Resurrecting the New-Keynesian Model: (Un)conventional Policy and the Taylor rule

Online Appendix

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C.1. Technical details

$\mathrm{d}a_t$ *on p.16:* The household can trade on Arrow securities (excluded to save on notation) and on a nominal government bonds b_t at a nominal interest rate of i_t. Let n_t denote the number of shares and p_t^b the equilibrium price of bonds. Suppose the household earns a disposable income of $i_t b_t + p_t w_t l_t + p_t T_t + p_t \digamma_t$, where p_t is the price level (or price of the consumption good), w_t is the real wage, T_t is a lump-sum transfer, and $\digamma_t$ are the profits of the firms in the economy; the household's budget constraint is:

$$\mathrm{d}n_t = \frac{i_t b_t - p_t c_t + p_t w_t l_t + p_t T_t + p_t \digamma_t}{p_t^b}\mathrm{d}t. \tag{C.1}$$

Let bond prices follow:

$$\mathrm{d}p_t^b = \alpha_t p_t^b \mathrm{d}t \tag{C.2}$$

in which α_t denotes a price change, which is determined in general equilibrium (in equilibrium prices are function of the state variables, for example, by fixing α_t the bond supply has to accommodate so as to permit the bond's nominal interest rate being admissible). The household's financial wealth, $b_t = n_t p_t^b$, is then given by:

$$\mathrm{d}b_t = (i_t b_t - p_t c_t + p_t w_t l_t + p_t T_t + p_t \digamma_t)\mathrm{d}t + \alpha_t b_t \mathrm{d}t, \tag{C.3}$$

Let prices p_t follow the process:

$$\mathrm{d}p_t = \pi_t p_t \mathrm{d}t \tag{C.4}$$

such that the (realized) rate of inflation is locally non-stochastic. We can interpret $\mathrm{d}p_t/p_t$ as the realized inflation over the period $[t, t+\mathrm{d}t]$ and π_t as the inflation rate.

Letting $a_t \equiv b_t/p_t$ denote real financial wealth and using Itô's formula, the household's real wealth evolves according to:

$$\mathrm{d}a_t = \frac{\mathrm{d}b_t}{p_t} - \frac{b_t}{p_t^2}\mathrm{d}p_t = \frac{i_t b_t - p_t c_t + p_t w_t l_t + p_t T_t + p_t \digamma_t + \alpha_t b_t}{p_t}\mathrm{d}t - \frac{b_t}{p_t^2}\pi_t p_t \mathrm{d}t$$

or:

$$\mathrm{d}a_t = ((i_t + \alpha_t - \pi_t)a_t - c_t + w_t l_t + T_t + \digamma_t)\,\mathrm{d}t \tag{C.5}$$

Since government bonds are in net zero supply, $b_t = 0$, it implies $\alpha_t = 0$ for all t. ■

$\#\,\mathrm{d}x_{1,t}$ *on p.18:* Differentiating $x_{1,t}$ in (18) with respect to time gives:

$$\begin{aligned}
\frac{1}{\mathrm{d}t}\,\mathrm{d}x_{1,t} &= -\lambda_t y_t + (\rho+\delta)x_{1,t} \\
&\quad (1-\varepsilon)\pi_t \mathbb{E}_t \int_t^\infty \lambda_\tau e^{-(\rho+\delta)(\tau-t)} \left(\frac{p_t}{p_\tau}\right)^{1-\varepsilon} e^{\int_t^\tau (1-\varepsilon)\chi\pi_s^*\,\mathrm{d}s} y_\tau\,\mathrm{d}\tau \\
&\quad + \mathbb{E}_t \int_t^\infty \lambda_\tau e^{-(\rho+\delta)(\tau-t)} \left(\frac{p_t}{p_\tau}\right)^{1-\varepsilon} \frac{\partial\left[e^{\int_t^\tau (1-\varepsilon)\chi\pi_s^*\,\mathrm{d}s}\right]}{\partial t} y_\tau\,\mathrm{d}\tau \\
&= -\lambda_t y_t + (\rho+\delta+(1-\varepsilon)\pi_t)x_{1,t} \\
&\quad + \mathbb{E}_t \int_t^\infty \lambda_\tau e^{-(\rho+\delta)(\tau-t)} \left(\frac{p_t}{p_\tau}\right)^{1-\varepsilon} \frac{\partial\left[e^{\int_t^\tau (1-\varepsilon)\chi\pi_s^*\,\mathrm{d}s}\right]}{\partial t} y_\tau\,\mathrm{d}\tau \\
&= -\lambda_t y_t + (\rho+\delta+(1-\varepsilon)\pi_t)x_{1,t} \\
&\quad + \mathbb{E}_t \int_t^\infty \lambda_\tau e^{-(\rho+\delta)(\tau-t)} \left(\frac{p_t}{p_\tau}\right)^{1-\varepsilon} e^{\int_t^\tau (1-\varepsilon)\chi\pi_s^*\,\mathrm{d}s} \frac{\partial\left[\int_t^\tau (1-\varepsilon)\chi\pi_s^*\,\mathrm{d}s\right]}{\partial t} y_\tau\,\mathrm{d}\tau \\
&= -\lambda_t y_t + (\rho+\delta+(1-\varepsilon)(\pi_t-\chi\pi_t^*))x_{1,t} \\
&\quad + \mathbb{E}_t \int_t^\infty \lambda_\tau e^{-(\rho+\delta)(\tau-t)} \left(\frac{p_t}{p_\tau}\right)^{1-\varepsilon} e^{\int_t^\tau (1-\varepsilon)\chi\pi_s^*\,\mathrm{d}s} (1-\varepsilon)\chi \int_t^\tau \frac{\partial\pi_s^*}{\partial t}\,\mathrm{d}s y_\tau\,\mathrm{d}\tau
\end{aligned}$$

