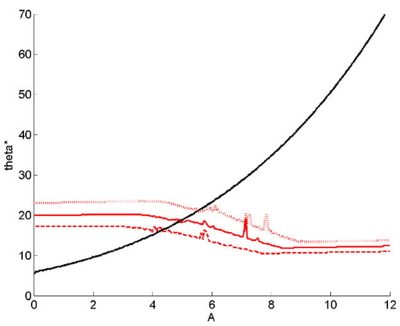
Figure 9b investigates the influence of the other component of the adaptation costs, the



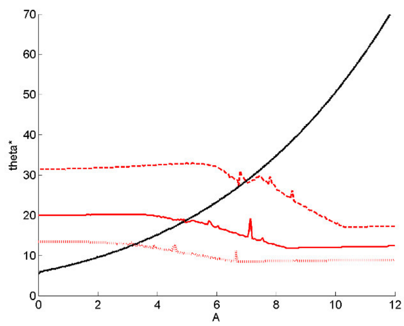
(a) Sensitivity to Alternative Adaptation Cost Parameters Generated by a ± 10% Variation: $\gamma_1 = 0.4$ (Solid Line, Base Calibration), $\gamma_1 = 0.36$ (Dashed Line), $\gamma_1 = 0.44$ (Dotted Line)



(b) Sensitivity to Alternative Adaptation Cost Parameters Generated by a ± 10% Variation: $\gamma_2 = 16.81$ (Solid Line, Base Calibration), $\gamma_2 = 15.13$ (Dashed Line), $\gamma_2 = 18.49$ (Dotted Line)



(c) Sensitivity to Alternative Mitigation Cost Parameters Generated by a ± 10% Variation: $\kappa_1 = 0.03$ (Solid Line, Base Calibration), $\kappa_1 = 0.027$ (Dashed Line), $\kappa_1 = 0.33$ (Dotted Line)



(d) Sensitivity to Alternative Mitigation Cost Parameters Generated by a ± 10% Variation: $\kappa_2 = 1.2$ (Solid Line, Base Calibration), $\kappa_2 = 1.08$ (Dashed Line), $\kappa_2 = 1.32$ (Dotted Line)

Figure 9: Sensitivity of the Optimal Policy Mix to the Calibration of the Cost Parameters

adjustment costs. This parameter γ_2 is amongst the factors that determine the costs of quick capital stock expansion, which can be interpreted as evidence of the economy's adaptive capacity. As expected, the timing of adaptation is insensitive to alternative adjustment costs.[26] In contrast, the adaptive capacity has an impact on the timing of mitigation. If the capability to adapt is poor, the policy maker should not rely on adaptation as a measure to fight climate impacts. Mitigation is thus adopted sooner to reduce the need to adapt in the future.

The final point to investigate is how the mitigation costs affect the thresholds. Figures 9c and 9d show that higher mitigation costs, i.e. higher κ_1 and lower κ_2, deter the policy maker from curbing emissions. On the contrary, the necessity to invest in adaptation is not influenced by the spending on mitigation, as it only depends on the magnitude of the marginal welfare of adaptation.[27]

4. Conclusion and Outlook

The optimal policy response to climate change has to account for a mix of mitigation and adaptation efforts. This paper considers this mix from the perspective of a continuous-time real options modelling framework, which allows the examination of the impacts of economic and ecological irreversibilities and intrinsic uncertainty in the future climate damage costs. To this end, a new framework for a portfolio of adaptation and mitigation options is developed. The mitigation option gives the opportunity to choose the optimal timing to commit to a certain emission reduction target. The form of adaptation that is considered can be categorized as proactive adaptation and is modelled as investments in an adaptation capital stock. Exercising the adaptation option means optimally expanding the adaptation stock. The model also features a stylized notion of adaptation capacity, which determines how quickly the adaptation proceeds and is assumed to be compromised by unabated climate damages.

The numerical simulations show the benefits of analysing the optimal climate policy decision from a real options perspective. It is not the existence of uncertainty in itself but the interaction with the irreversibilities that delays the adoption of both climate policy measures significantly. More precisely, it postpones the implementation of the first measure and it also prolongs the period until the second measure complements the policy mix. Hence, it points out that the policy maker is rather reluctant to adopt two measures that cause sunk costs and generate more or less uncertain benefits.

The optimal policy mix is determined by the differences in the characteristics of the measures. Among the most important distinguishing features are the different timescales on which the two measures work. The benefits evolve differently over time: while the investments in adaptation can pay off rather soon, the benefits of mitigation are expected to accumulate over a long time horizon. Consequently, the simulations demonstrate that

[26] As derived in Section 2.2, the timing of adaptation is determined by the values for which the marginal welfare of adaptation is greater than the "unit costs", i.e. the adjustment costs do not affect the adaptation threshold.

[27] More precisely, when taking the partial derivative of equations (14) and (15) to obtain the marginal welfare of adaptation, the mitigation costs drop out.

adaptation is the preferred measure if the economy is currently exposed to climate change impacts. If the marginal benefits of expanding the adaptation capital stock are sufficiently low, mitigation is given a higher priority so that the root causes of climate change can be fought. Another distinguishing feature is given by the magnitude of the incurred sunk costs. Curbing emissions incurs relatively high (at least) partially irreversible costs. The tension between these costs and the uncertainty, which grows over the time horizon in which the benefits accrue, nourishes the real option to mitigate. In contrast, the benefits of the investments in adaptation are of a shorter lifetime, i.e. the benefits are less subject to uncertainty. Adaptation allows small-scale investments to be made, which means that the incurred sunk costs are not necessarily high. Consequently, compared with other decision frameworks, the real options perspective grants adaptation more emphasis as the preferred measure in the portfolio.

The simulations also disclose significant asymmetry in the interaction of the two real options, which is again reasoned by the different timescales on which the two measures work. In particular, the simulations indicate that mitigation is delayed not only due to its own real options value but also due to the opportunity to invest in adaptation. In contrast, the timing of adaptation efforts is mainly determined by the present levels of climate change impacts and less so by the future developments of the atmospheric pollution level. Likewise, the today's investments in adaptation are only slightly affected by curbing emissions now. Hence, the real option to adapt is less affected by the presence of the opportunity to mitigate than vice versa.

