FTPL and the maturity structure of government debt in the New-Keynesian Model

Online Appendix

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C Linearized dynamics

C.1 Alternative parametrizations

For the alternative parametrizations in Tables D.1 and D.2, the equilibrium dynamics are

$$\begin{aligned}
\mathrm{d}c_t &= (i_t - \rho - \pi_t)(1 - s_g) y_{ss} \mathrm{d}t && \text{(A.1)}\\
\mathrm{d}\pi_t &= \left(\rho(\pi_t - \pi_{ss}) - \kappa x_t + \kappa_0 s_g/(1 - s_g)(g_t/g_{ss} - 1)\right) \mathrm{d}t && \text{(A.2)}\\
\mathrm{d}i_t &= \theta(\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_{ss}))\mathrm{d}t && \text{(A.3)}\\
\mathrm{d}a_t &= ((i_t - \rho - \pi_t) a_{ss} + \rho(a_t - a_{ss}) - (s_t - s_{ss}))\mathrm{d}t && \text{(A.4)}\\
\mathrm{d}T_t &= \left(\tau_a(a_t - a_{ss}) - (T_t - T_{ss})\right) \mathrm{d}t && \text{(A.5)}\\
\mathrm{d}g_t &= \left(\varphi_a(a_t - a_{ss}) - (g_t - g_{ss})\right) \mathrm{d}t && \text{(A.6)}\\
\mathrm{d}p_t^b &= \big((i_t - i_{ss}) + (\chi + \delta)(p_t^b - 1)\big) \mathrm{d}t, && \text{(A.7)}
\end{aligned}$$

where $s_t = T_t - g_t$, and $\kappa \equiv (1 + \vartheta(1 - s_g))\kappa_0$.

C.1.1 Term structure of interest rates

The term structure of interest rate reveals important insights into the expectations about the future path of macroeconomic aggregates and inflation. Given the general equilibrium prices (and dynamics), we may price any other asset. For example, a zero-coupon bond with maturity N satisfies the no-arbitrage condition

$$i_t \, \mathrm{d}t = (1/p_t^{(N)}) \, \mathrm{d}p_t^{(N)} - (1/p_t^{(N)})(\partial p_t^{(N)}/\partial N) \, \mathrm{d}t,$$

which shows that no-arbitrage considerations imply that the bond prices adjust such that the households will be indifferent in their investment decision. The zero-coupon bonds must inherit the *same* properties as the longer-term bonds introduced above. Hence, since the resulting maturity distribution is approximately exponential with a duration of $1/\delta$, longer-term bonds are interchangeable with zero-coupon bonds with maturity N.[1]

Let us consider a nominal (zero-coupon) bond with unity payoff at maturity N:

$$p_t^{(N)} = \mathbb{E}_t \left(e^{-\rho N} \lambda_{t+N}/\lambda_t e^{-\int_t^{t+N} \pi_s ds} \right),$$

where λ_t is the marginal value of wealth, or the current value shadow price, consistent with equilibrium dynamics of macro aggregates. The equilibrium bond price can be obtained

[1] Alternatively, consider the bond with nominal coupon payment in (7). The equilibrium price p_t^b can be computed along the same lines from the no-arbitrage condition as a function of the state variables.

from the fundamental pricing equation for the price $p_t^{(N)}$ (cf. Posch, 2020):

$$\mathbb{E}_t\left(\left(\mathrm{d}p_t^{(N)}\right)/p_t^{(N)}\right) - \left(1/p_t^{(N)}(\partial p_t^{(N)}/\partial N) + i_t\right)\mathrm{d}t = 0.$$

Observe that in equilibrium, the bond price $p_t^{(N)}$ is a function of the state variables, so $p_t^{(N)} = p_t^{(N)}(i_t, a_t, T_t, g_t)$, where from (14c), (14d), and (14e) we get

$$\begin{aligned}
\mathrm{d}p_t^{(N)} &= (\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))(\partial p_t^{(N)}/\partial i_t)\,\mathrm{d}t \\
&\quad +(\partial p_t^{(N)}/\partial a_t)((i_t - \pi_t)a_t - s_t)\mathrm{d}t + (\partial p_t^{(N)}/\partial T_t)(\tau_a(a_t - a_{ss}) - (T_t - T_t^*))\,\mathrm{d}t \\
&\quad +(\partial p_t^{(N)}/\partial g_t)(\varphi_a(a_t - a_{ss}) - (g_t - g_t^*))\,\mathrm{d}t
\end{aligned}$$

