Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

FAKULTÄT
FÜR WIRTSCHAFTS- UND
SOZIALWISSENSCHAFTEN

What Difference Does Long-Horizon Uncertainty Make? Housing Investment Choices in the Presence of Risky Steady State House Prices

Xi Chen

WiSo-HH Working Paper Series
Working Paper No. 31
April 2016

WiSo-HH Working Paper Series
Working Paper No. 31
April 2016

What Difference Does Long-Horizon Uncertainty Make? Housing Investment Choices in the Presence of Risky Steady State House Prices

Xi Chen, Universität Hamburg

ISSN 2196-8128

Font used: „TheSans UHH" / LucasFonts

Die Working Paper Series bieten Forscherinnen und Forschern, die an Projekten in Federführung oder mit der Beteiligung der Fakultät für Wirtschafts- und Sozialwissenschaften der Uni-versität Hamburg tätig sind, die Möglichkeit zur digitalen Publikation ihrer Forschungser-gebnisse. Die Reihe erscheint in unregelmäßiger Reihenfolge.

Jede Nummer erscheint in digitaler Version unter
https://www.wiso.uni-hamburg.de/de/forschung/working-paper-series/

Kontakt:
WiSo-Forschungslabor
Von-Melle-Park 5
20146 Hamburg
E-Mail: experiments@wiso.uni-hamburg.de
Web: http://www.wiso.uni-hamburg.de/forschung/forschungslabor/home/

What Difference Does Long-Horizon Uncertainty Make? Housing Investment Choices in the Presence of Risky Steady State House Prices

Xi Chen

Hamburg University
Department of Economics
xi.chen@wiso.uni-hamburg.de

Hamburg, December 2014

Abstract

This paper analyses, analytically and numerically, the consequences of risky house prices for housing investment within a risky steady state framework. I develop a stylised stochastic model to show that risky steady state house prices have a significant impact on housing investment choice. With increasing risk from aggregate income, the financial market and the housing market, the model predicts that agents tend to invest more in housing and financial assets. Moreover, the numerical analysis investigates the extents to which housing investment will go up when these uncertainties change at different scales. These predictions are shown to hold by using cross-country OECD data.

Acknowledgements

I record my gratitude to my supervisor Michael Funke for his full support throughout my research. I would also like to thank Lavan Mahadeva (Oxford Institute for Energy Studies, University of Oxford) for helpful comments and suggestions on an earlier draft.

Keywords: house prices, housing investment, house price uncertainty, risky steady state
JEL Classification: C61, D81, R31

1. Introduction

The global financial crisis in 2008–2009 led to increasing interest among academic researchers and policymakers alike in the dynamics of house prices. For many consumers, housing is the most important asset in their portfolio and a better understanding is required when making intertemporal investment decisions. I first provide a narrative description of the impact of the global financial crisis upon long-run house price growth. In the forecasting literature, substantially less attention has been dedicated to long-horizon forecasts. One reason for this might be the methodological challenges confronted in forecasting by taking future uncertainty into account.[1] Mueller and Watson (2013) generate a set of state-of-the-art econometric tools for evaluating long-run forecast uncertainty by focusing upon the low-frequency shape of the spectrum of a time series. Instead of predicting the exact point, they construct predictive sets that move towards probability distribution prediction. Unlike housing market models that relate house prices to a set of other variables in a structural modelling framework, they suggest a nonstructural approach. This univariate frequency-domain approach is a simple and effective way to make forecasts when long-run causal relationships are less clear.[2] In light of the challenges of obtaining reliable long-horizon predictions, I employ this technique to understand the risky steady state of real house prices across OECD countries. The probability distributions associated with the 25- and 50-year-horizon predictions are available in Figures 1 and 2, respectively. The probability distributions can be interpreted as a measure of how uncertain long-horizon house price forecasts are. Apparently, long-run risks imply a high degree of uncertainty around the predicted average growth rate of real house prices. The difference between the dashed red and solid blue lines is that the former refers to the sample period 1970Q1–2007Q4, while the latter uses the full sample up to 2013Q4.[3] It is shown that adding the after-crisis data tends to reduce the standard deviations of the long-run growth rate for most countries.[4] At the same time, predicted long-run house price growth rates are uneven. In some countries, such as the US, the UK, Japan and France, expected growth decreases, while in Germany, Finland and Switzerland, increments are clearly presented.

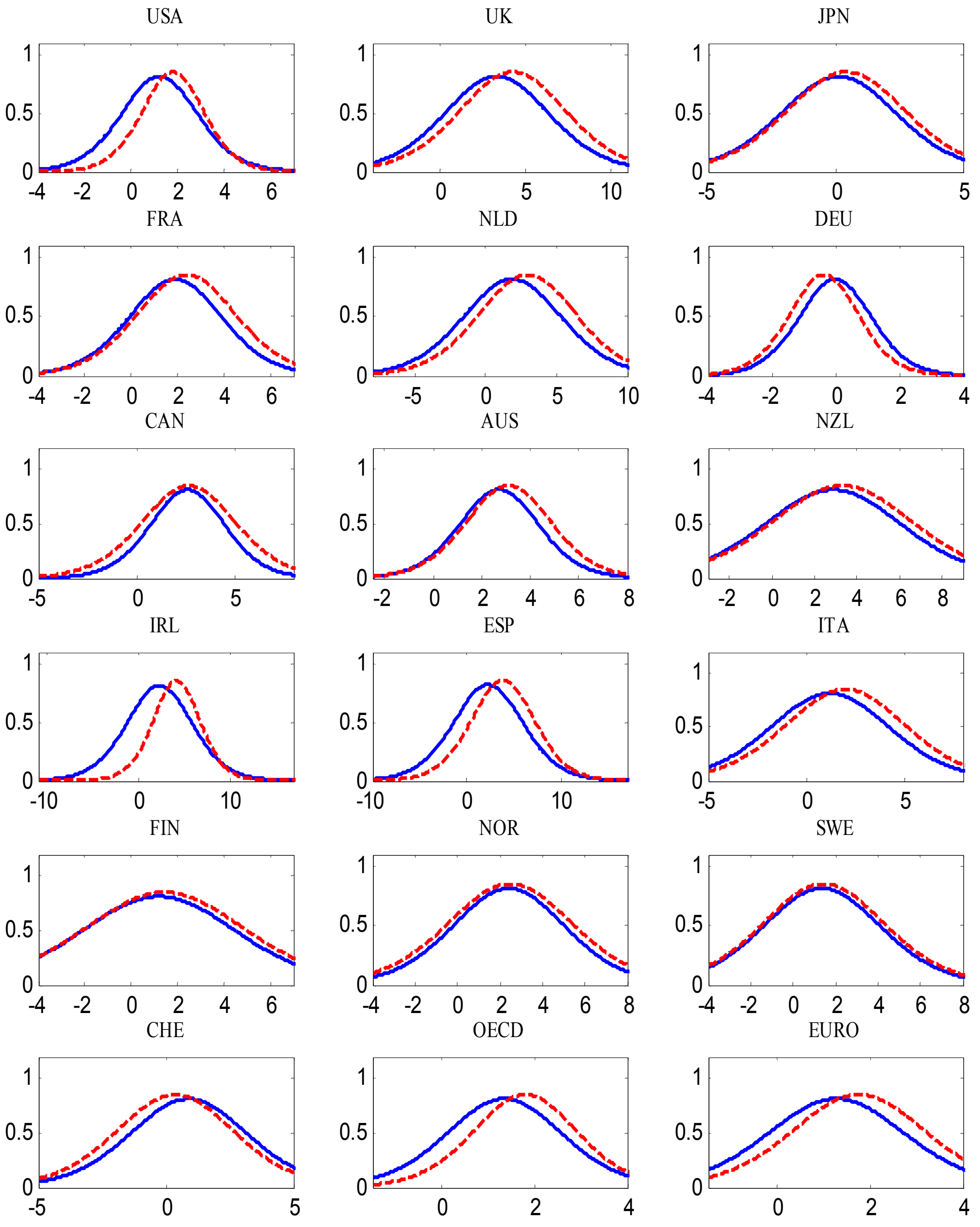
[1] Predictions are usually classified according to the timescale involved in the prediction. Short-term (high frequency) and long-term (low frequency) are the usual categories but the actual meaning of each will vary according to the economic question analysed. In the application below, we choose a forecasting horizon of 25 years and 50 years, respectively.

[2] It goes without saying that the main shortcoming of univariate time series methods is that they are purely statistical, mechanical filters. On the other hand, they only require time series data on real house prices, which makes them very easy to implement for a wide range of countries. For a recent structural modelling approach forecasting the US housing market, see Kouwenberg and Zwinkels (2014).

[3] The seasonally adjusted quarterly house price dataset employed in this paper stems from the Organization for Economic Cooperation and Development (OECD), which is a widely watched multi-country house price database.

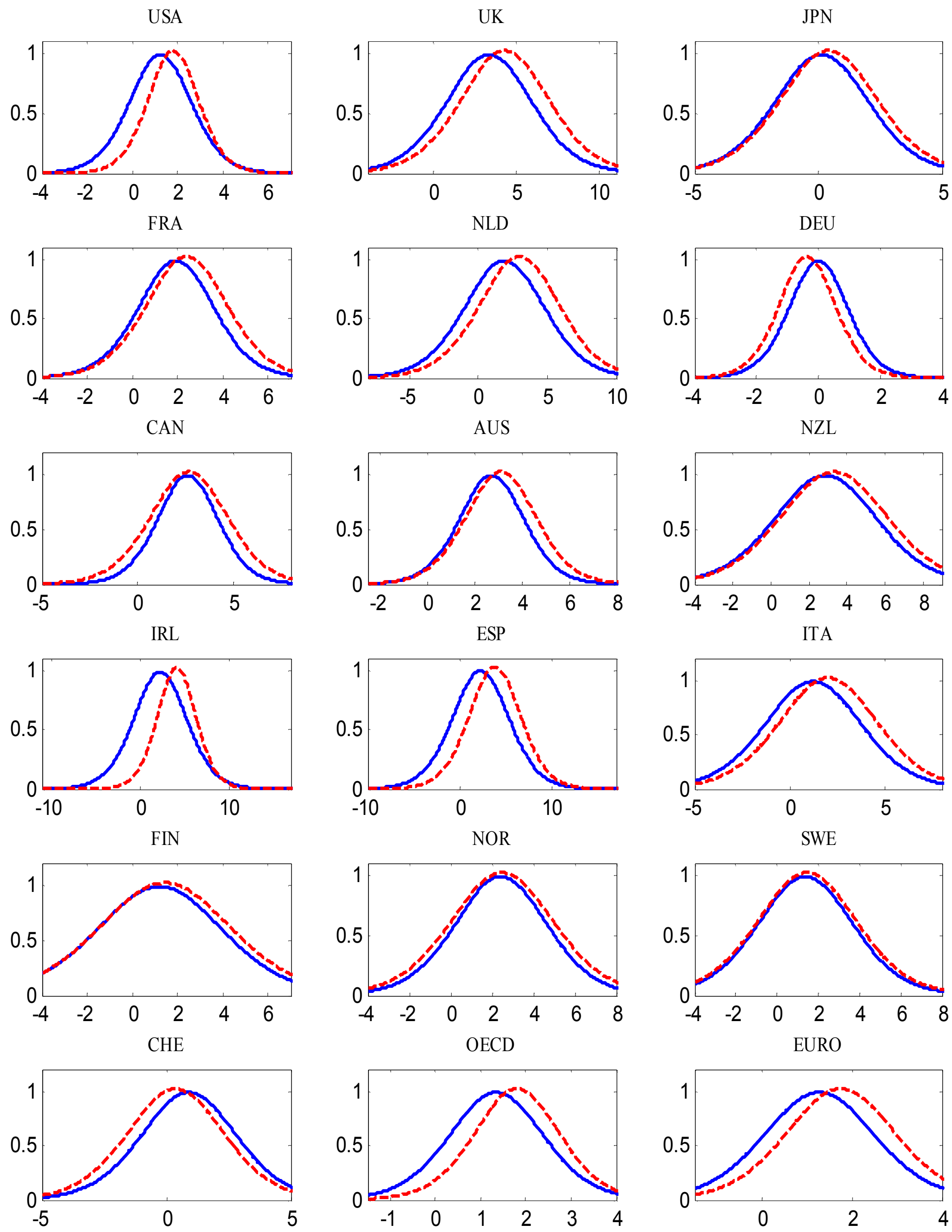
[4] Following Mueller and Watson (2013), the prediction sets are constructed by using the $I(d)$ model with d=0. Precise estimates for d are not readily available. Therefore, we also investigate the robustness and sensitivity of the results obtained by using the flat Bayes prior $d \in [-0.4; 1.4]$ suggested by Mueller and Watson (2013). The results match those in Figures 1 and 2. Supplementary graphs are available upon request.