An extensive sensitivity analysis reveals that the policy maker's weighting of future welfare is crucial for the optimal policy mix, because the discount rates determine the importance of emission cuts. The adaptation real option is less affected by discounting due to the above-mentioned short-term benefits of the involved investments. Further numerical simulations show that the adaptation option is exercised sooner and mitigation adopted later if adaptation depreciates less quickly. Higher "unit" costs of adaptation are demonstrated to increase the real options value of adaptation but to have no effects on mitigation. In contrast, a lower capacity to adapt accelerates mitigation.

The modelling framework is meant to be the first stepping stone towards real options models of holistic climate policy portfolios. The framework can be extended to incorporate options of Carbon Capture and Storage, options to promote technological progress or more specific adaptation options that allow the display of the manifoldness and complexity of adaptation in reality. Furthermore, it would be fruitful to account for adaptation measures that protect against catastrophic climate damage. Adaptation measures that grant different levels of flexibility are also worthwhile investigating in a real options model. Some adaptation measures have negative effects on mitigation efforts, while others have positive spillover effects, as outlined by IPCC (2007). There are adaptation measures that are inseparable from development policies, which would represent another real option. As a result of not covering all these and many more forms of adaptation, the model is rather stylized, but it grants the advantage of having a small model to explain the interaction of two climate policy instruments under uncertainty and irreversibility.

Appendices

A. Solution of the Optimal Adaptation Policy

The adaptation model needs to be solved in several steps. One possible way to proceed is to compute the marginal welfare of adaptation, in order to find the threshold between the area of inaction S_1 and the area of action S_2. The information about the marginal values w and the threshold location can then be used to derive the solution to the welfare function.

As already indicated in Section 2.2, the solution in the area of inaction consists of two parts. More precisely, the solution of W for S_1 is given by the expected present welfare of never investing into adaptation and the real option of investing in the future. Accordingly, the marginal welfare for S_1 consists of the respective marginal values. Both values can be derived from equation (16). The marginal expected present welfare of adaptation, from now on referred to as w^P, is the same as the particular solution to equation (16). The general solution of equation (16) is used to find the marginal real option of adaptation. As a by-product, the location of the threshold defined in terms of the marginal values is obtained. This information about the the threshold location is then used to determine the solution to equation (17). For S_2, the real options to adapt are exercised instantaneously and thus only the expected present welfare of optimal investment needs to be computed. In an analogous manner, only the particular solution of equation (17) needs to be computed, which can only be derived, because its value γ_1 in the threshold is known. The information about the threshold location and the marginal welfare for S_1 and S_2 is sufficient to derive the solution to equations (14) and (15). In the following, the above-outlined steps are described in more detail.

A.1. The Particular Solution of Equation (16)

The marginal expected present welfare of adaptation for S_1 equals:

$$w^P = \mathbb{E}_0 \left[\int_0^\infty \left(Y \frac{\alpha\phi\rho\theta(t)M(t)^\psi}{(1+\alpha A(t))^{\phi+1}} e^{-\frac{\rho\theta(t)M(t)^\psi}{(1+\alpha A(t))^\phi}} \right) e^{-rt} dt \right], \tag{A.1}$$

with $\theta(t)$ and $M(t)$ given by equations (4) and (5), and $A(t)$ is provided by equation (8) with $a(t) = 0$ for all t. The solution of (A.1) cannot be derived analytically but can be obtained by solving (16) numerically. To this end, the specification of the model needs to be enriched by some more information.

$$\text{upper boundary condition for } M \to \infty: \quad w^P = 0, \tag{A.2}$$

$$\text{lower boundary condition for } \theta = 0: \quad w^P = 0, \tag{A.3}$$

$$\text{upper boundary condition for } \theta \to \infty: \quad w^P = 0. \tag{A.4}$$

Condition (A.2) and (A.4) become clear by considering (A.1): for $M \to \infty$ as well as for $\theta \to \infty$, the exponential term converges to zero faster than its factor. Condition (A.3) explains that the integral is zero for $\theta = 0$ and it stays zero, as the geometric Brownian motion has an absorbing barrier at this point.

In the following, equation (14) is solved by applying the finite difference method, which gives the values of w^P in terms of a discrete choice of its function arguments. This means that the continuous function w^P is approximated by its discrete version $w^P(i\Delta\theta, j\Delta M, k\Delta A) = w^P_{i,j,k}$, where $0 \leq i \leq \mathscr{I}$, $0 \leq j \leq \mathscr{J}$ and $0 \leq k \leq \mathscr{K}$. The values are chosen so that $\mathscr{I}\Delta\theta = \theta_{\max}$, $\mathscr{J}\Delta M = M_{\max}$ and $\mathscr{K}\Delta A = A_{\max}$ with sufficiently large numbers $\theta_{\max}$, $M_{\max}$ and $A_{\max}$. The approximation of the partial derivatives by finite differences is crucial. In general, two types of finite difference schemes can be applied: the explicit and the implicit finite difference method. The explicit method has the disadvantage that the discretization must obey some constraints, which often turn out to be very restrictive. Especially for the equations at hand, the conditions for the number of steps and the length of the step sizes imply enormous computational effort. Therefore, the implicit finite difference method is applied in the following. More precisely, equation (16) is transformed into:

$$\begin{aligned}
(r+\xi) w^P_{i,j-1,k} = {} & Y \frac{\alpha\phi\rho i\Delta\theta \left((j-1)\Delta M\right)^{\psi}}{(1+\alpha k\Delta A)^{\phi+1}} e^{-\frac{\rho i\Delta\theta((j-1)\Delta M)^{\psi}}{(1+\alpha k\Delta A)^{\phi}}} \\
& + \left(\beta\epsilon(1-m)Y - \delta\left((j-1)\Delta M\right)\right) \frac{w^P_{i,j,k} - w^P_{i,j-1,k}}{\Delta M} \\
& + \mu i\Delta\theta \frac{w^P_{i+1,j-1,k} - w^P_{i-1,j-1,k}}{2\Delta\theta} \\
& + \frac{1}{2}\sigma^2 \left(i\Delta\theta\right)^2 \frac{w^P_{i+1,j-1,k} + w^P_{i-1,j-1,k} - 2w^P_{i,j-1,k}}{(\Delta\theta)^2} \\
& - \xi k\Delta A \frac{w^P_{i,j-1,k} - w^P_{i,j-1,k-1}}{\Delta A} \qquad \forall i,j,k,
\end{aligned} \tag{A.5}$$

which is the same as:

$$\begin{aligned}
w^P_{i,j,k} = {} & -qY \frac{\alpha\phi\rho i\Delta\theta \left((j-1)\Delta M\right)^{\psi}}{(1+\alpha k\Delta A)^{\phi+1}} e^{-\frac{\rho i\Delta\theta((j-1)\Delta M)^{\psi}}{(1+\alpha k\Delta A)^{\phi}}} + w^P_{i-1,j-1,k}\ x_1 \\
& + w^P_{i,j-1,k}\ x_2 + w^P_{i+1,j-1,k}\ x_3 + w^P_{i,j-1,k-1}\ x_4,
\end{aligned} \tag{A.6}$$

with

$$
\begin{aligned}
q &= \frac{\Delta M}{\beta\epsilon(1-m)Y - \delta(j-1)\Delta M} \\
x_1 &= q\left(\frac{1}{2}\mu i - \frac{1}{2}\sigma^2 i^2\right), \\
x_2 &= 1 + q\left(r + \sigma^2 i^2 + \xi(k+1)\right), \\
x_3 &= q\left(-\frac{1}{2}\mu i - \frac{1}{2}\sigma^2 i^2\right), \\
x_4 &= -q\xi k.
\end{aligned} \tag{A.7}
$$

As the values of $w^P_{i,\mathscr{J},k}$ for all i and k are given by (A.2), the values of $w^P_{i,\mathscr{J}-1,k}$ can be found by using its relation to $w^P_{i,\mathscr{J},k}$ as given by equation (A.6). Accordingly, all other values $w^P_{i,j-1,k}$ can thus be computed step by step. For $A = 0$, it should be noted that the partial derivative with respect to A vanishes and the 'out-of-the-grid' value $w^P_{i,j-1,-1}$ is not needed to approximate all the values in $k = 0$.

A.2. The General Solution of Equation (16)

Consider the value of the option to invest in additional adaptation capital W^G, which is described by the homogeneous part of equation (14):

$$
rW^G = \left(\beta\epsilon(1-m)Y - \delta M\right)\frac{\partial W^G}{\partial M} + \mu\theta\frac{\partial W^G}{\partial\theta} + \frac{1}{2}\sigma^2\theta^2\frac{\partial^2 W^G}{\partial\theta^2} - \xi A\frac{\partial W^G}{\partial A}. \tag{A.8}
$$

The real option to adapt loses value the more adaptation capital is installed. Hence, the partial derivative $\frac{\partial W^G}{\partial A}$ is negative. Defining w^G as $-\frac{\partial W^G}{\partial A}$, the marginal option can be expressed as:

$$
(r+\xi)w^G = \left(\beta\epsilon(1-m)Y - \delta M\right)w^G_M + \mu\theta w^G_\theta + \frac{1}{2}\sigma^2\theta^2 w^G_{\theta\theta} - \xi A w^G_A, \tag{A.9}
$$

which obeys the value-matching condition:

$$
w^G = \max\left\{w^P - \gamma_1, 0\right\}, \tag{A.10}
$$

at the threshold of taking action. Please note that w^P is the particular solution of equation (16), as described in Appendix A.1.

To approximate the marginal option, the following additional boundary conditions are then implied:

$$
\begin{aligned}
\text{upper boundary condition for } M \to \infty: &\quad w^G = 0, && \text{(A.11)} \\
\text{lower boundary condition for } \theta = 0: &\quad w^G = 0, && \text{(A.12)} \\
\text{upper boundary condition for } \theta \to \infty: &\quad w^P = 0. && \text{(A.13)}
\end{aligned}
$$

The conditions (A.11) and (A.13) can be explained by noting that w^P is zero for $M \to \infty$ and $\theta \to \infty$. Hence, welfare cannot be increased by additional investment in adaptation capital, which makes the real option worthless, and this does not change for a slightly higher value of A. Condition (A.12) is due to the absorbing barrier of the geometric Brownian motion.

Equation (A.9) is approximated in a similiar way to w^P in Appendix A.1:

$$w^G_{i,j,k} = w^G_{i-1,j-1,k}\ x_1 + w^G_{i,j-1,k}\ x_2 + w^G_{i+1,j-1,k}\ x_3 + w^G_{i,j-1,k-1}\ x_4, \quad \text{(A.14)}$$

with x_1, x_2, x_3 and x_4 as in (A.7). The numerical procedure, however, is more complex than the one in Appendix A.1. Implicit schemes for the free boundary problem given by (A.10) cannot be solved directly.[28] Therefore, the solution is derived iteratively by applying successive overrelaxation (SOR). The acceleration parameter is the value in which the spectral radius of the SOR matrix is the minimum, as explained in detail by Thomas (1999). This procedure provides the marginal real options values and as a by-product the threshold of taking action in adaptation.