and thus the partial differential equation (henceforth *PDE approach*) from (17) is:

$$\begin{aligned}
&(\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))(\partial p_t^{(N)}/\partial i_t) + (\tau_a(a_t - a_{ss}) - (T_t - T_t^*))(\partial p_t^{(N)}/\partial T_t) \\
&+((i_t - \pi_t)a_t - s_t)(\partial p_t^{(N)}/\partial a_t) + (\varphi_a(a_t - a_{ss}) - (g_t - g_t^*))(\partial p_t^{(N)}/\partial g_t) \\
&= (\partial p_t^{(N)}/\partial N) + i_t p_t^{(N)}.
\end{aligned}$$

The solution to the pricing equation implies the complete term structure of interest rate for any given interest rate and maturity:

$$y_t^{(N)} \equiv y^{(N)}(i_t, a_t, T_t, g_t) = -\log p_t^{(N)}(i_t, a_t, T_t, g_t)/N.$$

Our strategy is to use collocation to approximate the function $p_t^{(N)} \approx \Phi(N, i_t, a_t, T_t, g_t)v$, in which v is an n-vector of coefficients and Φ denotes the known $n \times n$ basis matrix, and can compute the unknown coefficients from a *linear* interpolation equation:

$$\begin{aligned}
&(\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))\Phi_2'(N, i_t, a_t, T_t, g_t)v + ((i_t - \pi_t)a_t - s_t)\Phi_3'(N, i_t, a_t, T_t, g_t)v \\
&+(\tau_a(a_t - a_{ss}) - (T_t - T_t^*))\Phi_4'(N, i_t, a_t, T_t, g_t)v \\
&+(\varphi_a(a_t - a_{ss}) - (g_t - g_t^*))\Phi_5'(N, i_t, a_t, T_t, g_t)v \\
=\;& \Phi_1'(N, i_t, a_t, T_t, g_t)v + i_t\Phi(N, i_t, a_t, T_t, g_t)v
\end{aligned}$$

or

$$\begin{aligned}
&\big((\tau_a(a_t - a_{ss}) - (T_t - T_t^*))\Phi_4' + (\varphi_a(a_t - a_{ss}) - (g_t - g_t^*))\Phi_5' + ((i_t - \pi_t)a_t - s_t)\Phi_3' \\
&\quad +(\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))\Phi_2' - \Phi_1' - i_t\Phi\big)v = 0_n,
\end{aligned}$$

where $n = n_1 \cdot n_2 \cdot n_3 \cdot n_4 \cdot n_5$ with the boundary condition $\Phi(0, i_t, a_t, T_t, g_t)v = 1_n$. So we concatenate the two matrices and solve the linear equation for the unknown coefficients.

C.1.2 Expected inflation

Inflation expectations are at the core of monetary policy, so it is useful to study the effects of monetary and fiscal policy shocks on the model-implied expected inflation as a benchmark within our framework. From the Phillips curve in (14b) it follows

$$\pi_t - \pi_t^* = \kappa \int_t^\infty e^{-\rho(v-t)} x_v \mathrm{d}v.$$

The inflation rate, π_t, denotes *current* expected inflation measured as deviation from its policy target rate π_t^*. Multiplying the differential equation for the inflation rate by the integrating factor and evaluating from t to $t+N$, we obtain

$$\pi_t^{(N)} \equiv \mathbb{E}_t(\pi_{t+N}) = \pi_t^* + e^{\rho N}(\pi_t - \pi_t^*) - \kappa e^{\rho N} \int_t^{t+N} e^{-\rho(s-t)} x_s \, \mathrm{d}s.$$

