$$\dot{D} = \frac{\partial D}{\partial p}\dot{p} + \frac{\partial D}{\partial m}\dot{m}$$

$$\frac{\dot{D}}{D} = \frac{\partial D}{\partial p}\frac{\dot{p}}{D} + \frac{\partial D}{\partial m}\frac{\dot{m}}{D}$$

$$= \frac{\partial D}{\partial p}\frac{\dot{p}}{D}\frac{p}{p} + \frac{\partial D}{\partial m}\frac{\dot{m}}{D}\frac{m}{m}$$

$$= \underbrace{\frac{p}{D}\frac{\partial D}{\partial p}}_{El_p D}\frac{\dot{p}}{p} + \underbrace{\frac{m}{D}\frac{\partial D}{\partial m}}_{El_m D}\frac{\dot{m}}{m}$$

$$F(x,y) = xy = 5$$

$$y = \frac{5}{x} = 5x^{-1}$$

$$y' = -5x^{-2} = -\frac{5}{x^2}$$

mit $y = \frac{5}{x} \Rightarrow y' = -\frac{y}{x}$

$$y' = -\frac{F'_x}{F'_y} = -\frac{y}{x}$$

$xf_1'(x,y) + yf_2'(x,y) = kf(x,y)$ (Eulers Theorem)

$f_1'(x,y)$ und $f_2'(x,y)$ sind jeweils homogen vom Grad $k-1$

$$f(tx, ty) = t^k f(x,y)$$

$$t f'_x(tx, ty) = t^k f'_x(x,y)$$

$$\Leftrightarrow f'_x(tx,ty) = t^{k-1} f'_x(x,y)$$

$f(x,y) = x^k f(1, y/x) = y^k f(x/y, 1)$ für $x > 0$ und $y > 0$

$$f(tx, ty) = t^k f(x,y) \quad \text{mit } t = \frac{1}{x}$$

$$\Leftrightarrow f\left(1, \frac{y}{x}\right) = \left(\frac{1}{x}\right)^k f(x,y) \quad | \cdot x^k$$

$$\Leftrightarrow x^k f\left(1, \frac{y}{x}\right) = f(x,y)$$

$$x^2 f''_{11}(x,y) + 2xy f''_{12}(x,y) + y^2 f''_{22}(x,y) = k(k-1) f(x,y)$$

f'_x & f'_y sind homogen vom Grad $(k-1)$

$\Rightarrow$ Eulers Theorem

$$x f''_{xx} + y f''_{xy} = (k-1) f'_x \quad \Big| \cdot x$$

$$x f''_{yx} + y f''_{yy} = (k-1) f'_y \quad \Big| \cdot y \qquad +$$

$$x^2 f''_{xx} + 2xy \underbrace{f''_{xy}}_{f''_{yx}} + y^2 f''_{yy} = (k-1) \underbrace{\left[x f'_x + y f'_y\right]}_{kf}$$

1. $$d(af+bg) = (af+bg)'dx$$
$$= (af'+bg')dx$$
$$= af'dx + bg'dx$$
$$= a\,df + b\,dg$$

2. $$d(fg) = (fg)'dx = (f'g+fg')dx$$
$$= f'g\,dx + fg'dx$$
$$= g\,df + f\,dg$$

3. $$d(f/g) = (f/g)'dx = \left(\frac{f'g-fg'}{g^2}\right)dx$$
$$= \frac{gf'dx - fg'dx}{g^2}$$