or (20) in the main text. A similar procedure gives (21). ■
$\#\,\mathrm{d}\pi_t$ *on p.19:* Differentiating (22), we obtain the inflation dynamics as:

$$\begin{aligned}
\mathrm{d}(\pi_t - \chi\pi_t^*) &= \delta\left(\Pi_t^*\right)^{-\varepsilon} \mathrm{d}\Pi_t^* \\
&= \delta\left(\Pi_t^*\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1}\left(1/x_{1,t}\,\mathrm{d}x_{2,t} - x_{2,t}/x_{1,t}^2\,\mathrm{d}x_{1,t}\right) \\
&= \delta\left(\Pi_t^*\right)^{1-\varepsilon}\left(1/x_{2,t}\,\mathrm{d}x_{2,t} - 1/x_{1,t}\,\mathrm{d}x_{1,t}\right) \\
&= -\delta\left(\Pi_t^*\right)^{1-\varepsilon}\left(\pi_t - \chi\pi_t^* + (mc_t/x_{2,t} - 1/x_{1,t})\lambda_t y_t\right)\,\mathrm{d}t \\
&= -(\delta + (1-\varepsilon)(\pi_t-\chi\pi_t^*))\left(\pi_t - \chi\pi_t^* + (mc_t/x_{2,t} - 1/x_{1,t})\lambda_t y_t\right)\,\mathrm{d}t
\end{aligned}$$

which is (23) in the main text. ■ $\#\,\mathrm{d}v_t$ *on p.21:* Differentiating (28), we get:

$$\begin{aligned}
\frac{1}{\mathrm{d}t}\mathrm{d}v_t &= \delta\left(\Pi_t^*\right)^{-\varepsilon} + \delta\int_{-\infty}^t \frac{1}{\mathrm{d}t}\mathrm{d}e^{-\delta(t-\tau)-\varepsilon\int_\tau^t \chi\pi_s^*\,\mathrm{d}s}\left(\frac{p_{i\tau}}{p_t}\right)^{-\varepsilon}\mathrm{d}\tau \\
&= \delta\left(\Pi_t^*\right)^{-\varepsilon} - (\delta+\varepsilon\chi\pi_t^*)\int_{-\infty}^t \delta e^{-\delta(t-\tau)-\varepsilon\int_\tau^t \chi\pi_s^*\,\mathrm{d}s}\left(\frac{p_{i\tau}}{p_t}\right)^{-\varepsilon}\mathrm{d}\tau \\
&\quad + \int_{-\infty}^t \delta e^{-\delta(t-\tau)-\varepsilon\int_\tau^t \chi\pi_s^*\,\mathrm{d}s} p_{i\tau}^{-\varepsilon}\varepsilon p_t^{\varepsilon-1}\frac{1}{\mathrm{d}t}\mathrm{d}p_t\mathrm{d}\tau \\
&= \delta\left(\Pi_t^*\right)^{-\varepsilon} + \left(\varepsilon(\pi_t-\chi\pi_t^*)-\delta\right)v_t. \qquad \text{(C.6)}
\end{aligned}$$

which is (29) in the main text. ■

#$\digamma_t$ on p.21: For aggregate profits, we use the demand of intermediate producers in (27):

$$\begin{aligned}
\digamma_t &= \int_0^1 \left(\frac{p_{it}}{p_t} - mc_t\right) y_{it} \mathrm{d}i \\
&= y_t \int_0^1 \left(\frac{p_{it}}{p_t} - mc_t\right) \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} \mathrm{d}i \\
&= \left(\int_0^1 \left(\frac{p_{it}}{p_t}\right)^{1-\varepsilon} \mathrm{d}i - mc_t v_t\right) y_t \\
&= (1 - mc_t v_t) y_t
\end{aligned}$$

which is (30) in the main text. ■

#$V(\mathbb{Z}_t, \mathbb{X}_t)$ on p.22: The HJB equation (32) in scalar notation reads

$$\begin{aligned}
\rho V(\mathbb{Z}_t; \mathbb{Y}_t) = & \max_{(c_t, l_t)} d_t \left\{\log c_t - \psi \frac{l_t^{1+\vartheta}}{1+\vartheta}\right\} \\
& + ((i_t - \pi_t) a_t - c_t + w_t l_t + T_t + \digamma_t) V_a \\
& + (\theta\phi_\pi(\pi_t - \pi_t^*) + \theta\phi_y(y_t/y_{ss} - 1) - \theta(i_t - i_t^*)) V_i + \tfrac{1}{2}\sigma_i^2 V_{ii} \\
& + \left(\delta \left(\Pi_t^*\right)^{-\varepsilon} + (\varepsilon(\pi_t - \chi\pi_t^*) - \delta) v_t\right) V_v \\
& - (\rho_d \log d_t - \tfrac{1}{2}\sigma_d^2) d_t V_d + \tfrac{1}{2}\sigma_d^2 d_t^2 V_{dd} \\
& - (\rho_A \log A_t - \tfrac{1}{2}\sigma_A^2) A_t V_A + \tfrac{1}{2}\sigma_A^2 A_t^2 V_{AA} \\
& - (\rho_g \log s_{g,t} - \tfrac{1}{2}\sigma_g^2) s_{g,t} V_g + \tfrac{1}{2}\sigma_g^2 s_{g,t}^2 V_{gg}. \qquad \text{(C.7)}
\end{aligned}$$