The rational expectation forecast π_{t+N} can be regarded as a function of the state variables (i_t, a_t, T_t and g_t) as well as the fixed forecasting horizon N. Hence, for the N-year ahead future expected inflation rate, we compute $\pi_t^{(N)}$

$$\begin{aligned} \partial\pi_t^{(N)}/\partial N \;=\; & (\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))(\partial\pi_t^{(N)}/\partial i_t)\,\mathrm{d}t + (\partial\pi_t^{(N)}/\partial a_t)((i_t - \pi_t)a_t - s_t)\mathrm{d}t \\ & + (\partial\pi_t^{(N)}/\partial T_t)(\tau_a(a_t - a_{ss}) - (T_t - T_t^*))\,\mathrm{d}t \\ & + (\partial\pi_t^{(N)}/\partial g_t)(\varphi_a(a_t - a_{ss}) - (g_t - g_t^*))\,\mathrm{d}t \end{aligned}$$

with boundary condition $\pi_t^{(0)} = \pi_t^{(0)}(i_t, a_t, T_t, g_t) = \pi_t$. The solution implies the term structure of N-years ahead inflation expectations for a given state $\pi_t^{(N)} = \pi_t^{(N)}(i_t, a_t, T_t, g_t)$. Our strategy is to use collocation to approximate the function $\pi_t^{(N)} \approx \Phi(N, i_t, a_t, T_t, g_t)v$. The n-vector v is a vector of coefficients and Φ denotes the known $n \times n$ basis matrix, and can compute the unknown coefficients from a linear interpolation equation

$$\begin{aligned} & \big((\tau_a(a_t - a_{ss}) - (T_t - T_t^*))\Phi_4' + (\varphi_a(a_t - a_{ss}) - (g_t - g_t^*))\Phi_5' \\ & + ((i_t - \pi_t)a_t - s_t)\Phi_3' + (\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))\Phi_2' - \Phi_1'\big)v = 0_n, \end{aligned}$$

using the solution to the FTPL-NK model (14a) to (14e) above.

D Tables and Figures

Table D.1: Parametrization 2 (cf. Kliem, Kriwoluzky, and Sarferaz, 2016)

ρ	0.03	subjective rate of time preference, $\rho = -4 \log 0.9925$
κ	0.4421	degree of price stickiness
y_{ss}	1	normalized steady-state output
ϑ	1	Frisch labor supply elasticity
ϕ_π	0.6	inflation response Taylor rule (fiscal regime)
ϕ_y	0	output response Taylor rule
θ	1	inertia Taylor rule
π_{ss}	0	inflation target rate
τ_y	0	output response fiscal tax rule
τ_a	0.01	debt response fiscal tax rule
ρ_τ	1	inertia of fiscal tax rule
φ_y	0	output response fiscal expenditure rule
φ_a	-0.01	debt response fiscal expenditure rule
ρ_g	1	inertia of fiscal expenditure rule
s_g	0.1534	output share for government expenditures (Bilbiie, Monacelli, and Perotti (2019))
s_{ss}	0.0324	steady-state primary surplus (to match US debt/GDP 2020Q1)
χ	0.03	net coupon payments (Del Negro and Sims, 2015)
$1/\delta$	6.8	average duration of government bonds (Del Negro and Sims, 2015)

Table D.2: Parametrization 3 (similar to Sims, 2011; Cochrane, 2018)

ρ	0.03	subjective rate of time preference
κ	0.4421	degree of price stickiness
y_{ss}	1	normalized steady state output
ϕ_π	0.6	inflation response Taylor rule (fiscal regime)
ϕ_y	0	output response Taylor rule
θ	1	inertia Taylor rule
π_{ss}	0	inflation target rate
τ_y	1	output response fiscal tax rule (Sims, 2011; Cochrane, 2018)
τ_a	0	debt response fiscal tax rule
ρ_τ	1	inertia of fiscal tax rule
φ_y	-0.1534	output response fiscal expenditure rule
φ_a	0	debt response fiscal expenditure rule
ρ_g	1	inertia of fiscal expenditure rule
s_g	0.1534	government consumption to output ratio (Bilbiie, Monacelli, and Perotti, 2019)
s_{ss}	0.0324	steady-state surplus (to match US debt/GDP 2020Q1)
χ	0.03	net coupon payments (Del Negro and Sims, 2015)
$1/\delta$	6.8	average duration of government bonds (Del Negro and Sims, 2015)