Figure 1: 25-Year-Ahead Predictive Density of Real House Price Growth across Countries



Note: Solid blue lines are predicted based on the after-crisis HPI 1970Q1–2013Q4 and the dashed red lines are based on pre-crisis HPI 1970Q1–2007Q4.

Figure 2: 50-Year-Ahead Predictive Density of Real House Price Growth across Countries



Note: Solid blue lines are predicted based on the after-crisis HPI 1970Q1–2013Q4 and the dashed red lines are based on pre-crisis HPI 1970Q1–2007Q4.

So far, we have had an impression of the impact of the global financial crisis on long-run real house price growth. In the next section, I describe how to think about such risky long-run house prices from a conceptual standpoint and analyse the implications for the housing market. To do this, I employ the concept of a risky steady state proposed by Coeurdacier et al. (2011, 2012). It is well known that many nonlinear dynamic macroeconomics problems do not have analytical solutions and can only be approximated numerically. One of the most frequently used methodologies is the standard perturbation method. Perturbation methods used to solve dynamic stochastic general equilibrium (DSGE) models are similar in spirit to Taylor series approximations.[5] Based on function derivatives, approximations are taken around a specific point or value of the parameter domain in order to approximate the function's corresponding value when these values are perturbed away by small degrees from that point around which the approximation is taken. This solution method lends itself very well to the approximation of the exact solution of the nonlinear systems of DSGE models for two reasons. First and foremost, the assumption that shocks (perturbations) are not too large in a real world economic system is a reasonable one most of the time. Second, while the general discussion on perturbation methods does not prescribe a preferred point around which the approximation should be taken, dynamic economic systems exhibiting a steady state provide a reasonable value around which the approximation should be taken. In the most recent literature, this point has been given additional attention in that researchers typically consider two possible candidate steady state equilibrium points around which to form the approximation. One of them is the traditional deterministic steady state to which the economic system gravitates when future shocks are assumed to be zero, while the other steady state, suitably called the risky or stochastic steady state, is the one where the system comes to rest, when agents know that future shocks will continue to occur based on the certain known distributions of those shocks. [6] In this paper, I want to draw attention to the second-order approximation of the equilibrium conditions to solve the stochastic steady state in a portfolio problem.

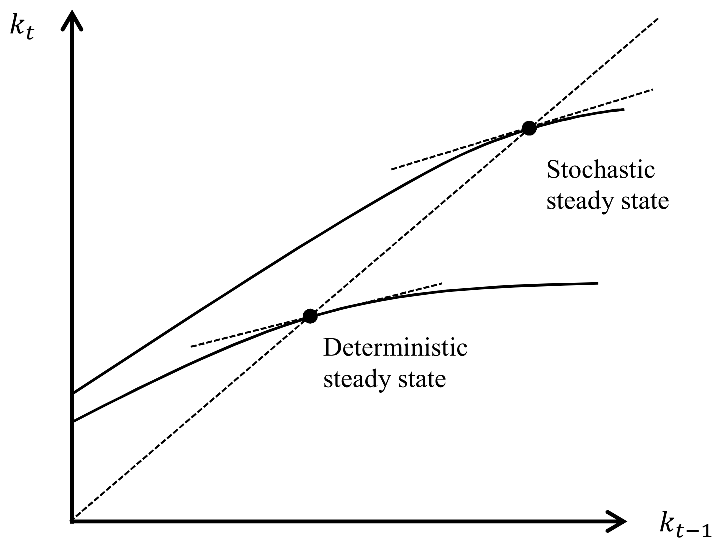
In fact, the risky steady state was first introduced by Juillard and Kamenik (2005). They argue that when perturbation methods are applied to stochastic general equilibrium models, because of nonlinearities, the centre of the ergodic distribution of the endogenous variables can be away from the deterministic steady state, making it not the best point around which to take the approximation. Coeurdacier et al. (2011, 2012) believe that risk-averse agents are aware of the existence of future shocks hitting the economy. Therefore, they anticipate the convergence of economic variables to some stochastic steady state, which incorporates information about expected future risk and the corresponding optimal decisions. Borrowing the example of a standard stochastic growth model presented by Coeurdacier et al. (2011, 2012), this concept is well illustrated in Figure 3. Anticipated uncertainty

[5] Standard perturbation relies on implicit function theorems, Taylor series expansions and techniques from bifurcation and singularity theory. Judd (1998) explains explicitly how to use these techniques to approximate the policy functions of dynamically stable stochastic control models near the steady state of their deterministic counterparts.

[6] De Groot (2013) extends this to general settings as a matrix quadratic problem.

leads to precautionary capital accumulation, which raises the level of the stock of capital in the stochastic steady state than that in the deterministic steady state. Specifically, the stochastic steady state is defined as the point where agents decide to stay in the absence of shocks, but taking into account the likelihood of future shocks. By contrast, the deterministic steady state is the point where agents decide to stay in the absence of shocks and ignoring future shocks. Unfortunately, the risky steady state cannot be found analytically for most DSGE models. In addition, there are only a few numerical methods available in previous studies. Juillard (2011) proposes a numerical algorithm to find the risky steady state, which is truly mathematically challenging because of the nonlinearity.[7] Alternatively, Coeurdacier et al. (2011, 2012) suggest another feasible strategy, which consists of postulating a linear decision rule for control variables around unknown risky steady states and their identifications along with the coefficients simultaneously.

Figure 3: Deterministic vs. Stochastic Steady State: Decision Rules for Capital Accumulation



The main purpose of this paper is to provide a risky steady state framework to demonstrate how long-horizon house price uncertainty effects housing investment choice. The model is designed to highlight the role of housing investment choice in the presence of stochastic labour income, a risky interest rate and risky house prices. Setting aside general equilibrium considerations, these risk sources will be treated as exogenous random processes, since in a risky steady state framework the mechanism of the calculation strategy can be more clearly demonstrated throughout this paper. According to my model,

[7] Based on this algorithm, Julliard (2011) finds that the approximation of the solution appears as quite different depending whether the approximation taken around the deterministic steady state or risky steady state. He uses the simple asset pricing model for which there exist a closed form solution to compare the accuracy of the approximation around the deterministic steady state with the one around the risky steady state.

the precautionary saving effect, the risk premium elicited by interest rate risk and house price risk and the crowding out effect are well reflected in the approximation equilibrium function. This implies that riskier countries will tend to have larger investment in housing and accumulate more financial assets than safer ones. In addition, I provide a numerical analysis of the theoretical model to show the impacts of a country's risk level on an agent's consumption and investment decision rules. Finally, I create proxy variables to denote the housing investment level and risk level of a country from three different perspectives, with which I provide evidence of positive relationships between them across 13 OECD countries.

The remainder of this paper is divided into five sections. Section 2 describes the details of the extended model and derives our solution system, with which I compute the risky steady states and postulated coefficients of the decision rule endogenously. Section 3 discusses the results of the numerical experiments. The empirical evidence is presented in section 4. In the final section, I summarise my conclusions.

2. Modelling Framework

To illustrate the idea, I present an extended model along the lines of the work by Coeurdacier et al. (2011, 2012) and I use their notation for convenience. A representative household lives forever and has preferences over current and future consumption and housing. In each period t, the individual needs to choose the amounts of non-housing consumption good c_t and housing service h_t. The price per unit of housing at time t is denoted by p_t, while the price of c_t is fixed and normalised to one. The representative individual's intertemporal utility is

$$E_t[\sum_{t=0}^{\infty} \beta^t u(c_t, h_t)], \tag{1}$$

where $E_t[\cdot]$ is the expectation operator based on the information available in period t, and β is the time discount factor for the future utility stream $u(c_t, h_t)$. The individual starts in period $t \geq 1$ with net worth NW_t given by

$$NW_t = HW_t + NHW_t, \tag{2}$$

where HW_t and NHW_t represent housing wealth and non-housing wealth, respectively.[8] Given the housing price in period t and housing investment from the previous period, the corresponding housing wealth in period t is then given as $p_t h_{t-1}$ assuming only one financial asset ω_{t-1} is involved in the

[8] Here, I ignore the maintenance cost of housing for simplicity.

non-housing investment from the last period at the risky interest rate r_t.[9] Therefore, the individual's net worth at the beginning of period t can be rewritten as

$$NW_t = p_t h_{t-1} + r_t \omega_{t-1}. \tag{3}$$

The resultant resources available for consumption and investment in period t, Q_t, are then defined as the sum of labour income plus net worth

$$Q_t = y_t + p_t h_{t-1} + r_t \omega_{t-1}. \tag{4}$$

Finally, in any period the individual faces the budget constraint

$$c_t + p_t h_t + \omega_t = y_t + p_t h_{t-1} + r_t \omega_{t-1}. \tag{5}$$

The meaning of the intertemporal budget constraint is straightforward. The right-hand side, again, represents the total resources of the household in period t. Ignoring the portfolio adjustment costs, these resources can be used for consumption, housing wealth and non-housing wealth accumulation, which is given by the left-hand side of the equation.[10] Putting general equilibrium considerations aside, I assume y_t, r_t and p_t are exogenous variables and are lognormally distributed stochastic AR(1) processes

$$ln(y_t) = \left(1-\rho_y\right)\bar{l}_y + \rho_y\, ln(y_{t-1}) + u_{y,t+1}, \tag{6}$$

$$ln(r_t) = (1-\rho_r)\bar{l}_r + \rho_r\, ln(r_{t-1}) + u_{r,t+1}, \tag{7}$$

$$ln(p_t) = \left(1-\rho_p\right)\bar{l}_p + \rho_p\, ln(p_{t-1}) + u_{p,t+1}, \tag{8}$$

where $\bar{l}_y$, $\bar{l}_r$ and $\bar{l}_p$ are the means of $ln(y_t)$, $ln(r_t)$ and $ln(p_t)$, $|\rho_y| < 1$, $|\rho_r| < 1$, and $|\rho_p| < 1$ are the AR(1) coefficients and the mean-zero random terms are defined as $u_{y,t} \sim N(0,\ \sigma^2_{u,y})$, $u_{r,t} \sim N(0,\ \sigma^2_{u,r})$ and $u_{p,t} \sim N(0,\ \sigma^2_{u,p})$ for $t \geq 1$. The correlations between all three variables are set to zero for simplicity. Given equations (1)–(5), the agent's problem is to maximise his or her discounted expected utility subject to the intertemporal budget constraint, given his or her initial values of asset holdings.