■

#$\mathrm{d}V_a(\mathbb{Z}_t, \mathbb{X}_t)$ on p.24: From C.7, the concentrated HJB equation in scalar notation reads

$$\begin{aligned}
\rho V(\mathbb{Z}_t; \mathbb{Y}_t) = & d_t \log c(\mathbb{Z}_t; \mathbb{Y}_t) - d_t \psi \frac{l(\mathbb{Z}_t; \mathbb{Y}_t)^{1+\vartheta}}{1+\vartheta} \\
& + ((i_t - \pi_t) a_t - c(\mathbb{Z}_t; \mathbb{Y}_t) + w_t l(\mathbb{Z}_t; \mathbb{Y}_t) + T_t + \digamma_t) V_a \\
& + (\theta\phi_\pi(\pi_t - \pi_t^*) + \theta\phi_y(y_t/y_{ss} - 1) - \theta(i_t - i_t^*)) V_i + \tfrac{1}{2}\sigma_i^2 V_{ii} \\
& + \left(\delta \left(\Pi_t^*\right)^{-\varepsilon} + (\varepsilon(\pi_t - \chi\pi_t^*) - \delta) v_t\right) V_v \\
& - (\rho_d \log d_t - \tfrac{1}{2}\sigma_d^2) d_t V_d + \tfrac{1}{2}\sigma_d^2 d_t^2 V_{dd} \\
& - (\rho_A \log A_t - \tfrac{1}{2}\sigma_A^2) A_t V_A + \tfrac{1}{2}\sigma_A^2 A_t^2 V_{AA} \\
& - (\rho_g \log s_{g,t} - \tfrac{1}{2}\sigma_g^2) s_{g,t} V_g + \tfrac{1}{2}\sigma_g^2 s_{g,t}^2 V_{gg}. \qquad \text{(C.8)}
\end{aligned}$$

Using the envelope theorem, we obtain the costate variable V_a as:

$$\begin{aligned}
\rho V_a &= (i_t - \pi_t)V_a + ((i_t - \pi_t)a_t - c_t + w_t l_t + T_t + F_t)\, V_{aa} \\
&+(\theta\phi_\pi(\pi_t - \pi_t^*) + \theta\phi_y(y_t/y_{ss} - 1) - \theta(i_t - i_t^*))V_{ia} + \tfrac{1}{2}\sigma_i^2 V_{iia} \\
&+ \left(\delta\,(\Pi_t^*)^{-\varepsilon} + (\varepsilon(\pi_t - \chi\pi_t^*) - \delta)\, v_t\right) V_{va} \\
&-(\rho_d \log d_t - \tfrac{1}{2}\sigma_d^2)d_t V_{da} + \tfrac{1}{2}\sigma_d^2 d_t^2 V_{dda} \\
&-(\rho_A \log A_t - \tfrac{1}{2}\sigma_A^2)A_t V_{Aa} + \tfrac{1}{2}\sigma_A^2 A_t^2 V_{AAa} \\
&-(\rho_g \log s_{g,t} - \tfrac{1}{2}\sigma_g^2)s_{g,t} V_{ga} + \tfrac{1}{2}\sigma_g^2 s_{g,t}^2 V_{gga}. \qquad \text{(C.9)}
\end{aligned}$$

An alternative formulation in terms of differentials is:

$$\begin{aligned}
(\rho - i_t + \pi_t)\, V_a \mathrm{d}t &= V_{aa}\mathrm{d}a_t + (\mathrm{d}i_t - \sigma_i \mathrm{d}B_{i,t})V_{ia} + \tfrac{1}{2}\sigma_i^2 V_{iia} + V_{va}\mathrm{d}v_t \\
&+ (\mathrm{d}d_t - \sigma_d d_t \mathrm{d}B_{d,t})\, V_{da} + \tfrac{1}{2}\sigma_d^2 d_t^2 V_{dda}\mathrm{d}t \\
&+ (\mathrm{d}A_t - \sigma_A A_t \mathrm{d}B_{A,t})\, V_{Aa} + \tfrac{1}{2}\sigma_A^2 A_t^2 V_{AAa}\mathrm{d}t + (\mathrm{d}s_{g,t} - \sigma_g s_{g,t}\mathrm{d}B_{g,t})\, V_{ga} + \tfrac{1}{2}\sigma_g^2 s_{g,t}^2 V_{gga}\mathrm{d}t
\end{aligned}$$

or

$$\begin{aligned}
&(\rho - i_t + \pi_t)\, V_a \mathrm{d}t + \sigma_d d_t V_{da}\mathrm{d}B_{d,t} + \sigma_A A_t V_{Aa}\mathrm{d}B_{A,t} + \sigma_g s_{g,t} V_{ga}\mathrm{d}B_{g,t} + \sigma_i i_t V_{ia}\mathrm{d}B_{i,t} \\
&= V_{aa}\mathrm{d}a_t + V_{ia}\mathrm{d}i_t + \tfrac{1}{2}\sigma_i^2 i_t^2 V_{iia} + V_{va}\mathrm{d}v_t \\
&\quad + V_{da}\mathrm{d}d_t + \tfrac{1}{2}\sigma_d^2 d_t^2 V_{da}\mathrm{d}t + V_{Aa}\mathrm{d}A_t + \tfrac{1}{2}\sigma_A^2 A_t^2 V_{Aa}\mathrm{d}t + V_{ga}\mathrm{d}s_{g,t} + \tfrac{1}{2}\sigma_g^2 s_{g,t}^2 V_{ga}\mathrm{d}t.
\end{aligned}$$