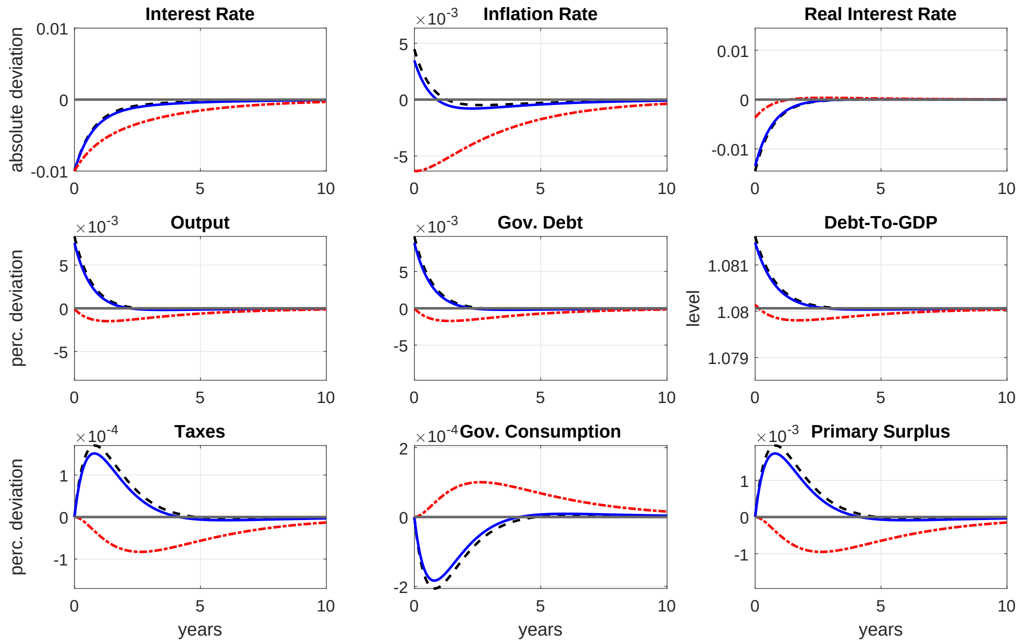

Figure D.1: Transitory monetary policy shock for the parametrization in Table D.1. Decrease in interest rate by 1 percentage point. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

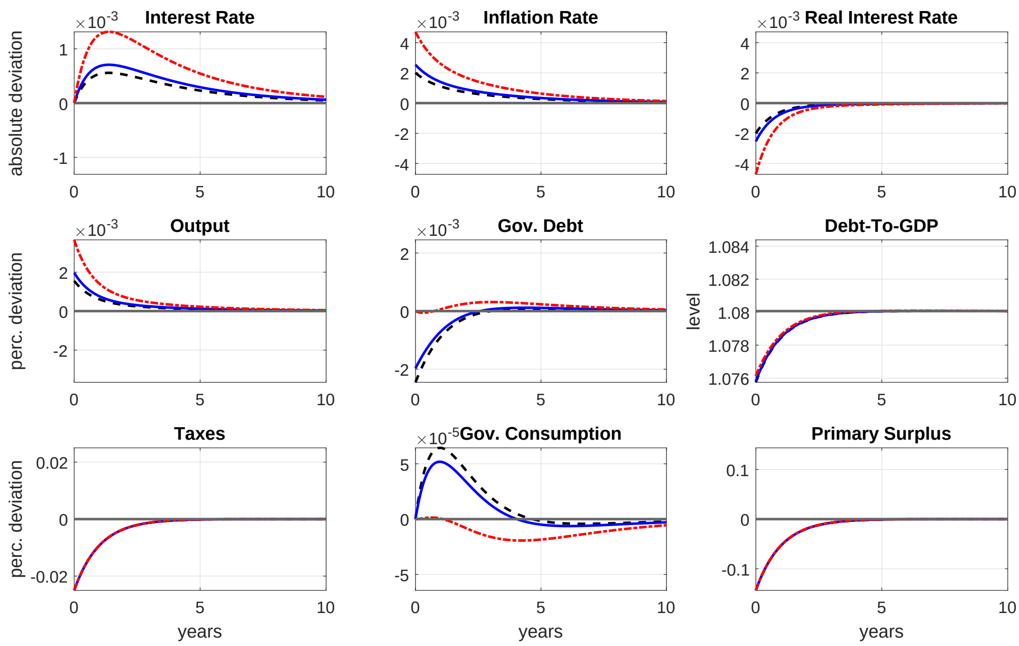

Figure D.2: Transitory fiscal policy shock for the parametrization in Table D.1. Decrease in taxes by 2.5 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