[9] One obvious omission from my analysis is that non-housing assets are not taken into account. Endogenous labour supply, the non-separability of c_t and h_t and bequest considerations are also ignored here.

[10] The difference $p_t(h_t - h_{t-1})$ in the budget constraint implies that I allow households to move up or down the housing ladder. For an empirical study of housing mobility and downsizing in older age in the US and the UK, see Banks et al. (2012).

Rearranging equation (1) gives us the value function of the individual's intertemporal consumption and investment problem

$$V_t(X_t) = \max_{Y_t}\{u(c_t, h_t) + \beta E_t V_{t+1}(X_{t+1})\},$$

where $X_t = (t,\ y_t,\ r_t,\ p_t, \omega_{t-1}, h_{t-1}\ ),$

$$Y_t = (c_t, h_t) \qquad \text{for} \qquad t \geq 0. \tag{9}$$

State vector X_t consists of the investor's labour income, the return on risky assets, the price per unit of housing services, the amount of existing risky non-housing assets and the size of existing housing. The first-order conditions of the value function with respect to c_t and h_t are given by the following three equations

$$\frac{\partial u/\partial c_t}{\partial u/\partial h_t} = \frac{1}{p_t}, \tag{10}$$

$$1 = \beta E_t[\frac{\partial u/\partial h_{t+1}}{\partial u/\partial h_t}\ r_{t+1}\ \frac{p_t}{p_{t+1}}], \tag{11}$$

$$1 = \beta E_t[\frac{\partial u/\partial c_{t+1}}{\partial u/\partial c_t}\ r_{t+1}]. \tag{12}$$

The interpretation of equations (10)–(12) is as follows. Optimal behaviour requires equating the marginal utilities of housing services in period t and the expected discounted marginal utilities of housing services in period $t+1$ with the relative housing prices in t and $t+1$. Notice that house price in $t+1$ is also discounted by the risky interest rate r_{t+1}. For ease of exposition and without loss of generality, the individual's preferences over the non-housing consumption good and housing services are parameterised as

$$u(c_t, h_t) = \frac{({c_t}^{1-\alpha}\ {h_t}^{\alpha})^{1-\gamma}}{1-\gamma}, \tag{13}$$

where α measures the relative importance of housing services versus non-housing and γ is the coefficient of relative risk aversion. By inserting equation (13) into (10)–(12), we have

$$h_t = M(\frac{{c_t}^{\alpha}}{p_t})^{\frac{1}{1-\alpha}}, \tag{14}$$

$$1 = \beta E_t[\ r_{t+1}\,(\frac{c_{t+1}}{c_t})^{-\gamma(1-\alpha)}(\frac{h_{t+1}}{h_t})^{-\gamma\alpha+\alpha-1}\ \frac{p_t}{p_{t+1}}], \tag{15}$$

$$1 = \beta E_t[\ r_{t+1}\,(\frac{c_{t+1}}{c_t})^{-\gamma(1-\alpha)-\alpha}(\frac{h_{t+1}}{h_t})^{-\gamma\alpha}\ ], \tag{16}$$

where

$$M = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\alpha}}. \tag{17}$$

Essentially, equations (15) and (16) are equivalent if I insert (14) into both of them. Therefore, one could focus on one of these two equations, say, equation (16) for further analysis. I define

$$g(c_{t+1}, c_t, r_{t+1}, h_{t+1}, h_t) \equiv \beta\, r_{t+1} \left(\frac{c_{t+1}}{c_t}\right)^{\Sigma} \left(\frac{h_{t+1}}{h_t}\right)^{\Pi} - 1,$$

where

$$\begin{aligned} \Sigma &= -\gamma(1-\alpha) - \alpha, \\ \Pi &= -\alpha\gamma. \end{aligned} \tag{18}$$

Then, equation (16) could be rewritten as

$$E_t\left[g(c_{t+1}, c_t, r_{t+1}, h_{t+1}, h_t)\right] = 0. \tag{19}$$

In order to take risk into account, equation (19) is replaced by its second-order Taylor expansion Ψ around the expected future variables

$$\begin{aligned} 0 &= E_t\left[g(c_{t+1}, c_t, r_{t+1}, h_{t+1}, h_t)\right] \\ &\approx \Psi[E_t\,(c_{t+1}), E_t\,(r_{t+1}), E_t\,(h_{t+1}), c_t, h_t], \end{aligned} \tag{20}$$

where

$$\begin{aligned} \Psi = {} & \beta E_t(r_{t+1}) \left(\frac{E_t(c_{t+1})}{c_t}\right)^{\Sigma} \left(\frac{E_t\,(h_{t+1})}{h_t}\right)^{\Pi} - 1 \\ & + \frac{\beta(\Sigma-1)\Sigma}{2} Var_t(c_{t+1}) \left(\frac{E_t(c_{t+1})}{c_t}\right)^{\Sigma} \left(\frac{E_t\,(h_{t+1})}{h_t}\right)^{\Pi} \frac{E_t(r_{t+1})}{E_t(c_{t+1})^2} \\ & + \frac{\beta(\Pi+1)\Pi}{2} Var_t(h_{t+1}) \left(\frac{E_t(c_{t+1})}{c_t}\right)^{\Sigma} \left(\frac{E_t\,(h_{t+1})}{h_t}\right)^{\Pi} \frac{E_t(r_{t+1})}{E_t(h_{t+1})^2} \\ & + \beta\Sigma\Pi Cov_t(c_{t+1}, h_{t+1}) \left(\frac{E_t(c_{t+1})}{c_t}\right)^{\Sigma} \left(\frac{E_t\,(h_{t+1})}{h_t}\right)^{\Pi} \frac{E_t(r_{t+1})}{E_t(c_{t+1})E_t(h_{t+1})} \\ & + \beta\Sigma Cov_t(c_{t+1}, r_{t+1}) \left(\frac{E_t(c_{t+1})}{c_t}\right)^{\Sigma} \left(\frac{E_t\,(h_{t+1})}{h_t}\right)^{\Pi} \frac{1}{E_t(c_{t+1})} \\ & + \beta\Pi Cov_t(h_{t+1}, r_{t+1}) \left(\frac{E_t(c_{t+1})}{c_t}\right)^{\Sigma} \left(\frac{E_t\,(h_{t+1})}{h_t}\right)^{\Pi} \frac{1}{E_t(h_{t+1})}. \end{aligned} \tag{21}$$

Multiplying equation (20) by the non-zero term $\beta^{-1} E_t(r_{t+1})^{-1} \left(\frac{E_t(c_{t+1})}{c_t}\right)^{-\Sigma} \left(\frac{E_t\,(h_{t+1})}{h_t}\right)^{-\Pi}$ and simplifying further allows me to obtain

$$\frac{1}{\beta}\left(\frac{E_t(c_{t+1})}{c_t}\right)^{-\Sigma} \left(\frac{E_t\,(h_{t+1})}{h_t}\right)^{-\Pi}$$

$$= E_t(r_{t+1})[1 + \frac{(\Sigma-1)\Sigma}{2}\frac{Var_t(c_{t+1})}{E_t(c_{t+1})^2} + \frac{(\Pi+1)\Pi}{2}\frac{Var_t(h_{t+1})}{E_t(h_{t+1})^2} + \Sigma\Pi\frac{Cov_t(c_{t+1},h_{t+1})}{E_t(c_{t+1})E_t(h_{t+1})}]$$
$$+\Sigma\frac{Cov_t(r_{t+1},c_{t+1})}{E_t(c_{t+1})} + \Pi\frac{Cov_t(r_{t+1},h_{t+1})}{E_t(h_{t+1})}. \quad (22)$$

I assume that agents are aware of future shocks hitting the economy and anticipating the economic variables converging to some stochastic steady states, i.e. the ergodic distribution of these variables. Instead of the deterministic steady state (X^*, Y^*), from now on I consider the local behaviour of an economy around the risky steady state $(\bar{X}, \bar{Y})$.[11] Therefore, at the risky steady state, this approximation becomes

$$\frac{1}{\beta} = \bar{r}\left[1 + \frac{(\Sigma-1)\Sigma}{2}\frac{\overline{Var_t}(c_{t+1})}{\bar{c}^2} + \frac{(\Pi+1)\Pi}{2}\frac{\overline{Var_t}(h_{t+1})}{\bar{h}^2} + \Sigma\Pi\frac{\overline{Cov_t}(c_{t+1},h_{t+1})}{\bar{c}\bar{h}}\right]$$
$$+\Sigma\frac{\overline{Cov_t}(r_{t+1},c_{t+1})}{\bar{c}} + \Pi\frac{\overline{Cov_t}(r_{t+1},h_{t+1})}{\bar{h}}, \quad (23)$$

where $\overline{Var_t}(\cdot)$ and $\overline{Cov_t}(\cdot)$ denote the second-order moments evaluated at the risky steady state. In period t, variance and covariance are evaluated at the risky steady state according to $\overline{Var_t}(c_{t+1}) = \overline{Cov_t}(c_{t+1}, c_{t+1}|c_t = \bar{c})$ and $\overline{Cov_t}(c_{t+1}, r_{t+1}) = \overline{Cov_t}(c_{t+1}, r_{t+1}|c_t = \bar{c}, r_t = \bar{r})$. The same occurs for $\overline{Var_t}(h_{t+1})$, $\overline{Cov_t}(c_{t+1}, h_{t+1})$ and $\overline{Cov_t}(h_{t+1}, r_{t+1})$. The pattern can now be inferred. Compared with Coeurdacier et al.'s (2011, 2012) results, I find common features and differences. In the absence of risk, the return on financial investment must be equal to the inverse of time preference, which is given by $\bar{r} = \frac{1}{\beta}$. The second term $\frac{(\Sigma-1)\Sigma}{2}\overline{Var_t}(c_{t+1})/\bar{c}^2$ in square brackets is the so-called precautionary saving term. Coeurdacier et al. (2011, 2012) point out that if uncertainty over future consumption increases, risk-averse agents will sacrifice consumption in the current period to ensure future consumption at the desired higher level. They also state that when financial assets are risky, an additional stabilising force on the consumption path is at work. This is reflected in the first term out of square brackets $\Sigma\overline{Cov_t}(r_{t+1}, c_{t+1})/\bar{c}$, i.e., the risk premium term elicited from the financial market. My extended model generates an extra three terms at the equilibrium: $\frac{(\Pi+1)\Pi}{2}\overline{Var_t}(h_{t+1})/\bar{h}^2$, $\Sigma\Pi\overline{Cov_t}(c_{t+1}, h_{t+1})/\bar{c}\bar{h}$ and $\Pi\overline{Cov_t}(r_{t+1}, h_{t+1})/\bar{h}$. The first one is similar to the precautionary effect. It implies that when future uncertainty in the housing market grows, clients anticipate the convergence of housing investment to a higher level in the long run. My second term comes from the risk premium associated with risky house prices. This stresses the fact that in the housing market, house price risk serves as the third stabilising force other than precautionary savings and the risky interest rate, since the covariance between consumption and housing reduces the persistence of shocks in the housing market. When the economy reaches the risky steady state, countries with higher house price risk tend to invest

[11] For the rest of the paper, Z^* and $\bar{Z}$ denote the deterministic and risky steady states for any variable Z, respectively.