Observe that the costate variable in general evolves according to:

$$\begin{aligned}
\mathrm{d}V_a &= V_{aa}\mathrm{d}a_t + V_{ia}\mathrm{d}i_t + \tfrac{1}{2}\sigma_i^2 V_{iia}\mathrm{d}t + V_{va}\mathrm{d}v_t \\
&\quad + V_{da}\mathrm{d}d_t + \tfrac{1}{2}\sigma_d^2 d_t^2 V_{dda}\mathrm{d}t + V_{Aa}\mathrm{d}A_t + \tfrac{1}{2}\sigma_A^2 A_t^2 V_{AAa}\mathrm{d}t + V_{ga}\mathrm{d}s_{gt} + \tfrac{1}{2}\sigma_g^2 s_{g,t}^2 V_{gga}\mathrm{d}t \\
&= (\rho - i_t + \pi_t)V_a \mathrm{d}t \\
&\quad + \sigma_d d_t V_{da}\mathrm{d}B_{d,t} + \sigma_A A_t V_{Aa}\mathrm{d}B_{A,t} + \sigma_g s_{g,t} V_{ga}\mathrm{d}B_{g,t} + \sigma_i V_{ia}\mathrm{d}B_{i,t},
\end{aligned}$$

which is (38) in the main text. ■

 # m_s/m_t (SDF) on p.24: Starting from (38):

$$\begin{aligned}
\mathrm{d}\ln V_a &= \frac{1}{V_a}\mathrm{d}V_a - \tfrac{1}{2}\sigma_d^2 d_t^2 \frac{V_{da}^2}{V_a^2}\mathrm{d}t - \tfrac{1}{2}\sigma_A^2 A_t^2 \frac{V_{Aa}^2}{V_a^2}\mathrm{d}t - \tfrac{1}{2}\sigma_g^2 s_{g,t}^2 \frac{V_{ga}^2}{V_a^2}\mathrm{d}t - \tfrac{1}{2}\sigma_m^2 \frac{V_{ia}^2}{V_a^2}\mathrm{d}t \\
&= (\rho - i_t + \pi_t)\mathrm{d}t + \sigma_d d_t \frac{V_{da}}{V_a}\mathrm{d}B_{d,t} + \sigma_A A_t \frac{V_{Aa}}{V_a}\mathrm{d}B_{A,t} + \sigma_g s_{g,t} \frac{V_{ga}}{V_a}\mathrm{d}B_{g,t} \\
&\quad + \sigma_i \frac{V_{ia}}{V_a}\mathrm{d}B_{i,t} - \tfrac{1}{2}\sigma_d^2 d_t^2 \frac{V_{da}^2}{V_a^2}\mathrm{d}t - \tfrac{1}{2}\sigma_A^2 A_t^2 \frac{V_{Aa}^2}{V_a^2}\mathrm{d}t - \tfrac{1}{2}\sigma_g^2 s_{g,t}^2 \frac{V_{ga}^2}{V_a^2}\mathrm{d}t - \tfrac{1}{2}\sigma_i^2 \frac{V_{ia}^2}{V_a^2}\mathrm{d}t.
\end{aligned}$$

For $s > t$, we may write:

$$e^{-\rho(s-t)}\frac{V_a(\mathbb{Z}_s;\mathbb{Y}_s)}{V_a(\mathbb{Z}_t;\mathbb{Y}_t)} =$$

$$\exp\left(\begin{array}{c} -\int_t^s (i_u-\pi_u)\mathrm{d}u - \frac{1}{2}\int_t^s \frac{V_{da}^2}{V_a^2}\sigma_d^2 d_u^2 \mathrm{d}u - \frac{1}{2}\int_t^s \frac{V_{Aa}^2}{V_a^2}\sigma_A^2 A_u^2 \mathrm{d}u \\ -\frac{1}{2}\int_t^s \frac{V_{ga}^2}{V_a^2}\sigma_g^2 s_{g,u}^2 \mathrm{d}u - \frac{1}{2}\int_t^s \frac{V_{ia}^2}{V_a^2}\sigma_i^2 \mathrm{d}u \\ +\int_t^s \frac{V_{da}}{V_a}\sigma_d d_u \mathrm{d}B_{d,u} + \int_t^s \frac{V_{Aa}}{V_a}\sigma_A A_u \mathrm{d}B_{A,u} + \int_t^s \frac{V_{ga}}{V_a}\sigma_g s_{g,u} \mathrm{d}B_{g,u} + \int_t^s \frac{V_{ia}}{V_a}\sigma_i \mathrm{d}B_{i,u} \end{array}\right).$$

which denotes the equilibrium SDF m_s/m_t in (39). ■

#PDE approach on p.33: Using Itô's lemma:

$$\begin{aligned} dP_t^{(N)} \;=\; & \theta(\phi_\pi(\pi_t-\pi_t^*)+\phi_y(y_t/y_{ss}-1)-(i_t-i_t^*))(\partial P_t^{(N)}/\partial i_t)\,\mathrm{d}t + \tfrac{1}{2}\sigma_i^2(\partial^2 P_t^{(N)}/(\partial i_t)^2)\,\mathrm{d}t \\ & +(\delta(1-(\varepsilon-1)(\pi_t-\chi\pi_t^*)/\delta)^{-\frac{\varepsilon}{1-\varepsilon}} + (\varepsilon(\pi_t-\chi\pi_t^*)-\delta)v_t)(\partial P_t^{(N)}/\partial v_t)\,\mathrm{d}t \\ & -\left(\rho_d\log d_t - \tfrac{1}{2}\sigma_d^2\right) d_t(\partial P_t^{(N)}/\partial d_t)\,\mathrm{d}t + \tfrac{1}{2}\sigma_d^2 d_t^2(\partial^2 P_t^{(N)}/(\partial d_t)^2)\,\mathrm{d}t \\ & -\left(\rho_A\log A_t - \tfrac{1}{2}\sigma_A^2\right) A_t(\partial P_t^{(N)}/\partial A_t)\,\mathrm{d}t + \tfrac{1}{2}\sigma_A^2 A_t^2(\partial^2 P_t^{(N)}/(\partial A_t)^2)\,\mathrm{d}t \\ & -\left(\rho_g\log s_{g,t} - \tfrac{1}{2}\sigma_g^2\right) s_{g,t}(\partial P_t^{(N)}/\partial s_{g,t})\,\mathrm{d}t + \tfrac{1}{2}\sigma_g^2 s_{g,t}^2(\partial^2 P_t^{(N)}/(\partial s_{g,t})^2)\,\mathrm{d}t \\ & +(\partial P_t^{(N)}/\partial i_t)\sigma_i dB_{i,t} + (\partial P_t^{(N)}/\partial d_t)\sigma_d d_t\,\mathrm{d}B_{d,t} + (\partial P_t^{(N)}/\partial A_t)\sigma_A A_t\,\mathrm{d}B_{A,t} \\ & +(\partial P_t^{(N)}/\partial s_{g,t})\sigma_g s_{g,t}\,\mathrm{d}B_{g,t}, \end{aligned}$$

where the relevant equations are

$$\begin{aligned} \mathrm{d}\lambda_t &= (\rho - i_t + \pi_t)\lambda_t \mathrm{d}t \\ &\quad +\sigma_d d_t \lambda_d \mathrm{d}B_{d,t} + \sigma_A A_t \lambda_A \mathrm{d}B_{A,t} + \sigma_g s_{g,t}\lambda_g \mathrm{d}B_{g,t} + \sigma_i \lambda_i \mathrm{d}B_{i,t} \\ \mathrm{d}x_{1,t} &= ((\rho+\delta-(\varepsilon-1)(\pi_t-\chi\pi_t^*))x_{1,t} - d_t/(1-s_g s_{g,t}))\,\mathrm{d}t \\ \mathrm{d}x_{2,t} &= ((\rho+\delta-\varepsilon(\pi_t-\chi\pi_t^*))\,x_{2,t} - mc_t d_t/(1-s_g s_{g,t}))\,\mathrm{d}t \\ \mathrm{d}i_t &= \theta(\phi_\pi(\pi_t-\pi_t^*)+\phi_y(y_t/y_{ss}-1)-(i_t-i_t^*))\mathrm{d}t + \sigma_i \mathrm{d}B_{i,t} \\ \mathrm{d}v_t &= (\delta(1-(\varepsilon-1)(\pi_t-\chi\pi_t^*)/\delta)^{-\frac{\varepsilon}{1-\varepsilon}} + (\varepsilon(\pi_t-\chi\pi_t^*)-\delta)v_t)\mathrm{d}t \\ \mathrm{d}d_t &= -\left(\rho_d\log d_t - \tfrac{1}{2}\sigma_d^2\right) d_t\mathrm{d}t + \sigma_d d_t \mathrm{d}B_{d,t} \\ \mathrm{d}A_t &= -(\rho_A\log A_t - \tfrac{1}{2}\sigma_A^2)A_t\,\mathrm{d}t + \sigma_A A_t \mathrm{d}B_{A,t} \\ \mathrm{d}s_{g,t} &= -(\rho_g\log s_{g,t} - \tfrac{1}{2}\sigma_g^2)s_{g,t}\,\mathrm{d}t + \sigma_g s_{g,t}\mathrm{d}B_{g,t}. \end{aligned}$$

Plugging into the pricing equation and eliminate time, we obtain the PDE for the risk-free bond with $\lambda_i = -\tilde{c}_i\lambda_t$, $\lambda_g = -\tilde{c}_g\lambda_t/s_{g,t}$, $\lambda_A = -\tilde{c}_A\lambda_t/A_t$, and $\lambda_d = (1-\tilde{c}_d)\lambda_t/d_t$. ■

C.2. Obtaining the Euler equation

Using the first-order condition (33) and (38), we obtain the implicit Euler equation:

$$\mathrm{d}\left(\frac{d_t}{c_t}\right) = (\rho - i_t + \pi_t)\left(\frac{d_t}{c_t}\right)\mathrm{d}t$$
$$+\sigma_d d_t\left(\frac{1}{c_t} - \frac{d_t}{c_t^2}c_d\right)\mathrm{d}B_{d,t} - \sigma_A A_t \frac{d_t}{c_t^2}c_A \mathrm{d}B_{A,t} - \sigma_g s_{g,t}\frac{d_t}{c_t^2}c_g \mathrm{d}B_{g,t} - \sigma_m \frac{d_t}{c_t^2}c_i \mathrm{d}B_{i,t}.$$

$V_{ad} = -\left(d_t/c_t^2\right)c_d + 1/c_t$, $V_{Aa} = -\left(d_t/c_t^2\right)c_A$, $V_{ga} = -\left(d_t/c_t^2\right)c_g$, and $V_{ia} = -\left(d_t/c_t^2\right)c_i$ are expressed in terms of derivatives and levels of the consumption function. This equation has a simple interpretation: the change in the marginal utility of consumption depends on the rate of time preference minus the effective real interest rate and four additional terms that control for the innovations to the four shocks to the economy.