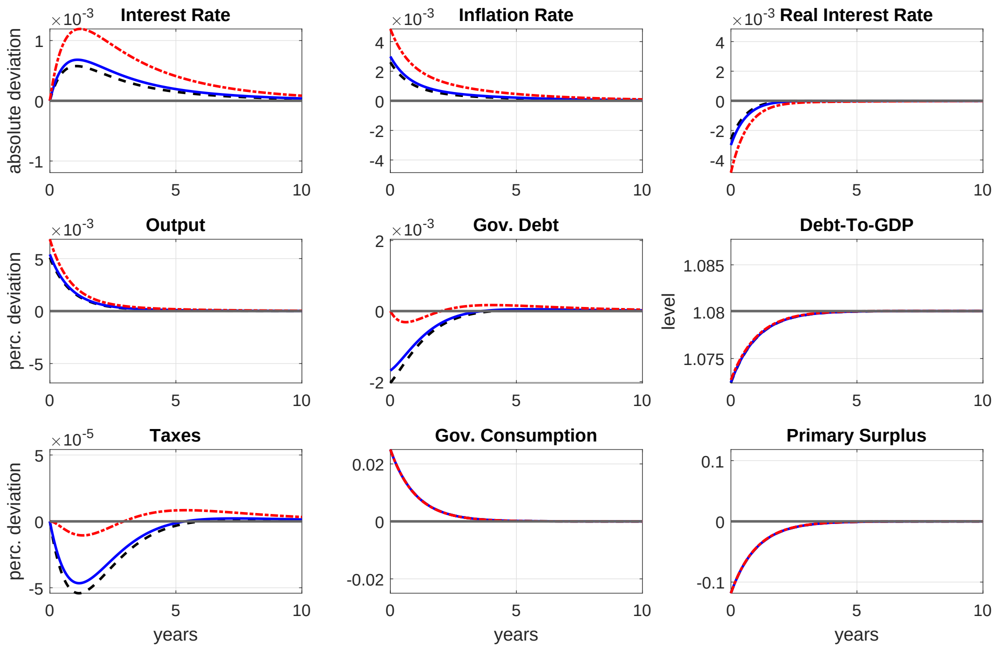

Figure D.3: Transitory fiscal policy shock for the parametrization in Table D.1. Increase in government consumption by 2.5 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

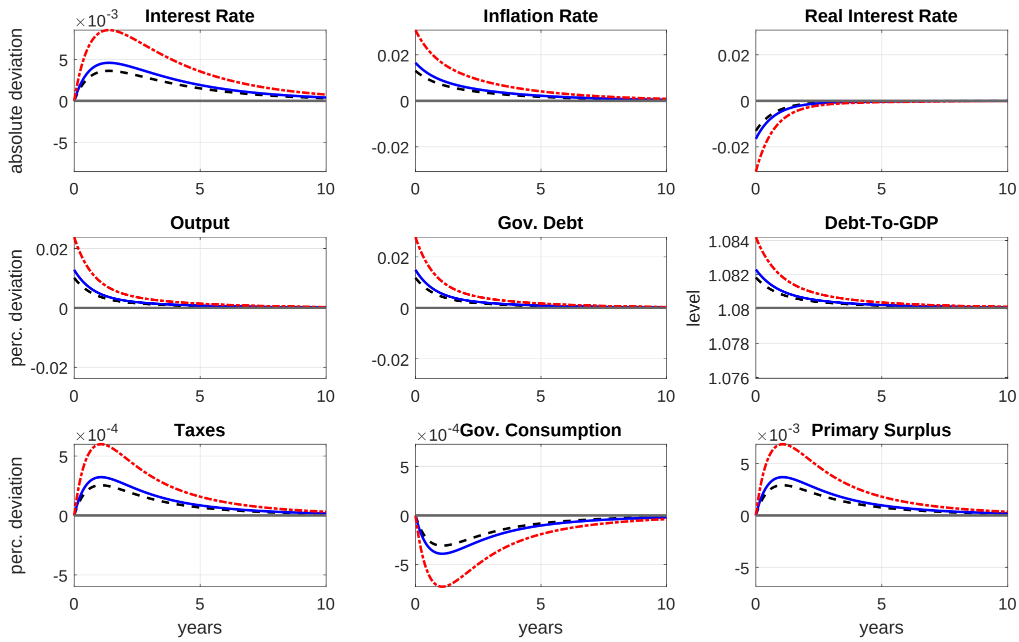

Figure D.4: Transitory fiscal policy shock for the parametrization in Table D.1. Increase in government debt by 3 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