A.3. The Particular Solution of Equation (17)

After conducting the numerical routine explained in Appendix A.1 and A.2, we can make use of the information about the threshold location. Denote the set of all values (θ, M, A) defining the threshold as $\mathscr{T} = \{(\theta, M, A) : w^G(\theta, M, A) = w^P(\theta, M, A) - \gamma_1\}$. Then, the required boundary conditions for the marginal expected present welfare for S_2 (henceforth referred to as w^{P2}) read:

$$\text{upper boundary condition for } M \to \infty: \quad w^{P2} = 0, \quad \text{(A.15)}$$

$$\text{upper boundary condition for } \theta \to \infty: \quad w^{P2} = 0. \quad \text{(A.16)}$$

$$\text{threshold condition, for } (\theta, M, A) \in \mathscr{T}: \quad w^{P2} = \gamma_1, \quad \text{(A.17)}$$

In the case of extremely high damage costs, i.e. $M \to \infty$ and/or $\theta \to \infty$, the welfare approaches zero and additional investment in adaptation will not change this.

In order to apply an implicit finite difference scheme to equation (17), it is neccessary to deal with two troubling characteristics of this partial differential equation. The first one relates to the non-linear terms, which render the matrix manipulations required to solve the implicit schemes impossible. As stated by Thomas (1995), there is no nice way out of this problem and the easiest and most common solution is to lag parts of the non-linear term. Accordingly, I choose to lag the values of w^{P2} in the non-linear terms of $\left(\frac{\Upsilon}{\gamma_2}\left(w^{P2} - \gamma_1\right) - \xi A\right) w^{P2}_A$ and $\Upsilon \frac{\alpha\phi\rho\theta M^{\psi}}{(1+\alpha A)^{\phi+1}} \frac{1}{2\gamma_2} \left(w^{P2} - \gamma_1\right)^2$. The other issue relates to the changing sign of the term $\left(\frac{\Upsilon}{\gamma_2}\left(w^{P2} - \gamma_1\right) - \xi A\right)$, which may cause instabilities in the routine. This problem can be elegantly handled by upwinding: whenever the sign is negative, w^{P2}_A is approximated by the backward finite difference scheme; whenever the

[28] A more detailed explanation of this problem and further useful information about the finite difference method is given by Brandimarte (2006).

sign is positive, the forward finite difference scheme is used. Additionally, the scheme is made conservative by refining the discretization of the term in the A-direction: whenever the sign is negative, the term is discretized at the point $(i, j, k - \frac{1}{2})$ instead of (i, j, k); whenever the sign is positive, the term is discretized at the point $(i, j, k + \frac{1}{2})$ instead of (i, j, k), see e.g. Wilmott (1998). Denoting $\Upsilon_{i,j,k}$ as the discretized version of equation (3), the scheme thus reads:

$$\begin{aligned} w^P_{i,j,k} = -q\Upsilon_{i,j,k}\frac{\alpha\phi\rho i\Delta\theta\left((j-1)\Delta M\right)^{\psi}}{(1+\alpha k\Delta A)^{\phi+1}}\left(Y + \frac{1}{2\gamma_2}\left(w^P_{i,j,k} - \gamma_1\right)^2\right) + w^P_{i-1,j-1,k}\ x_1 \\ + w^P_{i,j-1,k}\ x_5 + w^P_{i+1,j-1,k}\ x_3 + w^P_{i,j-1,k-1}\ x_6 + w^P_{i,j-1,k+1}\ x_7, \end{aligned} \tag{A.18}$$

with q, x_1 and x_3 as in (A.7). The remaining coefficients are given by:

$$x_5 = \begin{cases} 1 + q\left(r + \sigma^2 i^2 + \xi - \Pi_{i,j,k-\frac{1}{2}}\right) & \text{for} \quad \Pi \leq 0 \\ 1 + q\left(r + \sigma^2 i^2 + \xi + \Pi_{i,j,k+\frac{1}{2}}\right) & \text{for} \quad \Pi > 0, \end{cases} \tag{A.19}$$

$$x_6 = \begin{cases} q\Pi_{i,j,k-\frac{1}{2}} & \text{for} \quad \Pi \leq 0 \\ 0 & \text{for} \quad \Pi > 0 \end{cases} \tag{A.20}$$

and

$$x_7 = \begin{cases} 0 & \text{for} \quad \Pi \leq 0 \\ -q\Pi_{i,j,k+\frac{1}{2}} & \text{for} \quad \Pi > 0 \end{cases} \tag{A.21}$$

with Π being:

$$\Pi_{i,j,k} = \frac{\Upsilon_{i,j,k}}{\gamma_2\Delta A}\left(w^{P2}_{i,j,k} - \gamma_1\right) - \xi k. \tag{A.22}$$

The values $w^{P2}_{i,j,k-\frac{1}{2}}$ and $w^{P2}_{i,j,k+\frac{1}{2}}$ are the average values of their "neighbours" $w^{P2}_{i,j,k}$ and $w^{P2}_{i,j,k-1}$, and $w^{P2}_{i,j,k}$ and $w^{P2}_{i,j,k+1}$, respectively.

Please note that defining boundary conditions for A is not necessary. For $A = 0$, Π is positive and the values in $A = 0$ can be directly derived by the scheme. Likewise, the values $w^{P2}_{i,j,\mathscr{K}}$ directly result from the scheme, because the marginal welfare of adaptation for a very high A approaches zero and thus Π is certainly negative. Hence, in the A direction the scheme only uses values from inside the grid. The scheme is then iteratively solved for all the remaining values beyond the threshold of taking action.

A.4. The Particular Solution of Equation (14) *and Equation* (15)

Appendices A.1 and A.3 describe how to compute the marginal expected present welfare for $S1$ and $S2$, respectively. The idea is to insert these values for $\frac{\partial W}{\partial A}$ into the corresponding equations (14) and (15) and to apply an implicit finite difference scheme.

Equation (10) helps to find the boundary conditions for the particular solution of (14) (henceforth referred to as W^{P1}):

$$\text{upper boundary condition for } M \to \infty: \quad W^{P1} = -\frac{1}{r}\kappa_1 m^{\kappa_2} Y, \tag{A.23}$$

$$\text{lower boundary condition for } \theta = 0: \quad W^{P1} = \frac{Y}{r}\left(1 - \kappa_1 m^{\kappa_2}\right), \tag{A.24}$$

$$\text{upper boundary condition for } \theta \to \infty: \quad W^{P1} = -\frac{1}{r}\kappa_1 m^{\kappa_2} Y. \tag{A.25}$$

For an enormous amount of pollution $M \to \infty$ or for a high value of $\theta \to \infty$, the GDP net of damage tends to zero. The mitigation costs remain as the only term in equation (10). In the case of $\theta = 0$, the climate damage costs remain zero and $\Upsilon \equiv 1$. Then, the integral in equation (10) has the analytical solution (A.24).