Hence, by applying Itô's formula we obtain the Euler equation:

$$\mathrm{d}\left(\frac{c_t}{d_t}\right) = -\left(\frac{d_t}{c_t}\right)^{-2}\left[(\rho - i_t + \pi_t)\left(\frac{d_t}{c_t}\right)\mathrm{d}t\right.$$
$$\left.+\sigma_d\left(\frac{d_t}{c_t} - \frac{d_t^2}{c_t^2}c_d\right)\mathrm{d}B_{d,t} - \sigma_A A_t\frac{d_t}{c_t^2}c_A\mathrm{d}B_{A,t} - \sigma_g s_{g,t}\frac{d_t}{c_t^2}c_g\mathrm{d}B_{g,t} - \sigma_m\frac{d_t}{c_t^2}c_i\mathrm{d}B_{i,t}\right]$$
$$+\left(\frac{d_t}{c_t}\right)^{-3}\left(\sigma_d^2\left(\frac{d_t^2}{c_t^2} - 2\frac{d_t^3}{c_t^3}c_d + \frac{d_t^4}{c_t^4}c_d^2\right) + \sigma_A^2 A_t^2\frac{d_t^2}{c_t^4}c_A^2 + \sigma_g^2 s_{g,t}^2\frac{d_t^2}{c_t^4}c_g^2 + \sigma_i^2\frac{d_t^2}{c_t^4}c_i^2\right)\mathrm{d}t,$$

which simplifies to

$$\mathrm{d}\left(\frac{c_t}{d_t}\right) = -(\rho - i_t + \pi_t)\left(\frac{c_t}{d_t}\right)\mathrm{d}t$$
$$-\sigma_d\left(\frac{c_t}{d_t} - c_d\right)\mathrm{d}B_{d,t} + \sigma_A A_t d_t^{-1}c_A\mathrm{d}B_{A,t} + \sigma_g s_{g,t}d_t^{-1}c_g\mathrm{d}B_{g,t} + \sigma_m d_t^{-1}c_i\mathrm{d}B_{i,t}$$
$$+\left(\sigma_d^2\left(\frac{c_t}{d_t} - 2c_d + \frac{d_t}{c_t}c_d^2\right) + \sigma_A^2 A_t^2\frac{d_t^{-1}}{c_t}c_A^2 + \sigma_g^2 s_{g,t}^2\frac{d_t^{-1}}{c_t}c_g^2 + \sigma_i^2\frac{d_t^{-1}}{c_t}c_i^2\right)\mathrm{d}t,$$

or

$$\begin{aligned}\mathrm{d}c_t \;=\; & -(\rho - i_t + \pi_t)c_t\mathrm{d}t + \sigma_d^2\frac{d_t^2}{c_t}c_d^2\mathrm{d}t + \sigma_A^2\frac{A_t^2}{c_t}c_A^2\mathrm{d}t + \sigma_g^2\frac{s_{g,t}^2}{c_t}c_g^2\mathrm{d}t + \sigma_i^2\frac{1}{c_t}c_i^2\mathrm{d}t \\ & +\sigma_d c_d d_t\mathrm{d}B_{d,t} + \sigma_A A_t c_A\mathrm{d}B_{A,t} + \sigma_g s_{g,t}c_g\mathrm{d}B_{g,t} + \sigma_i c_i\mathrm{d}B_{i,t} \\ & -c_t\rho_d\log d_t\mathrm{d}t + \tfrac{1}{2}c_t\sigma_d^2\mathrm{d}t - c_d d_t\sigma_d^2\mathrm{d}t, \end{aligned} \tag{C.10}$$

which is (41), and $c_t = c(\mathbb{Z}_t; \mathbb{Y}_t)$ denotes the household's consumption function. A similar approach implies the Euler equation for the alternative shock process as:

$$
\begin{aligned}
\mathrm{d}c_t &= -(\rho - i_t + \pi_t)c_t\mathrm{d}t + \sigma_A^2 \frac{A_t^2}{c_t} c_A^2 \mathrm{d}t + \sigma_g^2 \frac{s_{g,t}^2}{c_t} c_g^2 \mathrm{d}t + \sigma_i^2 \frac{1}{c_t} c_i^2 \mathrm{d}t \\
&\quad + \sigma_A A_t c_A \mathrm{d}B_{A,t} + \sigma_g s_{g,t} c_g \mathrm{d}B_{g,t} + \sigma_i c_i \mathrm{d}B_{i,t} \\
&\quad c_t \rho_d (d_t - \bar{d})\,(1 - d_t)\,/(1 - \bar{d})/d_t \mathrm{d}t. \qquad \text{(C.11)}
\end{aligned}
$$

C.3. Equilibrium

We define the recursive-competitive equilibrium of the nonlinear NK model with shocks by the sequence $\{\lambda_t, l_t, a_t, mc_t, x_{1,t}, x_{2,t}, \digamma_t, w_t, i_t, i_t^*, g_t, T_t, \pi_t, \pi_t^*, \Pi_t^*, v_t, y_t, d_t, A_t, s_{g,t}\}_{t=0}^{\infty}$, which is determined by the following equations:

- Euler equation, the first-order conditions of the household, and budget constraint:

Equation 1

$$
\begin{aligned}
\mathrm{d}c_t = -(\rho - i_t + \pi_t - \sigma_A^2 \tilde{c}_A^2 - \sigma_g^2 \tilde{c}_g^2 - \sigma_i^2 \tilde{c}_i^2 + \rho_d \log d_t + (\tilde{c}_d(1 - \tilde{c}_d) - \tfrac{1}{2})\sigma_d^2)c_t \mathrm{d}t \\
+ \sigma_d \tilde{c}_d c_t \mathrm{d}B_{d,t} + \sigma_A \tilde{c}_A c_t \mathrm{d}B_{A,t} + \sigma_g \tilde{c}_g c_t \mathrm{d}B_{g,t} + \sigma_i \tilde{c}_i c_t \mathrm{d}B_{i,t}
\end{aligned}
$$

