

B Online Appendix

to “Peso Problems in the Estimation of the C-CAPM” (by Olaf Posch and Andreas Schrimpf)

B.1 General equilibrium prices in the Endowment economy

We use the stochastic differential for consumption implied by the Euler equation (40) and the market clearing condition $C_t = A_t$ together with the exogenous dividend process (7).

Proposition B.1 (Asset pricing) *In general equilibrium, market clearing implies*

$$\begin{aligned}\mu_M - r &= -\frac{u''(C_t)C_W W_t}{u'(C(W_t))}\sigma_M^2 - \frac{u'(e^{\bar{\nu}}C(W_t))}{u'(C(W_t))}((1 - e^{\kappa})q - \zeta_M)\lambda \\ \sigma_M &= \bar{\sigma}C_t/(C_W W_t) \\ r &= \rho - \frac{u''(C_t)C_t}{u'(C_t)}\bar{\mu} - \frac{1}{2}\frac{u'''(C_t)C_t^2}{u'(C_t)}\bar{\sigma}^2 + \lambda - (1 - (1 - e^{\kappa})q)\frac{u'(e^{\bar{\nu}}C_t)}{u'(C_t)}\lambda.\end{aligned}$$

as well as implicitly the portfolio jump-size

$$C((1 - \zeta_M(t))W_t) = \exp(\bar{\nu})C(W_t).$$

Proof. Using the inverse function, we are able to determine the path for consumption ($u'' \neq 0$). From the Euler equation (40), we obtain

$$\begin{aligned}dC_t &= ((\rho - \mu_M + \lambda)u'(C_t)/u''(C_t) - \sigma_M^2 W_t C_W - \frac{1}{2}u'''(C_t)/u''(C_t)\sigma_M^2 W_t^2 C_W^2 \\ &\quad - E^\zeta [u'(C((1 - \zeta_M(t))W_t))(1 - \zeta_M(t))] \lambda/u''(C_t))dt \\ &\quad + \sigma_M W_t C_W dB_t + (C((1 - \zeta_M(t))W_{t-}) - C(W_{t-}))dN_t,\end{aligned}\tag{67}$$

where we employed the inverse function $c = g(u'(c))$ which has

$$g'(u'(c)) = 1/u''(c), \quad g''(u'(c)) = -u'''(c)/(u''(c))^3.$$

Economically, concave utility ($u'(c) > 0$, $u''(c) < 0$) implies risk aversion, whereas convex marginal utility, $u'''(c) > 0$, implies a positive precautionary saving motive. Accordingly, $-u''(c)/u'(c)$ measures absolute risk aversion, whereas $-u'''(c)/u''(c)$ measures the degree of absolute prudence, i.e., the intensity of the precautionary saving motive.

Because output is perishable, using the market clearing condition $Y_t = C_t = A_t$, and

$$dC_t = \bar{\mu}C_t dt + \bar{\sigma}C_t dB_t + (\exp(\bar{\nu}) - 1)C_{t-} dN_t,$$

the parameters of price dynamics are pinned down in general equilibrium. In particular, we obtain J_t implicitly as function of $\bar{\nu}$, D_t , and the curvature of the consumption function,

where $\tilde{C}(W_t) \equiv C((1 - \zeta_M(t))W_t)/C(W_t)$ defines optimal consumption jumps. For market clearing we require the percentage jump in aggregate consumption to match the size of the disaster, $\exp(\bar{\nu}) = \tilde{C}(W_t)$, and thus $\exp(\bar{\nu}) = C((1 + (J_t - D_t)w_t + D_t)W_t)/C(W_t)$ implies a constant jump size. For consumption being linear homogeneous in wealth,

$$\zeta_M = e^{\bar{\nu}} - 1$$

Similarly, the market clearing condition pins down $\sigma_M W_t C_W = \bar{\sigma} C_t$, and

$$\mu_M - r = -\frac{u''(C_t)C_W W_t}{u'(C(W_t))}\sigma_M^2 - \frac{u'(e^{\bar{\nu}}C(W_t))}{u'(C(W_t))}((1 - e^\kappa)q - \zeta_M)\lambda.$$