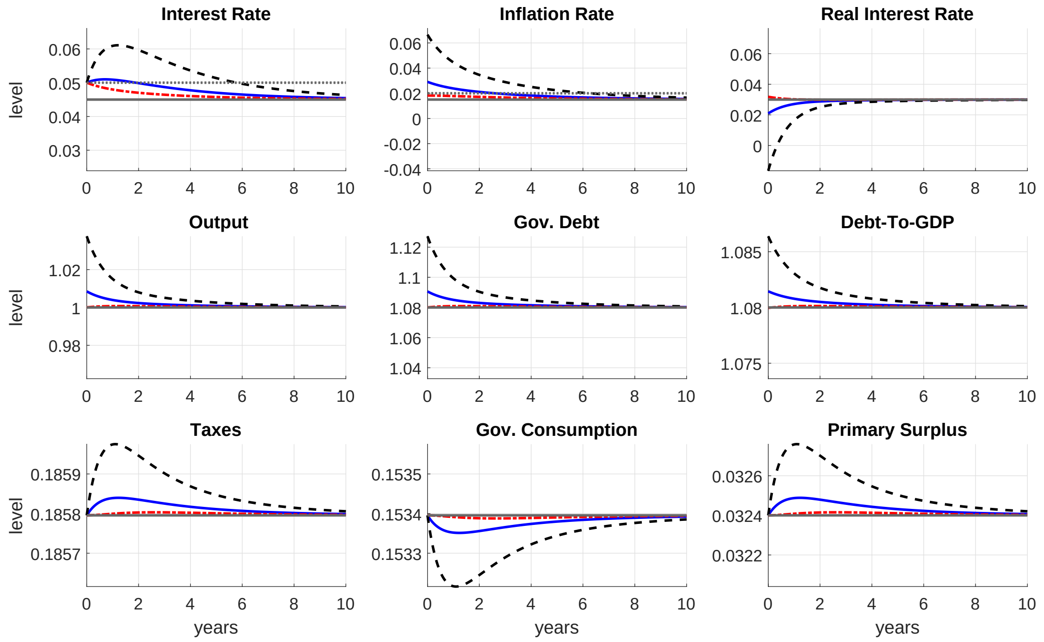

Figure D.5: Permanent monetary policy shock for the parametrization in Table D.1. Decrease $\pi_{ss} = 0.02$ by 50 bp to $\pi_{ss}^{new} = 0.015$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