Equation 2

$$\psi l_t^{\vartheta} c_t = w_t$$

Equation 3

$$d_t / c_t = \lambda_t$$

(redundant)

$$\mathrm{d}a_t = ((i_t - \alpha_t - \pi_t)a_t - c_t + w_t l_t + T_t + \digamma_t)\,\mathrm{d}t$$

- Profit maximization is given by:

Equation 4

$$\Pi_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{x_{2,t}}{x_{1,t}}$$

Equation 5

$$\mathrm{d}x_{1,t} = ((\rho + \delta + (1 - \varepsilon)(\pi_t - \chi \pi_t^*))x_{1,t} - \lambda_t y_t)\,\mathrm{d}t$$

Equation 6

$$\mathrm{d}x_{2,t} = ((\rho + \delta - \varepsilon(\pi_t - \chi \pi_t^*))x_{2,t} - \lambda_t mc_t y_t)\,\mathrm{d}t$$

Equation 7

$$\digamma_t = (1 - mc_t v_t) y_t$$

Equation 8

$$w_t = A_t mc_t$$

- Government policy:

Equation 9

$$\mathrm{d}i_t = (\theta\phi_\pi(\pi_t - \pi_t^*) + \theta\phi_y(y_t/y_{ss} - 1) - \theta(i_t - i_t^*))\mathrm{d}t + \sigma_i \mathrm{d}B_{i,t}$$

Equation 10

$$g_t = s_g s_{g,t} y_t$$

(redundant)

$$T_t = -i_t a_t - s_g s_{g,t} y_t$$

- Inflation evolution and price dispersion:

Equation 11

$$\pi_t - \chi\pi_t^* = \frac{\delta}{1-\varepsilon}\left((\Pi_t^*)^{1-\varepsilon} - 1\right)$$

Equation 12

$$\mathrm{d}v_t = \left(\delta\left(\Pi_t^*\right)^{-\varepsilon} + \left(\varepsilon(\pi_t - \chi\pi_t^*) - \delta\right)v_t\right)\mathrm{d}t$$

- Market clearing on goods and labor markets:

Equation 13

$$y_t = c_t + g_t \quad \text{(expenditure)}$$

Equation 14

$$y_t = \frac{A_t}{v_t} l_t \quad \text{(production)}$$

(redundant)

$$y_t = w_t l_t + F_t \quad \text{(income)}$$

- Stochastic processes follow:

Equation 15

$$\mathrm{d}d_t = -\left(\rho_d \log d_t - \tfrac{1}{2}\sigma_d^2\right) d_t \mathrm{d}t + \sigma_d d_t \mathrm{d}B_{d,t}$$

Equation 16

$$\mathrm{d}A_t = -\left(\rho_A \log A_t - \tfrac{1}{2}\sigma_A^2\right) A_t \mathrm{d}t + \sigma_A A_t \mathrm{d}B_{A,t}$$

Equation 17

$$\mathrm{d}s_{g,t} = -\left(\rho_g \log s_{g,t} - \tfrac{1}{2}\sigma_g^2\right) s_{g,t} \mathrm{d}t + \sigma_g s_{g,t} \mathrm{d}B_{g,t}$$

Note that using the household's budget constraint, we get in equilibrium:

$$\begin{aligned} \mathrm{d}a_t &= ((\alpha_t - \pi_t)a_t - c_t - g_t + y_t)\mathrm{d}t \\ &= (\alpha_t - \pi_t)a_t\mathrm{d}t, \end{aligned}$$

where for $\mathrm{d}a_t = 0$ either $\alpha_t = \pi_t$ and/or $a_t = 0$ for all t (here $a_t = 0$ because $b_t = 0$).

Moreover, in equilibrium the laws of motion for the discounted expected future profits, $x_{1,t}$ and discounted expected future costs $x_{2,t}$ are *not* direct functions of the controls:

$$\begin{aligned} \mathrm{d}x_{1,t} &= ((\rho + \delta - (\varepsilon - 1)(\pi_t - \chi\pi_t^*))x_{1,t} - \lambda_t y_t)\,\mathrm{d}t \\ &= ((\rho + \delta - (\varepsilon - 1)(\pi_t - \chi\pi_t^*))x_{1,t} - d_t/((1 - s_g s_{g,t})))\,\mathrm{d}t \end{aligned}$$

and similarly:

$$\begin{aligned} \mathrm{d}x_{2,t} &= ((\rho + \delta - \varepsilon(\pi_t - \chi\pi_t^*))\,x_{2,t} - \lambda_t y_t mc_t)\,\mathrm{d}t \\ &= ((\rho + \delta - \varepsilon(\pi_t - \chi\pi_t^*))\,x_{2,t} - mc_t d_t/(1 - s_g s_{g,t}))\,\mathrm{d}t \end{aligned}$$

Note that the TVC requires that $\lim_{t\to\infty} e^{-\rho t}\mathbb{E}_0 V(\mathbb{Z}_t^*) = 0$, in which $\mathbb{Z}_t^*$ denotes the state variables along the optimal path in line with general equilibrium conditions.