The same boundary conditions apply to the particular solution of (15) (henceforth referred to as W^{P2}). If the GDP net of the damage costs is close to zero, equation (12) shows that a^* becomes zero. If the climate damage costs remain zero, there is no need to invest in adaptation. Hence, for these extreme cases, a^* is zero and W^{P2} behaves in the same way as W^{P1}.

The scheme to approximate W^{P1} is then:

$$\begin{aligned} W^{P1}_{i,j,k} = &-q\left(Y\Upsilon_{i,j-1,k} - \kappa_1 m^{\kappa_2} - \xi k \Delta A w^{P1}_{i,j-1,k}\right) \\ &+ W^{P1}_{i-1,j-1,k}\ x_1 + W^{P1}_{i,j-1,k}\ x_8 + W^{P1}_{i+1,j-1,k}\ x_3, \end{aligned} \tag{A.26}$$

with w^{P1} being given by Appendix A.1, q, x_1 and x_3 as in (A.7) and x_8 is given by:

$$x_8 = 1 + q\left(r + \sigma^2 i^2\right). \tag{A.27}$$

The same coefficients are used for the scheme to approximate W^{P2}:

$$\begin{aligned} W^{P2}_{i,j,k} = &-q\left(Y\Upsilon_{i,j-1,k} - \kappa_1 m(t)^{\kappa_2} - \xi k \Delta A w^{P2}_{i,j-1,k} + \frac{\Upsilon_{i,j-1,k}}{2\gamma_1}\left(w^{P2}_{i,j-1,k} - \gamma_1\right)^2\right) \\ &+ W^{P2}_{i-1,j-1,k}\ x_1 + W^{P2}_{i,j-1,k}\ x_8 + W^{P2}_{i+1,j-1,k}\ x_3, \end{aligned} \tag{A.28}$$

with w^{P2} being given by Appendix A.3.

Along the same lines, the real options value as described by (A.8) can be derived. The boundary conditions are

$$\text{upper boundary condition for } M \to \infty: \quad W^{G} = 0, \tag{A.29}$$

$$\text{lower boundary condition for } \theta = 0: \quad W^{G} = 0, \tag{A.30}$$

$$\text{upper boundary condition for } \theta \to \infty: \quad W^{G} = 0. \tag{A.31}$$

The explanation again follows the same logic. In the situation of extremely high climate damage costs, investment in adaptation would no longer be beneficial. For $\theta = 0$, there is no need to invest in adaptation. Therefore, the real options value is zero in both extreme cases.

The marginal real options value w^G is then inserted into the partial differential equation (A.8), which is solved by the analogue to (A.26).

The full solution to Section 2.2 is then composed of the sum of the real options value W^G and the expected present welfare W^{P1} for $S1$ and the expected present welfare W^{P2} for S_2.

B. The Procedure to Solve the Real Option to Mitigate

The applied solution routine to find the values of the real option to mitigate does not fundamentally differ from the finite difference method outlined in Appendix A. To avoid needless repetitions, I only outline the most important steps that need to be considered when solving equation (18).

As in Appendix A.2, I opt to solve this free boundary problem by applying SOR. To save the computational costs of deriving the acceleration parameters, I take the ones derived in Appendix A.2. Although the involved spectral radii are not equal for the two routines, they are sufficiently close to guarantee quick convergence.

As in Appendix A.3, the non-linearity of the partial differential equation does not fit well with the implicit finite difference method. To solve it nonetheless, I rewrite equation (18) for S_2 as follows:

$$\begin{aligned} rW^M = {} & (\beta\epsilon Y - \delta M)\frac{\partial W^M}{\partial M} + \mu\theta\frac{\partial W^M}{\partial \theta} + \frac{1}{2}\sigma^2\theta^2\frac{\partial^2 W^M}{\partial \theta^2} \\ & + \left(\frac{\Upsilon}{2\gamma_2}\left(\frac{\partial W^M}{\partial A} - 2\gamma_1\right) - \xi A\right)\frac{\partial W^M}{\partial A} + \frac{\Upsilon\gamma_1^2}{2\gamma_2}, \end{aligned} \tag{B.1}$$

and opt to "lag" the discretized version of the partial derivative in $\left(\frac{\Upsilon}{2\gamma_2}\left(\frac{\partial W^M}{\partial A} - 2\gamma_1\right) - \xi A\right)$. Finding a boundary condition for $A = 0$ is far from being straightforward. Instead, I coarsely approximate the partial derivative $\frac{\partial W^M}{\partial A}$ in $A = 0$ by the derivative of the corresponding particular solution.

An issue of concern is caused by the switch of the functional form in equation (18). The resulting jump in the values may lead to errors in the approximated finite differences in the neighbourhood of the discontinuities. For instance, the real options values in the switch could drop to a suspiciously low level. With the aim of constraining the magnitude of these errors, I first solve the mitigation model that ignores the opportunity to adapt. The equation describing the real options value in that case is continuous and thus garantuees precise results. In the absence of adaptation, the urgency to mitigate is certainly higher than in the case in which the damage can be alleviated by adaptation.

Accordingly, the values computed thus then serve as a lower boundary in the SOR method that derives the real options values given by equation (18).

C. Calibration

The base calibration is as follows.