Inserting our results back into (67), we obtain that consumption follows,

$$\begin{aligned} dC_t = & (\rho - r + \lambda) \frac{u'(C_t)}{u''(C_t)} dt - \frac{1}{2} \frac{u'''(C_t)}{u''(C_t)} \sigma_M^2 W_t^2 C_W^2 dt - (1 - (1 - e^\kappa)q) \frac{u'(e^{\bar{\nu}}C_t)}{u''(C_t)} \lambda dt \\ & + \sigma_M W_t C_W dB_t + (C((1 - \zeta_M(t))W_{t-}) - C(W_{t-})) dN_t. \end{aligned}$$

This in turn determines the return on the riskless asset

$$r = \rho - \frac{u''(C_t)C_t}{u'(C_t)}\bar{\mu} - \frac{1}{2} \frac{u'''(C_t)C_t^2}{u'(C_t)}\bar{\sigma}^2 + \lambda - (1 - (1 - e^\kappa)q) \frac{u'(e^{\bar{\nu}}C_t)}{u'(C_t)}\lambda.$$

As a result, the higher the subjective rate of time preference, ρ , the higher is the general equilibrium interest rate to induce individuals to defer consumption (cf. Breeden, 1986). For convex marginal utility (decreasing absolute risk aversion), $u'''(c) > 0$, a lower conditional variance of dividend growth, $\bar{\sigma}^2$, and a higher conditional mean of dividend growth, $\bar{\mu}$, and a higher default probability, q , decrease the bond price and increases the interest rate. ■

B.2 Euler equation errors in the production economy

Consider two assets, i.e., the risky bond, R_{t+1}^b , and the claim on capital or output, R_{t+1}^c .

From the definition of Euler equation errors (3), for any asset i and CRRA preferences

$$\begin{aligned} e_{R|\alpha=\gamma}^i &= E_t \left[e^{-\int_t^{t+1} (r_s - \delta) ds + \lambda - e^{(1-\gamma)\nu} \lambda + \gamma \sigma^2 - \frac{1}{2} (\gamma \sigma)^2 - \gamma \sigma (Z_{t+1} - Z_t) - \gamma \nu (N_{t+1} - N_t)} R_{t+1}^i \right] - 1, \\ e_{R|\rho=\bar{\rho}}^i &= E_t \left[e^{-\int_t^{t+1} (r_s - \delta) ds + (1 - e^{(1-\alpha\gamma)\nu}) \lambda + (1 - e^{-\gamma\bar{\nu}}) \bar{\lambda} + \gamma \alpha \sigma^2 - \frac{1}{2} (\gamma \bar{\sigma})^2 - \frac{1}{2} (\alpha \gamma \sigma)^2} \right. \\ &\quad \left. \times e^{-\gamma \bar{\sigma} (B_{t+1} - B_t) - \alpha \gamma \sigma (Z_{t+1} - Z_t) - \alpha \gamma \nu (N_{t+1} - N_t) - \gamma \bar{\nu} (\bar{N}_{t+1} - \bar{N}_t)} R_{t+1}^i \right] - 1, \end{aligned}$$

where we inserted the SDFs from (53) and (54).

Risky assets. Inserting the one-period equilibrium returns for the bond yields

$$\begin{aligned} e_{R|\alpha=\gamma}^b &= E_t \left[e^{(1 - e^{-\gamma\nu}) \lambda - \frac{1}{2} (\gamma \sigma)^2 - \gamma \sigma (Z_{t+1} - Z_t) - \gamma \nu (N_{t+1} - N_t)} \right] - 1, \\ e_{R|\rho=\bar{\rho}}^b &= E_t \left[e^{(1 - e^{-\alpha\gamma\nu}) \lambda + (1 - e^{-\gamma\bar{\nu}}) \bar{\lambda} - \frac{1}{2} (\gamma \bar{\sigma})^2 - \frac{1}{2} (\alpha \gamma \sigma)^2} \right. \\ &\quad \left. \times e^{-\gamma \bar{\sigma} (B_{t+1} - B_t) - \alpha \gamma \sigma (Z_{t+1} - Z_t) - \alpha \gamma \nu (N_{t+1} - N_t) - \gamma \bar{\nu} (\bar{N}_{t+1} - \bar{N}_t)} \right] - 1. \end{aligned}$$