more in housing than safer ones. The last term in my equilibrium is similar to the crowding out effect highlighted by Cocco (2005). Unlike other risky assets, house price risk can hardly be avoided for most households, since everyone wants to purchase a home eventually. However, with limited resources, participation in the financial market is crowded out by risky house prices. Investment in housing assets increases alone with the covariance between housing investment and financial asset return at the risky steady state, as shown in equation (23). Note that the extended model renders an explicit analytical solution impossible. Therefore, I employ numerical techniques in the next section. Before turning to the numerical analysis, I need to rewrite the second-order expansion by defining

$$f(c_{t+1}, c_t, r_{t+1}, p_{t+1}, p_t) \equiv \beta\, r_{t+1} \left(\frac{c_{t+1}}{c_t}\right)^{\Delta} \left(\frac{p_t}{p_{t+1}}\right)^{\Lambda} - 1,$$

where

$$\Delta = -\left[\frac{\gamma}{1-\alpha}(\alpha^2 + (1-\alpha)^2) + \alpha\right],$$
$$\Lambda = -\frac{\alpha\gamma}{1-\alpha}. \quad (24)$$

Collecting the terms, another form of the approximation of our equilibrium around the risky steady state could be given as follows[12]

$$\frac{1}{\beta}\left(\frac{E_t(c_{t+1})}{c_t}\right)^{-\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{-\Lambda}$$
$$= E_t(r_{t+1})[1 + \frac{(\Delta-1)\Delta}{2}\frac{Var_t(c_{t+1})}{E_t(c_{t+1})^2} + \frac{(\Lambda+1)\Lambda}{2}\frac{Var_t(p_{t+1})}{E_t(p_{t+1})^2}$$
$$-\Delta\Lambda\frac{Cov_t(c_{t+1},p_{t+1})}{E_t(c_{t+1})E_t(p_{t+1})}] + \Delta\frac{Cov_t(c_{t+1},r_{t+1})}{E_t(c_{t+1})}. \quad (25)$$

In fact, this is equivalent to equation (22), which provides information between future consumption and housing investment. However, equation (25) contains only one non-predetermined variable, consumption c_t, which makes our numerical exercise easier, since both interest rate r_t and housing price p_t are assumed to be exogenous stochastic processes. Following Coeurdacier et al.'s (2011, 2012) strategy, we also postulate a linear decision rule for ω

$$\omega_t = \bar{\omega} + G_{\omega\omega}(\omega_{t-1} - \bar{\omega}) + G_{\omega r}(r_t - \bar{r}) + G_{\omega y}(y_t - \bar{y}) + G_{\omega p}(p_t - \bar{p}), \quad (26)$$

where $\bar{\omega}$ is the unknown risky steady state value for financial assets and $G_{\omega\omega}$, $G_{\omega r}$, $G_{\omega y}$ and $G_{\omega p}$ are the four coefficients needed to be calculated endogenously. By inserting equations (14) and (26) into budget constraint (5) and then linearising it, one obtains the approximations of conditional expectation and variance of consumption

[12] The calculation details are given in Appendixes A and B.

$$E_t(c_{t+1}) = K_1\bar{\omega}(G_{\omega\omega} - 1) + K_1 G_{\omega r}\bar{r} + K_1 G_{\omega y}\bar{y} + K_1 G_{\omega p}\bar{p} + K_1(1 - G_{\omega y})E_t(y_{t+1})$$
$$+K_1(\omega_t - G_{\omega r})E_t(r_{t+1}) - K_1 G_{\omega\omega}\omega_t + \left[K_1(h_t - G_{\omega p}) + K_2\right]E_t(p_{t+1}) + K_3, \quad (27)$$

$$Var_t(c_{t+1}) = {K_1}^2(1 - G_{\omega y})^2 Var_t(y_{t+1}) + {K_1}^2(\omega_t - G_{\omega r})^2 Var_t(r_{t+1})$$
$$+\left[K_1(h_t - G_{\omega p}) + K_2\right]^2 Var_t(p_{t+1}),$$

where

$$\Gamma = \frac{\alpha}{1-\alpha},\ K_1 = \frac{1}{1+\Gamma},\ K_2 = \frac{\Gamma}{1+\Gamma},\ K_3 = \frac{1-lnM}{1+\Gamma}. \quad (28)$$

At the same time, I also know the conditional covariance of consumption and the interest rate and housing price

$$Cov_t(c_{t+1}, r_{t+1}) = K_1(\omega_t - G_{\omega r})Var_t(r_{t+1}), \quad (29)$$
$$Cov_t(c_{t+1}, p_{t+1}) = [K_1(h_t - G_{\omega p}) + K_2]Var_t(p_{t+1}). \quad (30)$$

I assume y_t, r_t and p_t to be three exogenous variables and autocorrelated lognormally distributed stochastic processes. Therefore, their risky steady states are calculated as follows[13]

$$\bar{y} = e^{\bar{l}_y + \frac{1}{2}\frac{\sigma^2_{u,y}}{1-\rho_y{}^2}}, \quad (31)$$
$$\bar{r} = e^{\bar{l}_r + \frac{1}{2}\frac{\sigma^2_{u,r}}{1-\rho_r{}^2}}, \quad (32)$$
$$\bar{p} = e^{\bar{l}_p + \frac{1}{2}\frac{\sigma^2_{u,p}}{1-\rho_p{}^2}}, \quad (33)$$

and their corresponding conditional expectations are

$$E_t(y_{t+1}) = e^{(1-\rho_y)\bar{l}_y + \rho_y ln y_t + \frac{\sigma^2_{u,y}}{2}}, \quad (34)$$
$$E_t(r_{t+1}) = e^{(1-\rho_r)\bar{l}_r + \rho_r ln r_t + \frac{\sigma^2_{u,r}}{2}}, \quad (35)$$
$$E_t(p_{t+1}) = e^{(1-\rho_p)\bar{l}_p + \rho_p ln p_t + \frac{\sigma^2_{u,p}}{2}}. \quad (36)$$

Compared with conditional expectations, it is apparent that in the case of the risky steady state, the response to positive and negative shocks is stronger. In addition, their conditional variances are given as

[13] The risky steady states of y_t, p_t and r_t are the unconditional expectations of the associated ergodic distribution. Following the assumption, they are lognormal-distributed processes, i.e.,$y_t \sim lnN(\bar{l}_y, \frac{\sigma^2_{u,y}}{1-\rho_y{}^2})$, $r_t \sim lnN(\bar{l}_r, \frac{\sigma^2_{u,r}}{1-\rho_r{}^2})$ and $p_t \sim lnN(\bar{l}_p, \frac{\sigma^2_{u,p}}{1-\rho_p{}^2})$. Therefore, $E(y_t) = e^{\bar{l}_y + \frac{1}{2}\frac{\sigma^2_{u,y}}{1-\rho_y{}^2}}$, $E(r_t) = e^{\bar{l}_r + \frac{1}{2}\frac{\sigma^2_{u,r}}{1-\rho_r{}^2}}$ and $E(p_t) = e^{\bar{l}_p + \frac{1}{2}\frac{\sigma^2_{u,p}}{1-\rho_p{}^2}}$.

$$Var_t(y_{t+1}) = e^{2(1-\rho_y)\bar{l}_y+2\rho_y lny_t+ \sigma^2_{u,y}}(e^{\ \sigma^2_{u,y}} - 1), \tag{37}$$

$$Var_t(r_{t+1}) = e^{2(1-\rho_r)\bar{l}_r+2\rho_r lnr_t+ \sigma^2_{u,r}}(e^{\ \sigma^2_{u,r}} - 1), \tag{38}$$

$$Var_t(p_{t+1}) = e^{2(1-\rho_p)\bar{l}_p+2\rho_p lnp_t+ \sigma^2_{u,p}}\left(e^{\ \sigma^2_{u,p}} - 1\right). \tag{39}$$

Finally, I have the following local conditions to identify the risky steady states and coefficients assumed in equation (26)

$$\hat{\Phi}(\bar{S}) = 0, \tag{40}$$

$$\frac{\partial\hat{\Phi}}{\partial S_t}|_{S_t=\bar{S}} = 0, \tag{41}$$

where

$$S_t = (X_t, Y_t), \ \ \bar{S} = (\bar{X}, \bar{Y}).$$

In my model, the system can be written as a five-dimension equation system.[14] The solutions of this system are the values of $\bar{\omega}, G_{\omega\omega}, G_{\omega y}, G_{\omega r}\ and\ G_{\omega p}$, with which we are able to calculate $\bar{c}$ and $\bar{h}$ as follows

$$\bar{c} = \bar{y} + \bar{\omega}(\bar{r} - 1), \tag{42}$$

$$\bar{h} = M(\frac{\bar{c}^{\alpha}}{\bar{p}})^{\frac{1}{1-\alpha}} \text{ with } M = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\alpha}}. \tag{43}$$

3. Numerical Analysis

In this section, I provide a numerical analysis of the theoretical model using MATLAB. By changing the standard deviations of labour income, the risky interest rate and house prices, the discussion focuses on the variations of consumption and investment at the risky steady states. Specifically, I design three experiments to analyse the decision rules given the different levels of aggregate income risk, financial market risk and housing market risk. The purpose of first experiment is to replicate part of Coeurdacier et al.'s (2011, 2012) earlier work and extend the numerical analysis to test whether their conclusions are still robust in the presence of a risky housing market. Moreover, by changing the standard deviation of the risky interest rate and house prices, the last two experiments aim to demonstrate the risky housing-related effects, as presented in equation (23).

To do this, I begin by discussing a baseline setup of the parameters in my model. Instead of estimating key parameters from the original data myself, I rely upon the US data-based estimations already widely used by most researchers. The time discount parameter is standard; I set it at β=0.96, which follows Cocco et al. (2005). The curvature parameter γ is equal to 3, below the upper bound of 10 considered to

[14] See Appendix C for the calculation details.

be plausible by Mehra and Prescott (1985). Parameter α measures the relative importance of housing services versus non-housing, and it is fixed at α=0.2, which is approximatively equal to the average proportion of household housing expenditure in the Consumer Expenditure Survey 2001 suggested by Yao and Zhao (2005) (see also U.S. Department of Labor (2003)). Similar to Cocco (2005), I set the standard deviation of labour income σ_y at 0.02 in the second and third experiments. As such, the test scale of σ_y in my first experiment will include this value as well and this is fixed between 0 and 0.1. In order to make my results comparable with those of Coeurdacier et al. (2011, 2012), I follow their suggestion and fix the mean of labour income $\bar{y}$ at 0.8 and AR(1) coefficient ρ_y at 0.9. Recall that in the absence of risk, the usual Euler equation implies $\bar{r} = \frac{1}{\beta}$, but under the consideration of the risky steady state, the value of $\bar{r}$ is no longer equal to the inverse of the time discount parameter and it varies away from $\frac{1}{\beta}$. Thus, I set $\bar{r}$ at 1.027 similar to Coeurdacier et al. (2011, 2012). In my specification of the house price process, I follow Nagaraja et al. (2011), who provide a sophisticated autoregressive approach to predict the parameter of risky house prices. Their estimations for the standard deviation of house prices and the AR(1) coefficient are 0.07[15] and 0.9, respectively. The values of the baseline parameters and test ranges in each experiment are given in Table 1.

Table 1: Baseline Parameters in the Model

Parameter	Value		
Time discount parameter β	0.96		
Curvature parameter γ	3.0		
Preference for housing α	0.2		
Risky steady state $\bar{y}$	0.8		
Risky steady state $\bar{r}$	1.027		
Risky steady state $\bar{p}$	0.8		
AR (1) coefficient ρ_y	0.9		
AR (1) coefficient ρ_r	0.9		
AR (1) coefficient ρ_p	0.9		
	Experiment 1: Aggregate Income	Experiment 2: Financial Market	Experiment 3: Housing Market
Sd.of labour income σ_y	(0,0.1)	0.02	0.02
Sd.of risky interest rate σ_r	0.02	(0.01,0.1)	0.02
Sd.of house price σ_p	0.07	0.07	(0,0.1)

[15] This value is also close to Cocco's (2005) prediction, which is 0.062.