Greek letters:

adaptation parameter	α	0.05
atmospheric retention ratio (in ppm per trillion of CO_2 metric tonnes)	β	9.09
natural rate of CO_2 depletion in the atmosphere	δ	0*
emission parameter (in CO_2 metric tonnes per US dollars PPP of GDP)	ϵ	4.18×10^{-4}
adaptation parameter	ϕ	4.5
adaptation cost parameter	γ_1	0.4
adaptation cost parameter	γ_2	16.81
mitigation cost parameter	κ_1	0.03
mitigation cost parameter	κ_2	1.2
drift term in the Brownian motion	μ	0
damage cost parameter	ρ	7.17×10^{-12}
variance term in the Brownian motion	σ	0.07
depreciation rate of adaptation capital	ξ	0.075
damage cost parameter	ψ	4.88

Annotation * : This calibration represents a valuable simplication to the numerical solution routine. The parameter β is parametrized to capture the depreciation, by making the crude assumption that the increase in atmospheric CO_2 follows a constant trend of 3 ppm per year.

Further parameters:

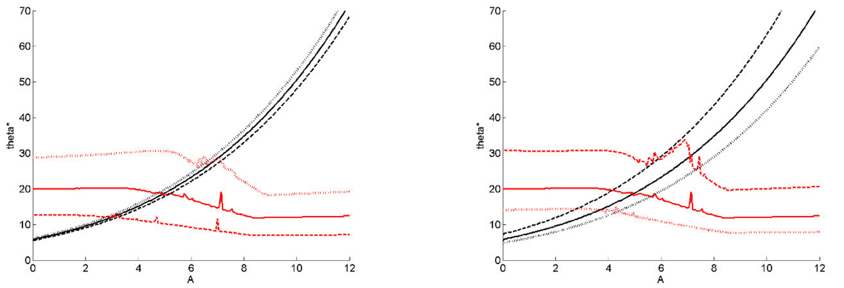
emissions (in trillion of CO_2 metric tonnes)	E	0.033
Global GDP in the absence of climate damages (in trillion US dollars PPP)	Y	78.97
emission reduction rate	m	0.09
discount rate	r	0.025

"Calibration" of the implicit finite difference method:

θ_{max}	100	$\Delta\theta$	0.2
M_{max}	420	ΔM	0.6
A_{max}	16.67	ΔA	0.05

D. Further Simulations for the Sensitivity Analysis

Here, the base calibration of the investigated parameter is varied by ± 20%. The thresholds resulting from the new simulations are then compared with the thresholds of the base calibration.



(a) Sensitivity to Alternative Discount Rates: $r = 0.025$ (Solid Line, Base Calibration), $r = 0.02$ (Dashed Line), $r = 0.03$ (Dotted Line)

(b) Sensitivity to Alternative GDP Values: $Y = 78.97$ (Solid Line, Base Calibration), $Y = 63.18$ (Dashed Line), $Y = 94.76$ (Dotted Line)

Figure 10: Sensitivity of the Optimal Policy Mix to Discounting and GDP, Depicted by the Threshold of Mitigation (Red) and the Threshold of Adaptation (Black) in $M = 40$

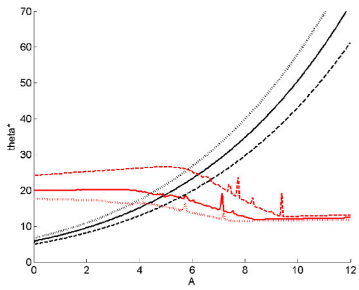
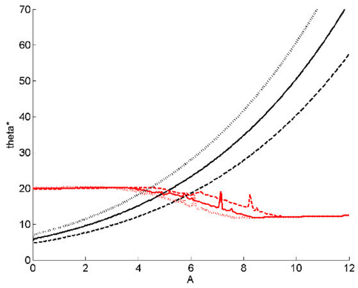
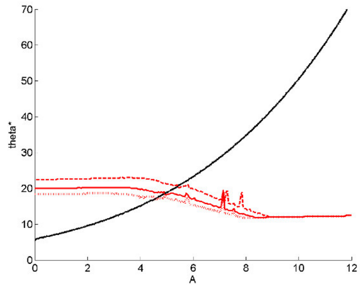


Figure 11: Sensitivity to Alternative Depreciation Rates Generated by a ±20% Variation: $\xi = 0.075$ (Solid line, Base Calibration), $\xi = 0.06$ (Dashed Line), $\xi = 0.09$ (Dotted Line)

Please note that a −20% parameter variation for κ_2 would make the mitigation cost curve concave. This case is thus ignored in the sensitivity analysis.

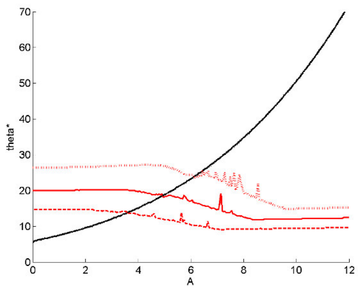


(a) Sensitivity to Alternative Adaptation Cost Parameters Generated by a ±20% Variation: $\gamma_1 = 0.4$ (Solid Line, Base Calibration), $\gamma_1 = 0.32$ (Dashed Line), $\gamma_1 = 0.48$ (Dotted Line)

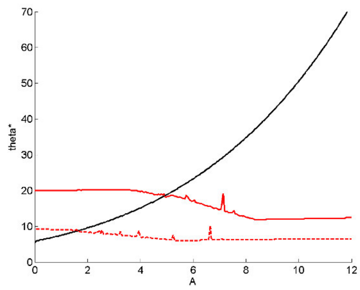


(b) Sensitivity to Alternative Adaptation Cost Parameters Generated by a ±20% Variation: $\gamma_2 = 16.81$ (Solid Line, Base Calibration), $\gamma_2 = 13.45$ (Dashed Line), $\gamma_2 = 20.17$ (Dotted Line)

Figure 12: Sensitivity of the Optimal Policy Mix to the Calibration of the Adaptation Parameters



(a) Sensitivity to Alternative Mitigation Cost Parameters Generated by a ±20% Variation: $\kappa_1 = 0.03$ (Solid Line, Base Calibration), $\kappa_1 = 0.024$ (Dashed Line), $\kappa_1 = 0.36$ (Dotted Line)



(b) Sensitivity to Alternative Mitigation Cost Parameters Generated by a +20% Variation: $\kappa_2 = 1.2$ (Solid Line, Base Calibration) and $\kappa_2 = 1.44$

Figure 13: Sensitivity of the Optimal Policy Mix to the Calibration of the Mitigation Costs

References

Adger, W., Dessai, S., Goulden, M., Hulme, M., Lorenzoni, I., Nelson, D., Naess, L., Wolf, J., Wreford, A., 2009. Are there social limits to adaptation to climate change? Climatic Change 93, 335–354.