Conditional on no disasters, on average we can rationalize Euler equation errors

$$\begin{aligned} e_{R|N_{t+1}-N_t=0}^b|_{\alpha=\gamma} &= \exp\left((1 - e^{-\gamma\nu})\lambda\right) - 1, \\ e_{R|N_{t+1}-N_t=0}^b|_{\rho=\bar{\rho}} &= \exp\left((1 - e^{-\alpha\gamma\nu})\lambda\right) - 1, \end{aligned}$$

or, conditional on no rare events, on average we can rationalize Euler equation errors

$$\begin{aligned} e_{R|N_{t+1}-N_t=\bar{N}_{t+1}-\bar{N}_t=0}^b|_{\alpha=\gamma} &= \exp\left((1 - e^{-\gamma\nu})\lambda\right) - 1, \\ e_{R|N_{t+1}-N_t=\bar{N}_{t+1}-\bar{N}_t=0}^b|_{\rho=\bar{\rho}} &= \exp\left((1 - e^{-\alpha\gamma\nu})\lambda + (1 - e^{-\gamma\bar{\nu}})\bar{\lambda}\right) - 1. \end{aligned}$$

Similarly, inserting the return on the claims on output (64) and capital (66) yields

$$\begin{aligned} e_R^c|_{\alpha=\gamma} &= E_t \left[e^{-\frac{1}{2}\bar{\sigma}^2 - (e^{\bar{\nu}} - 1)\bar{\lambda} + \bar{\sigma}(B_{t+1} - B_t) + \bar{\nu}(\bar{N}_{t+1} - \bar{N}_t)} \right] - 1, \\ e_R^c|_{\rho=\bar{\rho}} &= E_t \left[e^{-\frac{1}{2}(\gamma\bar{\sigma})^2 - (e^{-\gamma\bar{\nu}} - 1)\bar{\lambda} - \gamma\bar{\sigma}(B_{t+1} - B_t) - \gamma\bar{\nu}(\bar{N}_{t+1} - \bar{N}_t)} \right] - 1. \end{aligned}$$

Note that EE errors based on excess returns are obtained from $e_X^i = e_R^i - e_R^b$ for any asset i .