In Figure 4, the results of experiments 1–3 are presented in the three panels. By allowing the standard deviation of stochastic income σ_y to change between 0 and 0.1, the first panel shows the impact of labour income risk on consumption and investment decisions at the risky steady state. The overall qualitative feature of the variations is consistent with Coeurdacier et al.'s (2011, 2012) results: consumption, financial assets and housing investment all increase monotonously alone with the risk level of labour income at the risky steady state. A rise in consumption is not a surprising outcome, since the precautionary effect is at work. People living in riskier countries usually anticipate bigger shocks hitting the economy. Typically, risk-averse agents will save more resources in exchange for a stable life in the long run. Therefore, higher future uncertainty caused either by income risk or by financial market/housing market risk induces higher consumption at the risky steady state. These patterns can be observed in the first graph of each panel.

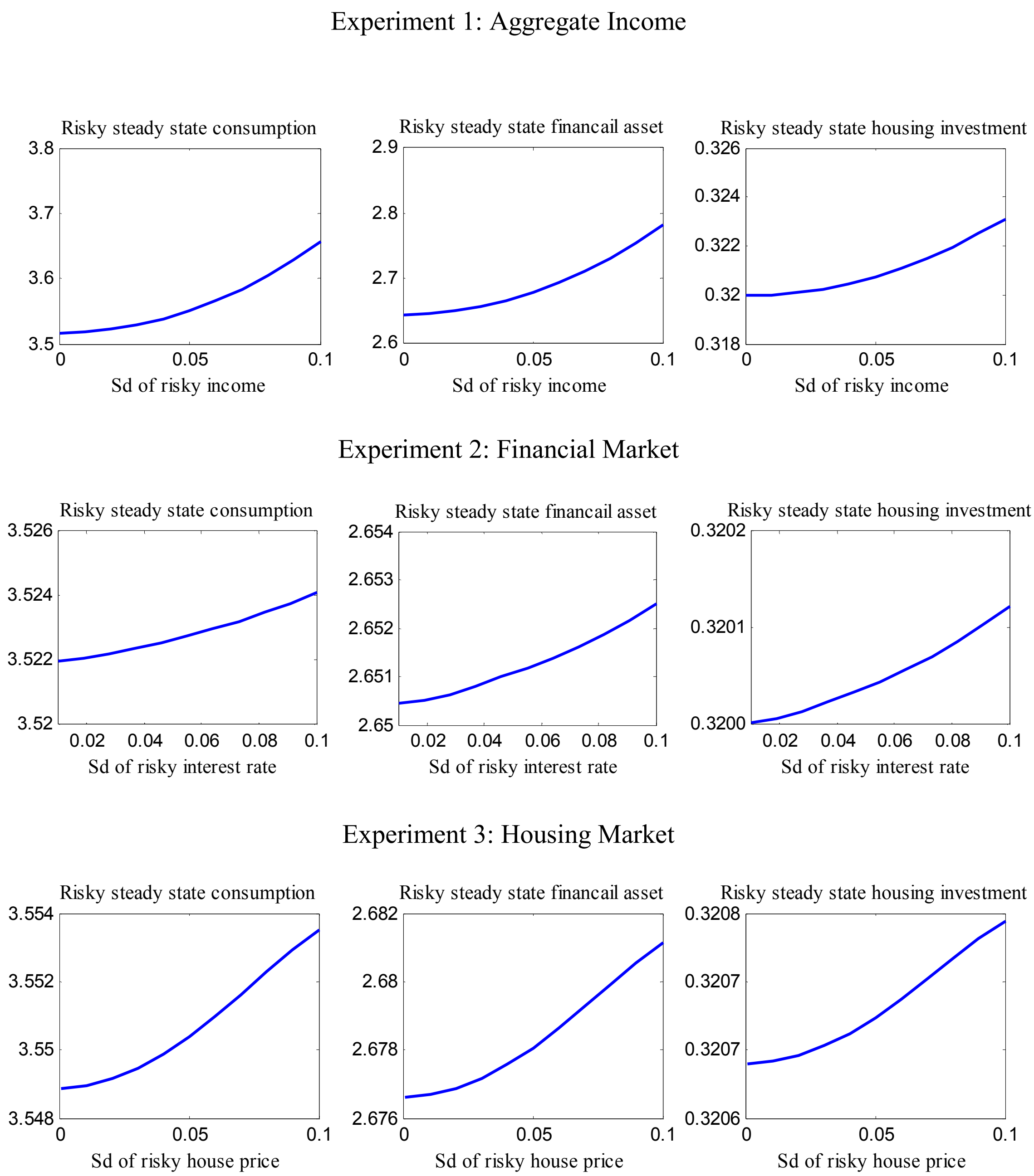
Meanwhile, some quantitative differences are also worth noticing. In my experiments, the variations of aggregate income risk, financial risk and housing risk have a relatively small impact on the risky steady states of consumption. For instance, in the first experiment, it goes up from 3.5166 to 3.5506, only a 0.97% increase, when aggregate risk changes from 0.01 to 0.05. These small dispersions lie on the risk premium effects induced by the risky interest rate and house prices. Coeurdacier et al. (2011, 2012) extend their numerical analysis by differentiating the precautionary effect and risk premium effect separately. Under the assumption that financial returns are risk-free, small changes to labour income risk have a strong impact on the risky steady state for net foreign assets; however, this feature will disappear if the assets are risky,[16] since a risky financial return reduces the persistence of shocks on risky assets compared with non-stochastic scenarios and acts as an additional stabilising force on the consumption path. In my model, I also include the housing market under the assumption that house prices change randomly. In this scenario, the risk premium effect associated with risky house prices is working. Similar to risky financial returns, uncertainty in house prices lowers the persistence of shocks on housing assets and also has a stabilising effect on consumption. This smoothing caused by the housing market has already been pointed out by Hurst and Stafford (2004). They use an empirical study to document the extent to which homeowners use housing equity to smooth their consumption over time.

As for financial assets level, I find it positively associated with the country's risk levels evaluated in three aspects. This is also highlighted roughly by Coeurdacier et al. (2011, 2012). Influenced by the level of aggregate income risk in the economy, the precautionary motivation induces a well-defined risky steady state for financial assets. Therefore, a riskier country tends to accumulate more wealth than safer one in the long-term. However, the extents of accumulated wealth are different from case to case. If the aggregate income risk level changes from 0 to 0.1, this causes a 4.97% increase in financial assets

[16] In Coeurdacier et al.'s (2011, 2012) numerical example, the risky steady state for consumption $\bar{c}$ jumps from 0.4 to 2.0 when σ_y varies from 0.01 to 0.05, a 400% increase under the assumption of a risk-free financial return. However, when financial return is risky, $\bar{c}$ changes only from 0.995 to 1.017, a 2.2% increase.

(second graph, panel 1). With the same amount of risk changing in the housing market, my model predicts lower growth from 2.6766 to 2.6799, a 0.12% increase (second graph, panel 3). This difference can be explained by the so-called crowding out effect induced by risky housing. House price risk can be substantial, but unlike other risky assets that people can avoid, most households keep investing in housing in order to own their home eventually (Banks et al. (2010)).

Figure 4: Results of the Numerical Analysis



Participation in the financial market is squeezed out by risky housing.[17] Although long-horizon uncertainty triggers more financial wealth accumulation, the increment will be smaller if this uncertainty comes from the housing market. Following a similar logic, I am also able to explain the changing of housing investment when the standard deviation of the risky interest rate increases, as shown in the third graph, panel 2. As the financial market becomes more volatile, households will adjust their investment strategy by investing less in risky assets and more in housing. High uncertainty in the financial market then serves as a motivation to invest in the housing market. Moreover, another investment incentive can be identified in my experiments as well. With increasing house price risk, the model predicts that housing investment also grows. This impact is at work through the channel of the hedging effect associated with future house price risk (Han (2010)). If households expect higher uncertainty in the housing market, they may have an incentive to invest more in housing assets, since these serve as an insurance against price fluctuations for future movements up the housing ladder (Banks et al. (2010)). By comparing the second experiment with the third one, the general impression is that uncertainty rooted in the housing market has a stronger impact on household consumption and investment decisions than that rooted in the financial market.

4. Empirical Analysis

In the previous section, I used three numerical experiments to discuss the effects of certain parameters qualitatively and quantitatively. Next, I present some tentative empirical evidence for each idea. The model implies that, at equilibrium with the expectation of future risk, a country with a higher risk level tends to invest more in housing assets than a safer one. In this sense, I want to visualise this relationship between risk level and housing investment level by creating a scatter plot of two proxy variables from three different perspectives across OECD countries. More specifically, I use the 25-year-ahead predicted standard deviations of the real GDP growth rate to represent a country's aggregate risk level. These predictions are constructed by the $I(0)$ model following the earlier work of Mueller and Watson (2013) using the real GDP growth rate 1970Q1–2010Q4 for 13 OECD countries.[18] Their method aims to quantify the uncertainty in the long-run forecasting of economic variables. Therefore, past history-based standard deviations could be considered to be proxies of countries' aggregate risk levels. In the same spirit, the paper constructs a country's risk level in the financial market and housing market by predicting the 25-year-ahead standard deviations of the real interest rate and real house price index growth rates, respectively. To keep the forecast results as consistent as possible, my samples of the real interest rate and real house price index are taken between 1970Q1 and 2010Q4 as well. Given the potential reverse causality, the proxy variable for housing investment level needs to be measured

[17] This is similar to Cocco's (2005) crowding out effect. He provides empirical evidence to show that house price risk can crowd out stockholdings, and this effect is significant for investors that have limited financial wealth.

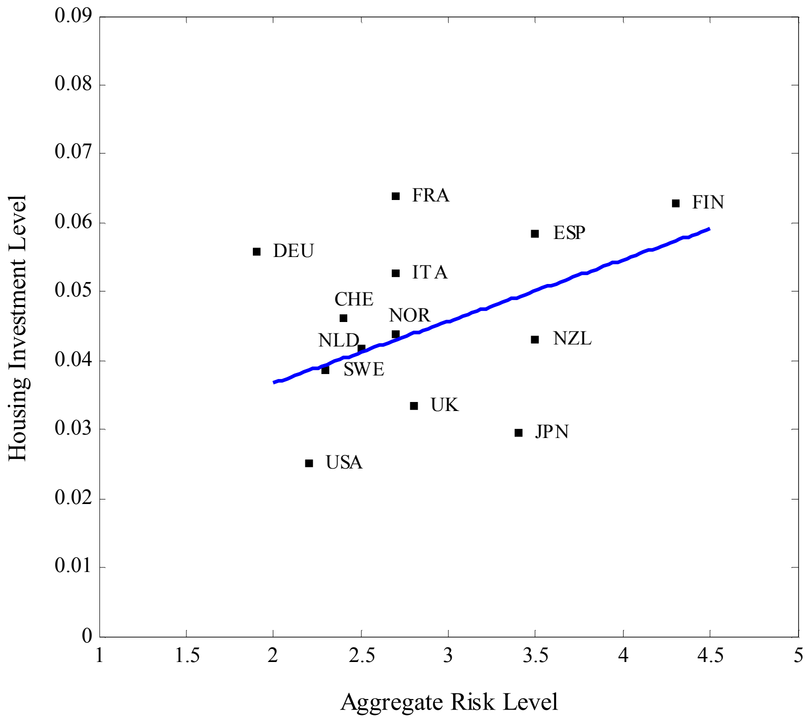
[18] Owing to the availability of data in OECD's iLibrary, I use the data sample between 1970Q1–2011Q4 for 13 OECD countries.

carefully. To do this, I use OECD data to compute the housing asset-to-GDP ratio in 2011 to capture the level of housing investment in that year. According to the modelling setup, this paper takes the risk resources as given and the standard reversal causality issue is less of a concern in my discussion. The measures of risk used in my scatter plots are forecasted from historical data between 1970Q1 and 2010Q4, which are most unlikely to be affected by the housing asset level evaluated in 2011.