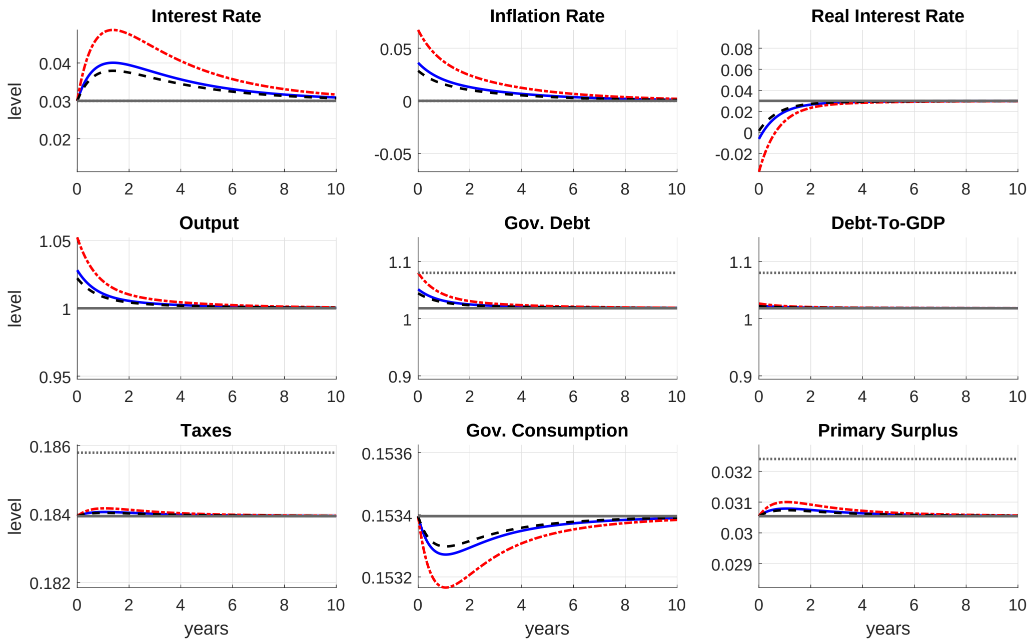

Figure D.6: Permanent fiscal policy shock for parametrization in Table D.1. Permanent decrease of T_{ss} by 1 percent to $T_{ss}^{new} = 0.99T_{ss}$, together with a transitory shock that decreases taxes by 1 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

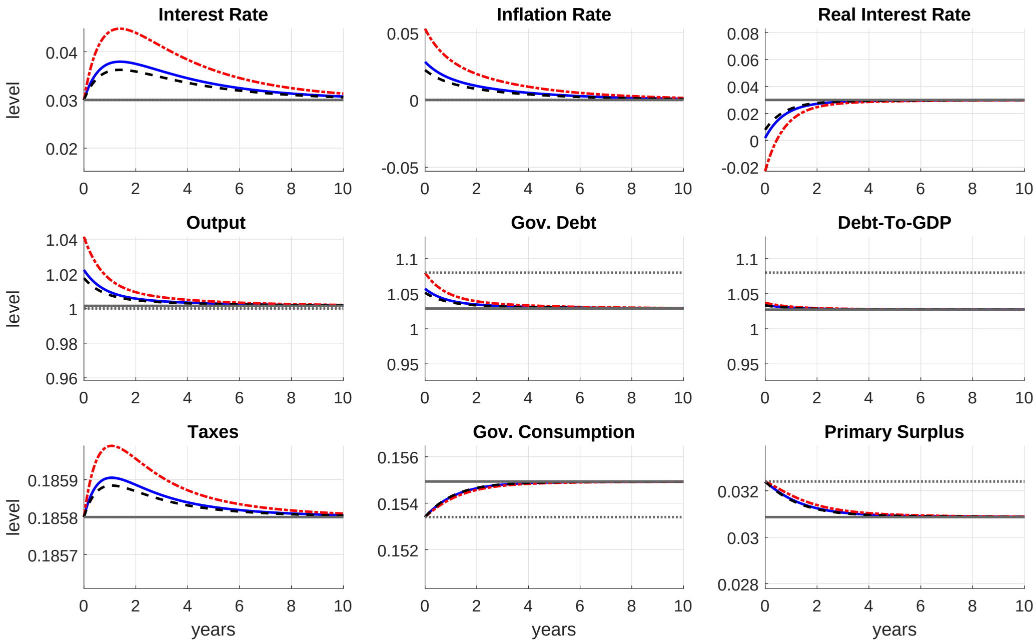

Figure D.7: Permanent fiscal policy shock for the parametrization in Table D.1. Increase of g_{ss} by 1 percent to $g_{ss}^{new} = 1.01 g_{ss}$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

References

BILBIIE, F. O., T. MONACELLI, AND R. PEROTTI (2019): "Is Government Spending at the Zero Lower Bound Desirable?," *American Economic Journal: Macroeconomics*, 11(3), 147–173.

COCHRANE, J. H. (2018): "Stepping on a rake: The fiscal theory of momentary policy," *European Economic Review*, 101, 354–375.

DEL NEGRO, M., AND C. A. SIMS (2015): "When does a central bank's balance sheet require fiscal support?," *Journal of Monetary Economics*, 73, 1–19.

KLIEM, M., A. KRIWOLUZKY, AND S. SARFERAZ (2016): "Monetary–fiscal policy interaction and fiscal inflation: A tale of three countries," *European Economic Review*, 88, 158–184.

POSCH, O. (2020): "Resurrecting the New-Keynesian Model: (Un)conventional Policy and the Taylor rule," *Working Paper*, *https://dx.doi.org/10.2139/ssrn.3130903*.

SIMS, C. A. (2011): "Stepping on a rake: The role of fiscal policy in the inflation of the 1970s," *European Economic Review*, 55(1), 48–56.