B.3 Tables and Figures

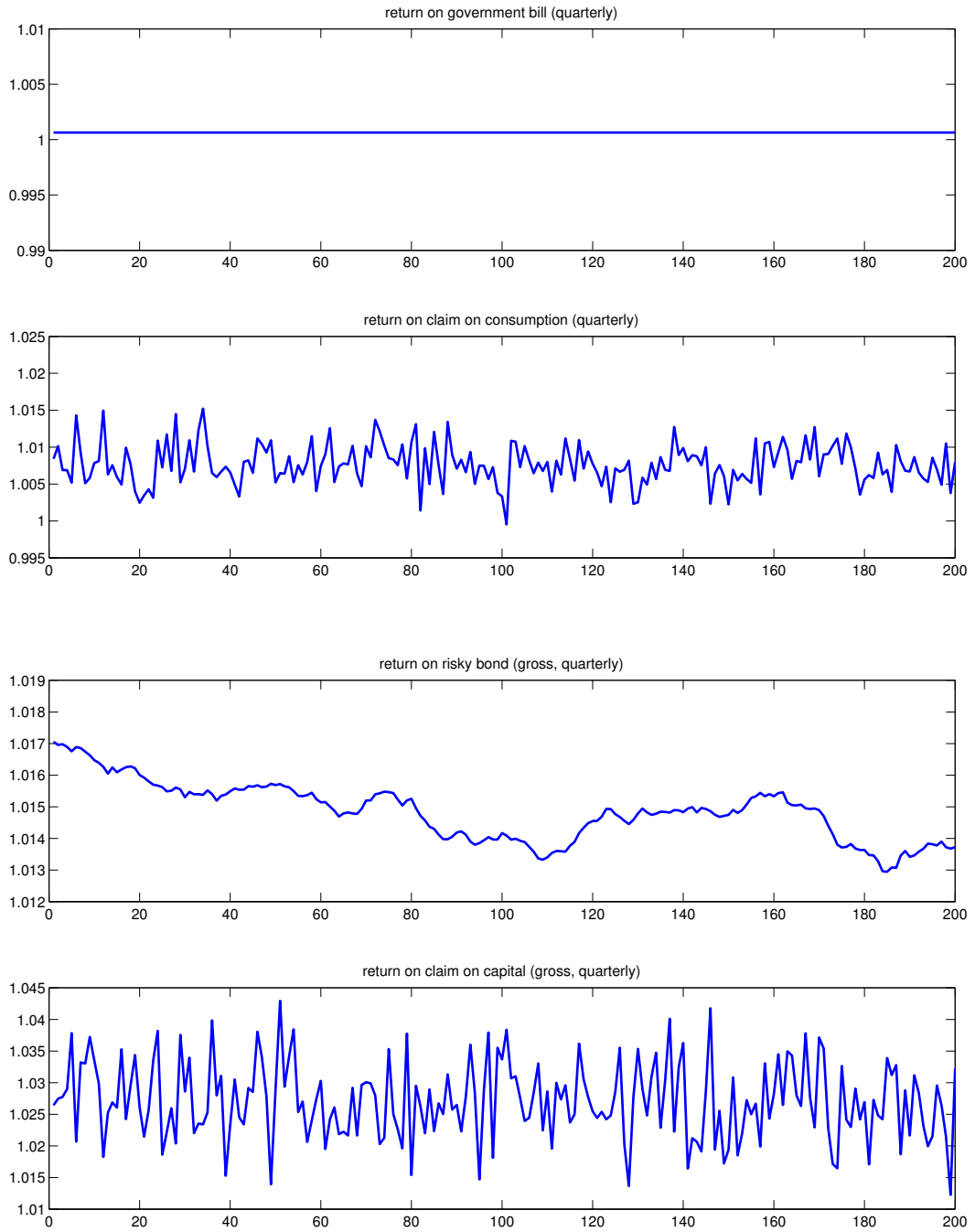
Table B.9: Robustness: Simulation study (endowment economy)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ρ rate of time preference	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
γ coef. of relative risk aversion	0.5	4	4	4	1	2	2	3
$\bar{\mu}$ consumption growth	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
$\bar{\sigma}$ consumption noise	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
$-\bar{\nu}$ size of consumption disaster	0.4	0.4	0.4	0	0.4	0.4	0.55	0.4
λ consumption disaster probability	0.017	0.017	0.017	0	0.017	0.017	0.017	0.017
$-\kappa$ size of government default	0	0	0.3	0	0	0	0	0
q default probability	0	0	0.5	0	0	0	0	0

Table B.10: Robustness: Simulation study (production economy)

	(1)	(2)	(3)	(4)	(5)
ρ rate of time preference	0.03	0.024	0.017	0.016	0.03
γ coef. of relative risk aversion	0.5	4	4	4	4
α output elasticity of capital	0.5	0.6	0.6	0.6	0.6
δ capital depreciation	0.025	0.025	0.025	0.025	0.05
$\bar{\mu}$ productivity growth	0.02	0.01	0.01	0.01	0.01
$\bar{\sigma}$ productivity noise	0.01	0.01	0.01	0.01	0.01
$-\bar{\nu}$ size of productivity slump	0.01	0.01	0.01	0	0
$\bar{\lambda}$ productivity jump probability	0.2	0.2	0.2	0	0
σ capital stochastic depreciation	0.005	0.005	0.005	0.005	0.005
$-\nu$ size of capital disaster	0.55	0.55	0.4	0.55	0
λ capital disaster probability	0.017	0.017	0.017	0.017	0

Figure B.1: General equilibrium asset returns



Notes: This figure illustrates the equilibrium asset returns and shows one realization of the return to the bonds and the risky assets in the endowment economy (upper two panels, parameterization (2) in Table B.9) and the production economy (lower two panels, parameterization (2) in Table B.10), respectively.