Finally, the relationships between housing investment and the three different risk measures are demonstrated in Figure 5, panels 1–3. The general impression is that there are positive associations across OECD countries.[19] Agents living in riskier countries are aware of the existence of future uncertainty and therefore anticipate bigger shocks hitting the economy. For instance, in 2010, based on historical information between 1970 and 2010, agents from Spain presuppose their housing market risk as high as 28. In such a case, they tend to invest more in housing compared with a safer county such as Sweden (22.4) or Switzerland (18.6). In Spain, the housing asset-to-GDP ratio was almost 6% in 2011.

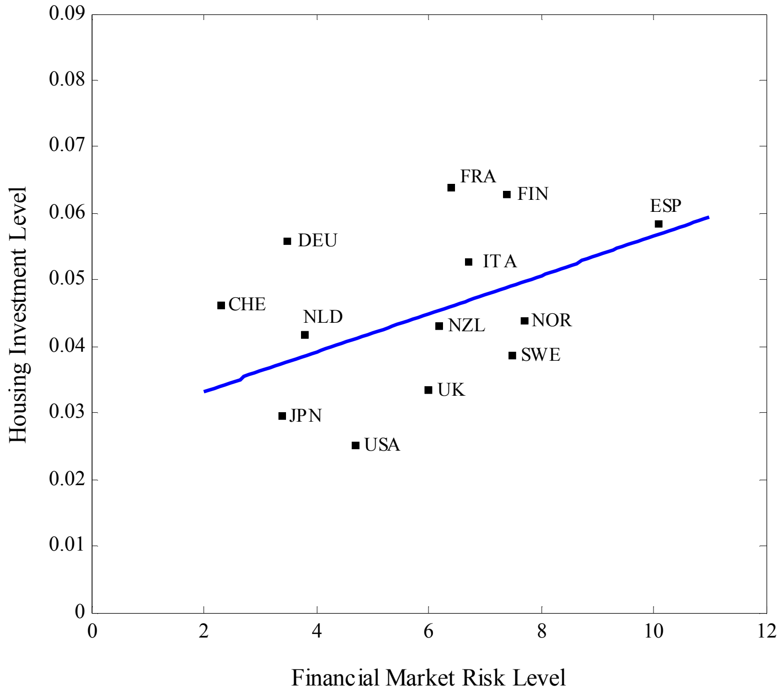
Figure 5: Relationship between Housing Investment Level and Risk Level Evaluated from Three Aspects across OECD Countries

Panel 1: Aggregate Income

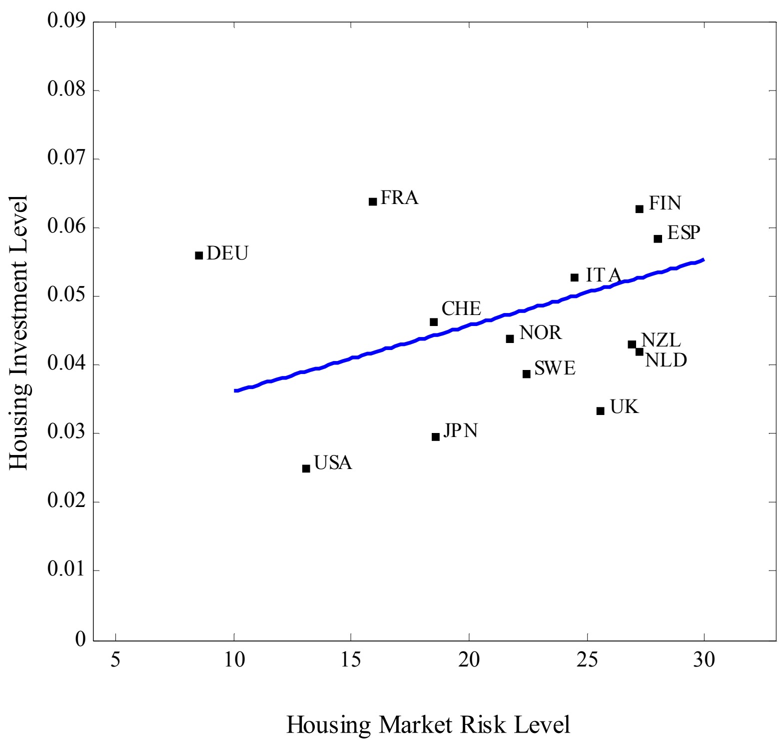


[19] I could also use 50-year-ahead predicted standard deviation and this will not change the distribution of the points in the scatter plot too much. The relationship will remain positive.

Panel 2: Financial Market



Panel 3: Housing Market



Meanwhile, housing investment in the same year accounted for 3.6% and 4.8% of GDP in Sweden and Switzerland, respectively (panel 3 in Figure 5). A similar positive relationship can also be observed in the financial market in panel 2 of Figure 5, which provides empirical evidence of the crowding out effect triggered by risky house prices. Unlike other risky assets, purchasing a home is the single most important financial decision that cannot be bypassed for typical households (Han (2010)). Therefore, they prefer housing assets to financial assets, since most people want to own their home eventually and thereby create insurance demand for housing ownership (Banks et al. (2010)).

5. Conclusion

This paper analyses the impact of long-horizon house price uncertainty on housing investment choices in the risky steady state framework. Typically, risk-averse agents will spend less on risky assets as volatility increases. However, in this paper I show that housing investment in the presence of risky steady state house prices is an exception to this rule. Taking future risk into account, the precautionary saving effect, crowding out effect and risk premium elicited by interest rate risk and house price risk are well reflected in my approximated risky steady state equilibrium function. Therefore, the model stresses analytically and conceptually why long-horizon house price uncertainty acts as an incentive to invest more in housing assets in a riskier country. Moreover, the numerical results also show the extents to which housing investment will go up when uncertainty changes at different scales, measured by different aspects: aggregate income risk, the risky interest rate and risky house prices. This also stresses that in a volatile economy housing serves not only as a durable consumption good but also as a self-insurance instrument, through which a household has leverage against future house price fluctuations. Finally, the empirical evidence across OECD countries sheds light on the role of the risky housing market in an agent's intertemporal consumption and investment decisions and broadly confirms my discussions of the theoretical results within the risky steady state framework.

What is missing in the modelling setup above and what warrants further investigation? My model above makes the extreme assumption that house prices are an exogenous AR(1) process. However, the true stochastic process is likely to be much more complex than the one I have assumed, involving higher-order autoregressive or moving average terms (Cocco (2005)). Another important limitation is the assumption of the correlation between aggregate income and house prices. I focus mainly on the covariance of consumption with housing investment, house prices and the interest rate. The fact that I ignored this correlation may have an important impact on the covariance terms in my model. Further ignored features are financial frictions and the resulting borrowing constraints. At a more fundamental level, general equilibrium effects are missing in the simple representative agent model. Conventional wisdom is that house price fluctuations are driven by technology, tastes and various macroeconomic shocks in general equilibrium models. I leave such richer modelling frameworks for future research.

Appendix A: Derivation of Equation (25)

From equations (14), (15) or (16) in the text, the Euler equation is given by

$$1 = \beta E_t[\ r_{t+1}\,(\frac{c_{t+1}}{c_t})^{\Delta}(\frac{p_t}{p_{t+1}})^{\Lambda}],$$

with

$$\Delta = -\left[\frac{\gamma}{1-\alpha}(\alpha^2 + (1-\alpha)^2) + \alpha\right],$$
$$\Lambda = -\frac{\alpha\gamma}{1-\alpha}. \tag{A1}$$

Next, we define

$$f(c_{t+1}, c_t, r_{t+1}, p_{t+1}, p_t) \equiv \beta\, r_{t+1}\left(\frac{c_{t+1}}{c_t}\right)^{\Delta}\left(\frac{p_t}{p_{t+1}}\right)^{\Lambda} - 1, \tag{A2}$$

The Euler equation (A1) can be rewritten as

$$E_t\,[f(c_{t+1}, c_t, r_{t+1}, p_{t+1}, p_t)] = 0. \tag{A3}$$

Following Coeurdacier et al.'s (2011) risky steady state strategy, we replace equation (A3) with its second-order Taylor expansion Φ around the expected future variable

$$\begin{aligned} 0 &= E_t\,[f(c_{t+1}, c_t, r_{t+1}, p_{t+1}, p_t)] \\ &\approx \Phi[E_t\,(c_{t+1}), E_t\,(r_{t+1}), E_t\,(p_{t+1}), c_t, p_t], \end{aligned} \tag{A4}$$

where

$$\begin{aligned} \Phi = {} & \beta E_t(r_{t+1})\left(\frac{E_t(c_{t+1})}{c_t}\right)^{\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{\Lambda} - 1 \\ & + \frac{\beta(\Delta-1)\Delta}{2} Var_t(c_{t+1})\left(\frac{E_t(c_{t+1})}{c_t}\right)^{\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{\Lambda}\frac{E_t(r_{t+1})}{E_t(c_{t+1})^2} \\ & + \frac{\beta(\Lambda+1)\Lambda}{2} Var_t(p_{t+1})\left(\frac{E_t(c_{t+1})}{c_t}\right)^{\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{\Lambda}\frac{E_t(r_{t+1})}{E_t(p_{t+1})^2} \\ & - \beta\Delta\Lambda Cov_t(c_{t+1}, p_{t+1})\left(\frac{E_t(c_{t+1})}{c_t}\right)^{\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{\Lambda}\frac{E_t(r_{t+1})}{E_t(c_{t+1})E_t(p_{t+1})} \\ & + \beta\Delta Cov_t(c_{t+1}, r_{t+1})\left(\frac{E_t(c_{t+1})}{c_t}\right)^{\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{\Lambda}\frac{1}{E_t(c_{t+1})}. \end{aligned} \tag{A5}$$

Multiplying equation (A5) by the non-zero $\beta^{-1}E_t(r_{t+1})^{-1}\left(\frac{E_t(c_{t+1})}{c_t}\right)^{-\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{-\Lambda}$ term gives us

$$
\begin{aligned}
0 &\approx \widehat{\Phi}[E_t\,(c_{t+1}), E_t\,(r_{t+1}), E_t\,(p_{t+1}), c_t, p_t] \\
&= 1 - \frac{1}{\beta E_t(r_{t+1})}\left(\frac{E_t(c_{t+1})}{c_t}\right)^{-\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{-\Lambda} \\
&+ \frac{(\Delta-1)\Delta}{2}\frac{Var_t(c_{t+1})}{E_t(c_{t+1})^2} + \frac{(\Lambda+1)\Lambda}{2}\frac{Var_t(p_{t+1})}{E_t(p_{t+1})^2} \\
&- \Delta\Lambda\frac{Cov_t(c_{t+1},p_{t+1})}{E_t(c_{t+1})E_t(p_{t+1})} + \Delta\frac{Cov_t(c_{t+1},r_{t+1})}{E_t(c_{t+1})E_t(r_{t+1})}. \qquad \text{(A6)}
\end{aligned}
$$