Agrawala, S., Bosello, F., Carraro, C., Bruin, K. D., Cian, E. D., Dellink, R., Lanzi, E., 2011a. Plan or react? analysis of adaptation costs and benefits using integrated assessment models. Climate Change Economics 2 (03), 175–208.

Agrawala, S., Bosello, F., Carraro, C., de Cian, E., Lanzi, E., 2011b. Adapting to climate change: Costs, benefits, and modelling approaches. International Review of Environmental and Resource Economics 5 (3), 245–284.

Agrawala, S., Fankhauser, S., 2008. Economic aspects of adaptation to climate change: Costs, benefits and policy instruments. OECD.

Anda, J., Golub, A., Strukova, E., 2009. Economics of climate change under uncertainty: Benefits of flexibility. Energy Policy 37 (4), 1345 – 1355.

Bahn, O., Chesney, M., Gheyssens, J., 2012. The effect of proactive adaptation on green investment. Environmental Science & Policy 18 (0), 9 – 24.

Baranzini, A., Chesney, M., Morisset, J., 2003. The impact of possible climate catastrophes on global warming policy. Energy Policy 31 (8), 691 – 701.

Barrieu, P., Chesney, M., 2003. Optimal timing to adopt an environmental policy in a strategic framework. Environmental Modeling and Assessment 8, 149–163.

Black, F., Scholes, M., 1973. The pricing of options and corporate liabilities. Journal of Political Economy 81 (3), pp. 637–654.

Bosello, F., Carraro, C., Cian, E. D., 2010. Climate policy and the optimal balance between mitigation, adaptation and unavoided damage. Climate Change Economics 1 (2), 71–92.

Bosello, F., Carraro, C., Cian, E. D., 2011. Adaptation can help mitigation: An integrated approach to post-2012 climate policy. Working Papers 2011.69, Fondazione Eni Enrico Mattei.

Bosello, F., Carraro, C., De Cian, E., 2009. An analysis of adaptation as a response to climate change. Working paper, University Ca' Foscari of Venice, Dept. of Economics Research Paper Series No. 26.

Bosello, F., Chen, C., 2010. Adapting and mitigating to climate change: Balancing the choice under uncertainty. Working Papers 159, Fondazione Eni Enrico Mattei.

Brandimarte, P., 2006. Numerical Methods in Finance and Economics: A MATLAB-Based Introduction. John Wiley & Sons.

Chen, Y.-F., Funke, M., Glanemann, N., 2011a. Dark clouds or silver linings? Knightian uncertainty and climate change. Discussion paper, University of Dundee.

Chen, Y.-F., Funke, M., Glanemann, N., 2011b. Time is running out: The 2°C target and optimal climate policies. CESifo Working Paper Series 3664, CESifo Group Munich.

Clarke, L., Edmonds, J., Krey, V., Richels, R., Rose, S., Tavoni, M., 2009. International climate policy architectures: Overview of the {EMF} 22 international scenarios. Energy Economics 31, Supplement 2, S64 – S81.

Cline, W. R., 2011. Carbon Abatement Costs and Climate Change Finance. Peterson Institute for International Economics.

de Bruin, K. C., Dellink, R. B., Tol, R. S., 2009. AD-DICE: An implementation of adaptation in the DICE model. Climatic Change 95, 63–81.

de Zeeuw, A., Zemel, A., 2012. Regime shifts and uncertainty in pollution control. Journal of Economic Dynamics and Control 36 (7), 939 – 950.

Dixit, A. K., Pindyck, R. S., 1994. Investment under Uncertainty. Princeton University Press.

Dobes, L., 2008. Getting real about adapting to climate change: using 'real options' to address the uncertainties. Agenda 15 (3), 55–69.

Dobes, L., 2010. Notes on applying 'real options' to climate change adaptation measures, with examples from vietnam. CCEP Working Papers 0710, Centre for Climate Economics & Policy, Crawford School of Public Policy, The Australian National University.

Felgenhauer, T., Bruin, K. C. D., 2009. The optimal paths of climate change mitigation and adaptation under certainty and uncertainty. International Journal of Global Warming 1, 66–88.

Felgenhauer, T., Webster, M., 2013a. Modeling adaptation as a flow and stock decision with mitigation. Climatic Change.

Felgenhauer, T., Webster, M., 2013b. Multiple adaptation types with mitigation: A framework for policy analysis. Global Environmental Change in press, –.

Fuss, S., Johansson, D. J., Szolgayova, J., Obersteiner, M., 2009. Impact of climate policy uncertainty on the adoption of electricity generating technologies. Energy Policy 37 (2), 733 – 743.

Fuss, S., Szolgayova, J., Golub, A., Obersteiner, M., 2011. Options on low-cost abatement and investment in the energy sector: new perspectives on REDD. Environment and Development Economics 16, 507–525.

Gifford, R., 2011. The dragons of inaction: Psychological barriers that limit climate change mitigation and adaptation. American Psychologist 66 (4), 290–302.

Hertzler, G., 2007. Adapting to climate change and managing climate risks by using real options. Australian Journal of Agricultural Research 58 (10), 985–992.

Hof, A. F., de Bruin, K. C., Dellink, R. B., den Elzen, M. G., van Vuuren, D. P., 2009. The effect of different mitigation strategies on international financing of adaptation. Environmental Science & Policy 12 (7), 832 – 843.

Ingham, A., Ma, J., Ulph, A., 2005. Can adaption and mitigation be complements? Working Paper 79, Tyndall Centre.