By moving the second term to the left and multiplying both sides by $\frac{1}{E_t(r_{t+1})}$, equation (A6) will be transferred into

$$
\begin{aligned}
&\frac{1}{\beta}\left(\frac{E_t(c_{t+1})}{c_t}\right)^{-\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{-\Lambda} \\
&= E_t(r_{t+1})[1 + \frac{(\Delta-1)\Delta}{2}\frac{Var_t(c_{t+1})}{E_t(c_{t+1})^2} + \frac{(\Lambda+1)\Lambda}{2}\frac{Var_t(p_{t+1})}{E_t(p_{t+1})^2} \\
&- \Delta\Lambda\frac{Cov_t(c_{t+1},p_{t+1})}{E_t(c_{t+1})E_t(p_{t+1})}] + \Delta\frac{Cov_t(c_{t+1},r_{t+1})}{E_t(c_{t+1})}. \qquad \text{(A7)}
\end{aligned}
$$

At the risky steady state, this approximation becomes

$$
\frac{1}{\beta} = \bar{r}\left[1 + \frac{(\Delta-1)\Delta}{2}\frac{\overline{Var_t}(c_{t+1})}{\bar{c}^2} + \frac{(\Lambda+1)\Lambda}{2}\frac{\overline{Var_t}(p_{t+1})}{\bar{p}^2} - \Delta\Lambda\frac{\overline{Cov_t}(c_{t+1},p_{t+1})}{\bar{c}\bar{p}}\right] + \Delta\frac{\overline{Cov_t}(c_{t+1},r_{t+1})}{\bar{c}}. \qquad \text{(A8)}
$$

Appendix B: Second-Order Approximation of the Euler Equation

The Euler equation's second-order expansion around the expected future variable is given by

$$
\begin{aligned}
0 &= E_t\,[f(c_{t+1}, c_t, r_{t+1}, p_{t+1}, p_t)] \\
&\approx \Phi[E_t\,(c_{t+1}), E_t\,(r_{t+1}), E_t\,(p_{t+1}), c_t, p_t], \\
&= f(E_t(c_{t+1}), c_t, E_t(r_{t+1}), E_t(p_{t+1}), p_t) \\
&+ \frac{1}{2}f_{c_{t+1}c_{t+1}}E_t(c_{t+1} - E_t(c_{t+1}))^2 \\
&+ \frac{1}{2}f_{p_{t+1}p_{t+1}}E_t(p_{t+1} - E_t(p_{t+1}))^2 \\
&+ \frac{1}{2}f_{r_{t+1}r_{t+1}}E_t(r_{t+1} - E_t(r_{t+1}))^2 \\
&+ f_{c_{t+1}p_{t+1}}E_t[(c_{t+1} - E_t(c_{t+1}))(p_{t+1} - E_t(p_{t+1}))] \\
&+ f_{c_{t+1}r_{t+1}}E_t[(c_{t+1} - E_t(c_{t+1}))(r_{t+1} - E_t(r_{t+1}))] \\
&+ f_{r_{t+1}p_{t+1}}E_t[(r_{t+1} - E_t(r_{t+1}))(p_{t+1} - E_t(p_{t+1}))] \\
&= \beta E_t(r_{t+1})\left(\frac{E_t(c_{t+1})}{c_t}\right)^{\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{\Lambda} - 1
\end{aligned}
$$

$$+\frac{1}{2}f_{c_{t+1}c_{t+1}}Var_t(c_{t+1})+\frac{1}{2}f_{p_{t+1}p_{t+1}}Var_t(p_{t+1})+\frac{1}{2}f_{r_{t+1}r_{t+1}}Var_t(r_{t+1})$$

$$+f_{c_{t+1}p_{t+1}}Cov_t(c_{t+1},p_{t+1})+f_{c_{t+1}r_{t+1}}Cov_t(c_{t+1},r_{t+1})+f_{r_{t+1}p_{t+1}}Cov_t(p_{t+1},r_{t+1})$$

$$=\beta E_t(r_{t+1})\left(\frac{E_t(c_{t+1})}{c_t}\right)^{\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{\Lambda}-1$$

$$+\frac{\beta(\Delta-1)\Delta}{2}Var_t(c_{t+1})\left(\frac{E_t(c_{t+1})}{c_t}\right)^{\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{\Lambda}\frac{E_t(r_{t+1})}{E_t(c_{t+1})^2}$$

$$+\frac{\beta(\Lambda+1)\Lambda}{2}Var_t(p_{t+1})\left(\frac{E_t(c_{t+1})}{c_t}\right)^{\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{\Lambda}\frac{E_t(r_{t+1})}{E_t(p_{t+1})^2}$$

$$-\beta\Delta\Lambda Cov_t(c_{t+1},p_{t+1})\left(\frac{E_t(c_{t+1})}{c_t}\right)^{\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{\Lambda}\frac{E_t(r_{t+1})}{E_t(c_{t+1})E_t(p_{t+1})}$$

$$+\beta\Delta Cov_t(c_{t+1},r_{t+1})\left(\frac{E_t(c_{t+1})}{c_t}\right)^{\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{\Lambda}\frac{1}{E_t(c_{t+1})}\,. \tag{B1}$$

Appendix C: Derivation of the Solution System and the Risky Steady State

The optimality conditions are given by a five-dimensional equation system. The first equation is equation (A8) in Appendix A

$$1-\frac{1}{\beta\bar{r}}+\frac{(\Delta-1)\Delta}{2}\frac{\overline{Var_t}(c_{t+1})}{\bar{c}^2}+\frac{(\Lambda+1)\Lambda}{2}\frac{\overline{Var_t}(p_{t+1})}{\bar{p}^2}-\Delta\Lambda\frac{\overline{Cov_t}(c_{t+1},p_{t+1})}{\bar{c}\bar{p}}+\Delta\frac{\overline{Cov_t}(c_{t+1},r_{t+1})}{\bar{c}\bar{r}}=0. \tag{C1}$$

The remaining equations are given as follows

$$\frac{d\hat{\Phi}}{d\omega_{t-1}}|_{S_t=\bar{S}}=\frac{\partial\hat{\Phi}}{\partial\omega_t}\frac{\partial\omega_t}{\partial\omega_{t-1}}+\frac{\partial\hat{\Phi}}{\partial E_t(c_{t+1})}\frac{\partial E_t(c_{t+1})}{\partial\omega_t}\frac{\partial\omega_t}{\partial\omega_{t-1}}+\frac{\partial\hat{\Phi}}{\partial c_t}\frac{\partial c_t}{\partial\omega_{t-1}}=0, \tag{C2}$$

$$\frac{d\hat{\Phi}}{dy_t}|_{S_t=\bar{S}}=\frac{\partial\hat{\Phi}}{\partial\omega_t}\frac{\partial\omega_t}{\partial y_t}+\frac{\partial\hat{\Phi}}{\partial E_t(c_{t+1})}\frac{\partial E_t(c_{t+1})}{\partial\omega_t}\frac{\partial\omega_t}{\partial y_t}+\frac{\partial\hat{\Phi}}{\partial E_t(c_{t+1})}\frac{\partial E_t(c_{t+1})}{\partial E_t(y_{t+1})}\frac{\partial E_t(y_{t+1})}{\partial y_t}$$
$$+\frac{\partial\hat{\Phi}}{\partial c_t}\frac{\partial c_t}{\partial y_t}+\frac{\partial\hat{\Phi}}{\partial Var_t(y_{t+1})}\frac{\partial Var_t(y_{t+1})}{\partial y_t}=0, \tag{C3}$$

$$\frac{d\hat{\Phi}}{dr_t}|_{S_t=\bar{S}}=\frac{\partial\hat{\Phi}}{\partial\omega_t}\frac{\partial\omega_t}{\partial r_t}+\frac{\partial\hat{\Phi}}{\partial E_t(c_{t+1})}\frac{\partial E_t(c_{t+1})}{\partial\omega_t}\frac{\partial\omega_t}{\partial r_t}+\frac{\partial\hat{\Phi}}{\partial E_t(c_{t+1})}\frac{\partial E_t(c_{t+1})}{\partial E_t(r_{t+1})}\frac{\partial E_t(r_{t+1})}{\partial r_t}$$
$$+\frac{\partial\hat{\Phi}}{\partial E_t(r_{t+1})}\frac{\partial E_t(r_{t+1})}{\partial r_t}+\frac{\partial\hat{\Phi}}{\partial c_t}\frac{\partial c_t}{\partial r_t}+\frac{\partial\hat{\Phi}}{\partial Var_t(r_{t+1})}\frac{\partial Var_t(r_{t+1})}{\partial r_t}=0, \tag{C4}$$

$$\frac{d\hat{\Phi}}{dp_t}|_{S_t=\bar{S}}=\frac{\partial\hat{\Phi}}{\partial\omega_t}\frac{\partial\omega_t}{\partial p_t}+\frac{\partial\hat{\Phi}}{\partial E_t(c_{t+1})}\frac{\partial E_t(c_{t+1})}{\partial\omega_t}\frac{\partial\omega_t}{\partial p_t}+\frac{\partial\hat{\Phi}}{\partial E_t(c_{t+1})}\frac{\partial E_t(c_{t+1})}{\partial E_t(p_{t+1})}\frac{\partial E_t(p_{t+1})}{\partial p_t}$$
$$+\frac{\partial\hat{\Phi}}{\partial E_t(p_{t+1})}\frac{\partial E_t(p_{t+1})}{\partial p_t}+\frac{\partial\hat{\Phi}}{\partial c_t}\frac{\partial c_t}{\partial p_t}+\frac{\partial\hat{\Phi}}{\partial Var_t(p_{t+1})}\frac{\partial Var_t(p_{t+1})}{\partial p_t}+\frac{\partial\hat{\Phi}}{\partial p_t}=0. \tag{C5}$$

In order to obtain the exact expression of this equation system, we need to deduce the partial derivatives of $\widehat{\Phi}$ given by equation (A6) with respect to ω_t, c_t, p_t, $E_t(c_{t+1})$, $E_t(r_{t+1})$, $E_t(p_{t+1})$, $Var_t(y_{t+1})$, $Var_t(r_{t+1})$ and $Var_t(p_{t+1})$

$$\frac{\partial\widehat{\Phi}}{\partial\omega_t}=(\Delta-1)\Delta\frac{{K_1}^2(\omega_t-G_{\omega r})Var_t(r_{t+1})}{E_t(c_{t+1})^2}+\Delta\frac{K_1 Var_t(r_{t+1})}{E_t(c_{t+1})E_t(r_{t+1})}, \tag{C6}$$

$$\frac{\partial\widehat{\Phi}}{\partial c_t}=-\frac{\Delta}{\beta E_t(r_{t+1})c_t}\left(\frac{E_t(c_{t+1})}{c_t}\right)^{-\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{-\Lambda}, \tag{C7}$$

$$\frac{\partial\widehat{\Phi}}{\partial p_t}=\frac{\Lambda}{\beta E_t(r_{t+1})p_t}\left(\frac{E_t(c_{t+1})}{c_t}\right)^{-\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{-\Lambda}, \tag{C8}$$

$$\begin{aligned}\frac{\partial\widehat{\Phi}}{\partial E_t(c_{t+1})}=&\frac{\Delta}{\beta E_t(r_{t+1})E_t(c_{t+1})}\left(\frac{E_t(c_{t+1})}{c_t}\right)^{-\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{-\Lambda}+\Delta(\Delta-1)\frac{K_1(c_{t+1})}{E_t(c_{t+1})^3}\\&+\Delta\Lambda\frac{Cov_t(c_{t+1},p_{t+1})}{E_t(c_{t+1})^2E_t(p_{t+1})}-\Delta\frac{Cov_t(c_{t+1},r_{t+1})}{E_t(c_{t+1})^2E_t(r_{t+1})},\end{aligned} \tag{C9}$$