IPCC, 2007. Climate Change 2007: Impacts, Adaptation and Vulnerability. Cambridge University Press.

Jensen, S., Traeger, C., 2013. Mitigation under long-term growth uncertainty: Growing emissions but outgrowing its consequences–sure? Working Paper Series 0-65, University of California, Center fro Energy and Environmental Economics.

Kane, S., Shogren, J. F., 2000. Linking adaptation and mitigation in climate change policy. Climatic Change 45, 75–102.

Kolstad, C. D., 1996. Fundamental irreversibilities in stock externalities. Journal of Public Economics 60 (2), 221 – 233.

Lecocq, F., Shalizi, Z., 2007. Balancing expenditures on mitigation of and adaptation to climate change : an exploration of issues relevant to developing countries. Policy Research Working Paper Series 4299, The World Bank.

Lin, T. T., Ko, C.-C., Yeh, H.-N., 2007. Applying real options in investment decisions relating to environmental pollution. Energy Policy 35 (4), 2426 – 2432.

Linquiti, P., Vonortas, N., 2012. The value of flexibility in adapting to climate change: A real options analysis of investments in coastal defense. Climate Change Economics 3 (2), 1250008–1 – 1250008–33.

Maybee, B. M., Packey, D. J., Ripple, R. D., 2012. Climate change policy: The effect of real options valuation on the optimal mitigation-adaptation balance. Economic Papers: A journal of applied economics and policy 31 (2), 216–224.

McDonald, R., Siegel, D., 1986. The value of waiting to invest. The Quarterly Journal of Economics 101 (4), 707–727.

Mendelsohn, R., 2000. Efficient adaptation to climate change. Climatic Change 45 (3-4), 583–600.

Merton, R. C., 1973. Theory of rational option pricing. The Bell Journal of Economics and Management Science 4 (1), pp. 141–183.

Nishide, K., Ohyama, A., 2009. Using real options theory to a countryŠs environmental policy: considering the economic size and growth. Operational Research 9, 229–250.

Nordhaus, W. D., 1994. Managing the global commons : the economics of climate change. MIT Press, Cambridge.

Nordhaus, W. D., 2010. Economic aspects of global warming in a post-Copenhagen environment. Proceedings of the National Academy of Sciences 107 (26), 11721–11726.

Nordvik, V., Lisø, K. R., 2004. A primer on the building economics of climate change. Construction Management and Economics 22 (7), 765–775.

Ohyama, A., Tsujimura, M., 2006. Political measures for strategic environmental policy with external effects. Environmental and Resource Economics 35, 109–135.

Ohyama, A., Tsujimura, M., 2008. Induced effects and technological innovation with strategic environmental policy. European Journal of Operational Research 190 (3), 834 – 854.

Pindyck, R. S., 1988. Irreversible investment, capacity choice, and the value of the firm. American Economic Review 78 (5), 969–85.

Pindyck, R. S., 1993. Investments of uncertain cost. Journal of Financial Economics 34 (1), 53 – 76.

Pindyck, R. S., 2000. Irreversibilities and the timing of environmental policy. Resource and Energy Economics 22 (3), 233–259.

Pindyck, R. S., 2002. Optimal timing problems in environmental economics. Journal of Economic Dynamics and Control 26 (9-10), 1677 – 1697.

Reilly, J., Hohmann, N., Kane, S., 1994. Climate change and agricultural trade: Who benefits, who loses? Global Environmental Change 4 (1), 24 – 36.

Smit, B., Burton, I., Klein, R. J., Wandel, J., 2000. An anatomy of adaptation to climate change and variability. Climatic Change 45, 223–251.

Smith, J., Schellnhuber, H., Monirul Qader Mirza, M., Fankhauser, S., Leemans, R., Erda, L., Ogallo, L., Pittock, B., Richels, R., Rosenzweig, C., Safriel, U., Tol, R., Weyant, J., Yohe, G., 2001. Vulnerability to climate change and reasons for concern: A synthesis. In: McCarthy, J. J., White, K. S., Canziani, O., Leary, N., Dokken, D. J. (Eds.), Climate change 2001: impacts, adaption and vulnerability. Cambridge University Press, Cambridge, pp. 913–970.

Stokey, N. L., 2009. The Economics of Inaction: Stochastic Control Models with Fixed Costs. Princeton University Press.

Strand, J., 2011. Implications of a lowered damage trajectory for mitigation in a continuous-time stochastic model. Policy Research Working Paper Series 5724, The World Bank.

Thomas, J., 1995. Numerical Partial Differential Equations: Finite Difference Methods. Springer, New York.

Thomas, J., 1999. Numerical Partial Differential Equations: Conservation Laws and Elliptic Equations. Springer, New York.

Tol, R. S., 2005a. Emission abatement versus development as strategies to reduce vulnerability to climate change: an application of fund. Environment and Development Economics 10 (05), 615–629.

Tol, R. S., 2005b. The marginal damage costs of carbon dioxide emissions: an assessment of the uncertainties. Energy Policy 33 (16), 2064 – 2074.

Tol, R. S., Fankhauser, S., Smith, J. B., 1998. The scope for adaptation to climate change: what can we learn from the impact literature? Global Environmental Change 8 (2), 109 – 123.

Tsur, Y., Withagen, C., 2013. Preparing for catastrophic climate change. Journal of Economics 110 (3), 225–239.

Ulph, A., Ulph, D., 1997. Global warming, irreversibility and learning. The Economic Journal 107 (442), 636–650.

Watkiss, P., Hunt, A., Blyth, W., 2013. Real options analysis: Decision support methods for adaptation. Briefing note 4, MEDIATION Project.

Wilmott, P., 1998. Derivatives: The Theory and Practice of Financial Engineering (Frontiers in Finance). John Wiley & Sons, New York.

Wirl, F., 2006. Consequences of irreversibilities on optimal intertemporal CO_2 emission policies under uncertainty. Resource and Energy Economics 28 (2), 105–123.