$$\frac{\partial\widehat{\Phi}}{\partial E_t(r_{t+1})}=\frac{\Lambda}{\beta E_t(r_{t+1})^2}\left(\frac{E_t(c_{t+1})}{c_t}\right)^{-\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{-\Lambda}-\Delta\frac{Cov_t(c_{t+1},r_{t+1})}{E_t(c_{t+1})E_t(r_{t+1})^2}, \tag{C10}$$

$$\begin{aligned}\frac{\partial\widehat{\Phi}}{\partial E_t(p_{t+1})}=&-\frac{\Lambda}{\beta E_t(r_{t+1})E_t(p_{t+1})}\left(\frac{E_t(c_{t+1})}{c_t}\right)^{-\Delta}\left(\frac{p_t}{E_t(p_{t+1})}\right)^{-\Lambda}\\&-\Lambda(\Lambda+1)\frac{Var_t(p_{t+1})}{E_t(p_{t+1})^3}+\Delta\Lambda\frac{Cov_t(c_{t+1},p_{t+1})}{E_t(c_{t+1})E_t(p_{t+1})^2},\end{aligned} \tag{C11}$$

$$\frac{\partial\widehat{\Phi}}{\partial Var_t(y_{t+1})}=\frac{(\Delta-1)\Delta}{2}\frac{{K_1}^2(1-G_{\omega y})^2}{E_t(c_{t+1})^2}, \tag{C12}$$

$$\frac{\partial\widehat{\Phi}}{\partial Var_t(r_{t+1})}=\frac{(\Delta-1)\Delta}{2}\frac{{K_1}^2(\omega_t-G_{\omega r})^2}{E_t(c_{t+1})^2}+\frac{K_1(\omega_t-G_{\omega r})}{E_t(c_{t+1})E_t(r_{t+1})}, \tag{C13}$$

$$\frac{\partial\widehat{\Phi}}{\partial Var_t(p_{t+1})}=\frac{(\Delta-1)\Delta}{2}\frac{[K_1(h_t-G_{\omega p})+K_2]^2}{E_t(c_{t+1})^2}+\frac{1}{2}\frac{\Lambda(\Lambda+1)}{E_t(p_{t+1})^2}-\Delta\Lambda\frac{K_1(h_t-G_{\omega p})+K_2}{E_t(c_{t+1})E_t(p_{t+1})}. \tag{C14}$$

Next, we differentiate the conditional expectation of consumption (27) in terms of the expected values of $y_{t+1}, r_{t+1}, p_{t+1}$

$$\frac{\partial E_t(c_{t+1})}{\partial E_t(y_{t+1})}=K_1(1-G_{\omega y}), \tag{C15}$$

$$\frac{\partial E_t(c_{t+1})}{\partial E_t(r_{t+1})}=K_1(\omega_t-G_{\omega r}), \tag{C16}$$

$$\frac{\partial E_t(c_{t+1})}{\partial E_t(p_{t+1})} = K_1(h_t - G_{\omega p}) + K_2, \tag{C17}$$

and

$$\frac{\partial E_t(c_{t+1})}{\partial \omega_t} = K_1[E_t(r_{t+1}) - G_{\omega\omega}]. \tag{C18}$$

Then, we also need the differentiations of expectation and variances in income, the interest rate and housing prices with respect to y_t, r_t and p_t

$$\frac{\partial E_t(y_{t+1})}{\partial y_t} = E_t(y_{t+1})\frac{\rho_y}{y_t}, \tag{C19}$$

$$\frac{\partial E_t(r_{t+1})}{\partial r_t} = E_t(r_{t+1})\frac{\rho_r}{r_t}, \tag{C20}$$

$$\frac{\partial E_t(p_{t+1})}{\partial p_t} = E_t(p_{t+1})\frac{\rho_p}{p_t}, \tag{C21}$$

$$\frac{\partial Var_t(y_{t+1})}{\partial y_t} = 2Var_t(y_{t+1})\frac{\rho_y}{y_t}, \tag{C22}$$

$$\frac{\partial Var_t(r_{t+1})}{\partial r_t} = 2Var_t(r_{t+1})\frac{\rho_r}{r_t}, \tag{C23}$$

$$\frac{\partial Var_t(p_{t+1})}{\partial p_t} = 2Var_t(p_{t+1})\frac{\rho_p}{p_t}. \tag{C24}$$

After substituting equation (26) into the budget constraint, we obtain the derivatives of c_t to y_t, r_t, p_t and ω_{t-1}

$$\frac{\partial c_t}{\partial y_t} = K_1(1 - G_{\omega y}), \tag{C25}$$

$$\frac{\partial c_t}{\partial r_t} = K_1(\omega_{t-1} - G_{\omega r}), \tag{C26}$$

$$\frac{\partial c_t}{\partial p_t} = K_1(h_t - G_{\omega p}) + K_2, \tag{C27}$$

$$\frac{\partial c_t}{\partial \omega_{t-1}} = K_1(r_t - G_{\omega\omega}). \tag{C28}$$

Finally, we differentiate ω_t again with respect to y_t, r_t, p_t and ω_{t-1}

$$\frac{\partial \omega_t}{\partial y_t} = G_{\omega y}, \tag{C29}$$

$$\frac{\partial \omega_t}{\partial r_t} = G_{\omega r}, \tag{C30}$$

$$\frac{\partial \omega_t}{\partial p_t} = G_{\omega p}, \tag{C31}$$

$$\frac{\partial \omega_t}{\partial \omega_{t-1}} = G_{\omega\omega}. \tag{C32}$$

By inserting equations (C6– (C32) into equations (C1)–(C5) and evaluating at the risky steady state, we can get an equation system containing five unknown variables: $\bar{\omega}$, $G_{\omega\omega}$, $G_{\omega y}$, $G_{\omega r}$, $G_{\omega p}$, with which we are able to calculate $\bar{c}$ and $\bar{h}$ as follows

$$\bar{c} = \bar{y} + \bar{\omega}(\bar{r} - 1) \tag{C33}$$

$$\bar{h} = M(\frac{\bar{c}^{\alpha}}{\bar{p}})^{\frac{1}{1-\alpha}} \tag{C34}$$

Appendix D: Optimal Housing Investment

An agent's maximisation problem gives us the optimal housing investment

$$h_t = M(\frac{{c_t}^{\alpha}}{p_t})^{\frac{1}{1-\alpha}} \text{ with } M = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\alpha}}. \tag{D1}$$

Taking ln on both sides of equation (D1) leads to

$$lnh_t = lnM + \frac{\alpha}{1-\alpha} lnc_t + \frac{1}{\alpha-1} lnp_t, \tag{D2}$$

which is approximately a lognormally distributed process under our assumptions. Then, expectation and variance of lnh_t can be computed immediately as follows

$$E[lnh_t] = lnM + \frac{\alpha}{1-\alpha} E[lnc_t] + \frac{1}{\alpha-1} E[lnp_t], \tag{D3}$$

$$Var[lnh_t] = (\frac{\alpha}{1-\alpha})^2 Var[lnc_t] + \frac{1}{(1-\alpha)^2} Var[lnp_t] - \frac{2\alpha}{(1-\alpha)^2} Cov[lnc_t, lnp_t]. \tag{D4}$$

Variance in h_t is then

$$Var[h_t] = e^{2E[lnh_t]+Var[lnh_t]}\left(e^{Var[lnh_t]} - 1\right). \tag{D5}$$

This implies that an increase in house price risk will increase housing investment volatility in my model.

References

Banks, J., Blundell, R., Oldfield, Z. and J.P. Smith (2010) "House Price Volatility and the Housing Ladder", *IZA Discussion Papers* 5173, Institute for the Study of Labor (IZA).

Banks, J., Blundell, R., Oldfield, Z. and J.P. Smith (2012) "Housing Mobility and Downsizing at Older Ages in Britain and the USA", *Economica* 79, 1–2.

Bansal, R., Kiku, D. and A. Yaron (2010) "Long-run Risks, the Macroeconomy and Asset Prices", *American Economic Review, p&p*, 542–546.

Blundell, R., Pistaferri, L. and I. Preston (2008) "Consumption Inequality and Partial Insurance", *American Economic Review*, 98(5), 1887–1921.

Campbell, J. Y. and J. F. Cocco (2007) "How do House Prices affect Consumption? Evidence from Micro Data", *Journal of Monetary Economics*, *Elsevier*, 54(3), 591-621.

Chen, Y. F., Funke, M. and A. N. Mehrotra (2014) "What drives Urban Consumption in Mainland China? The Role of Property Price Dynamics*", Pacific Economic Review,* forthcoming.

Cocco, J. F. (2005) "Portfolio Choice in the Presence of Housing", *Review of Financial Studies,* 18(2), 535–567.

Cocco, J. F., Gomes, F.J. and P.J. Maenhout (2005) "Consumption and Portfolio Choice over the Life-Cycle", *Review of Financial Studies,* 18(2).

Coeurdacier, N., Rey, H. and P. Winant (2011, 2012) "The Risky Steady State", *American Economic Review,* 101(3), 398–401.

De Groot, O. (2013) "Computing the Risky Steady State of DSGE Models", *Economics Letters* 120, 566–569.

Han, L. (2010) "The Effects of Price Risk on Housing Demand: Empirical Evidence from US markets", *Review of Financial Studies*, 23(11), 3889–3928.

Hurst, E. and F. Stafford (2004) "Home Is where the Equity Is: Mortgage Refinancing and Household Consumption", *Journal of Money, Credit and Banking*, 36(6), 985–1014.

Iacoviello, M. (2011) “Housing Wealth and Consumption”, *FRB International Finance Discussion Papers No.*1027.

Judd, K. L. (1998) “Numerical Methods in Economics”, *MIT Press*, 1998.

Juillard, M. (2011) “Local Approximation of DSGE Models around the Risky Steady State”, *wp. Comunite* 0087, Department of Communication, University of Teramo.

Juillard, M. and O. Kamenik (2005) “Solving SDGE Models: Approximation about the Stochastic Steady State”, *Computing in Economics and Finance* 2005, 106.

Kouwenberg, R. and R. Zwinkels (2014) “Forecasting the US Housing Market”, *International Journal of Forecasting* 30, 415–425.

Lucas, R.E. (1987) “Models of Business Cycles”, *Wiley-Blackwell*, 1987.

Mahadeva, L. (2013) “Notes on Solving for the Risky Steady State”, *Working Paper Series*.

Mehra, R. and E. Prescott (1985) “The Equity Premium Puzzle”, *Journal of Monetary Economics*, 15, 145–161.

Mueller, U. and M.W. Watson (2013) “Measuring Uncertainty about Long-Run Prediction”, *NBER Working Paper No.* 18870, Cambridge (Mass.).

Nagaraja, C. H., Browny, L.D., and L.H. Zhao (2011) “An Autoregressive Approach to House Price Modeling*”, Annals of Applied Statistics* 5(1), 124–149.

Schmitt-Grohé, S. and M. Uribe (2004) “Solving Dynamic General Equilibrium Models using a second-order Approximation to the Policy Function”, *Journal of Economic Dynamics and Control* 28, 755–775.

Yao, R. and H. Zhao (2005) “Optimal Consumption and Portfolio Choices with Risky Housing and Borrowing Constraints”, *Review of Financial Studies*, 18(1), 197–239.

U.S. Department of Labor, Bureau of Labor Statistics, 2003. Consumer Expenditures in 2001, Report 966.