C.4. Proof of Proposition 1

We insert $\mathrm{d}c_t$ from (41) and the law of motions for the state variables

$$\begin{gathered} -(\rho - i_t + \pi_t)c_t\mathrm{d}t + \sigma_d^2\frac{d_t^2}{c_t}c_d^2\mathrm{d}t + \sigma_A^2\frac{A_t^2}{c_t}c_A^2\mathrm{d}t + \sigma_g^2\frac{s_{g,t}^2}{c_t}c_g^2\mathrm{d}t + \sigma_i^2\frac{1}{c_t}c_i^2\mathrm{d}t \\ +\sigma_d c_d d_t \mathrm{d}B_{d,t} + \sigma_A A_t c_A \mathrm{d}B_{A,t} + \sigma_g s_{g,t} c_g \mathrm{d}B_{g,t} + \sigma_i c_i \mathrm{d}B_{i,t} \\ -c_t\rho_d \log d_t \mathrm{d}t + \tfrac{1}{2}c_t\sigma_d^2\mathrm{d}t - \sigma_d^2 d_t c_d \mathrm{d}t \\ -\tfrac{1}{2}c_{ii}\sigma_i^2\,\mathrm{d}t - \tfrac{1}{2}c_{dd}(\sigma_d d_t)^2\mathrm{d}t - \tfrac{1}{2}c_{AA}(\sigma_A A_t)^2\mathrm{d}t - \tfrac{1}{2}c_{gg}(\sigma_g s_{g,t})^2\mathrm{d}t = \\ c_a\left((i_t - \pi_t)a_t - c_t + w_t l_t + T_t + F_t\right)\mathrm{d}t \\ +c_i((\theta\phi_\pi(\pi_t - \pi_t^*) + \theta\phi_y(y_t/y_{ss} - 1) - \theta(i_t - i_t^*))\mathrm{d}t + \sigma_i\mathrm{d}B_{i,t}) \\ +c_v(\delta(1 - (\varepsilon - 1)(\pi_t - \chi\pi_t^*)/\delta)^{\frac{\varepsilon}{1-\varepsilon}} + (\varepsilon(\pi_t - \chi\pi_t^*) - \delta)v_t)\mathrm{d}t \\ +c_A(-\left(\rho_A \log A_t - \tfrac{1}{2}\sigma_A^2\right)A_t\mathrm{d}t + \sigma_A A_t \mathrm{d}B_{A,t}) \\ +c_d(-\left(\rho_d \log d_t - \tfrac{1}{2}\sigma_d^2\right)d_t\mathrm{d}t + \sigma_d d_t \mathrm{d}B_{d,t}) \\ +c_g(-\left(\rho_g \log s_{g,t} - \tfrac{1}{2}\sigma_g^2\right)s_{g,t}\mathrm{d}t + \sigma_g s_{g,t} \mathrm{d}B_{g,t}) \end{gathered}$$

Collecting terms we may eliminate time (and stochastic shocks) and arrive at

$$
\begin{gathered}
-(\rho - i_t + \pi_t) d_t V_a^{-1} \mathrm{d}t \\
+\sigma_d^2 \frac{d_t^2}{d_t V_a^{-1}} \left(V_a^{-2} - 2 d_t V_a^{-2} V_a^{-1} V_{ad} + d_t^2 V_a^{-4} V_{ad}^2 \right) \mathrm{d}t \\
+\sigma_A^2 \frac{A_t^2}{d_t V_a^{-1}} d_t^2 V_a^{-4} V_{aA}^2 \mathrm{d}t + \sigma_g^2 \frac{s_{g,t}^2}{d_t V_a^{-1}} d_t^2 V_a^{-4} V_{ag}^2 \mathrm{d}t + \sigma_i^2 \frac{1}{d_t V_a^{-1}} d_t^2 V_a^{-4} V_{ai}^2 \mathrm{d}t \\
+\sigma_d (V_a^{-1} - d_t V_a^{-2} V_{ad}) d_t \mathrm{d}B_{d,t} - \sigma_A A_t d_t V_a^{-2} V_{aA} \mathrm{d}B_{A,t} - \sigma_g s_{g,t} d_t V_a^{-2} V_{ag} \mathrm{d}B_{g,t} \\
-\sigma_i d_t V_a^{-2} V_{ai} \mathrm{d}B_{i,t} - d_t V_a^{-1} \rho_d \log d_t \mathrm{d}t + \tfrac{1}{2} d_t V_a^{-1} \sigma_d^2 \mathrm{d}t \\
-\sigma_d^2 d_t (V_a^{-1} - d_t V_a^{-2} V_{ad}) \mathrm{d}t - \tfrac{1}{2} \left(2 d_t V_a^{-3} V_{ai}^2 - d_t V_a^{-2} V_{aii} \right) \sigma_i^2 \, \mathrm{d}t \\
-\tfrac{1}{2} \left(-2 V_a^{-2} V_{ad} + 2 d_t V_a^{-3} V_{ad}^2 - d_t V_a^{-2} V_{add} \right) (\sigma_d d_t)^2 \mathrm{d}t \\
-\tfrac{1}{2} \left(2 d_t V_a^{-3} V_{aA}^2 - d_t V_a^{-2} V_{aAA} \right) (\sigma_A A_t)^2 \mathrm{d}t \\
-\tfrac{1}{2} \left(2 d_t V_a^{-3} V_{ag}^2 - d_t V_a^{-2} V_{agg} \right) (\sigma_g s_{g,t})^2 \mathrm{d}t = \\
-d_t V_a^{-2} V_{aa} \left((i_t - \pi_t) a_t - c_t + w_t l_t + T_t + F_t \right) \mathrm{d}t \\
-d_t V_a^{-2} V_{ai} ((\theta \phi_\pi (\pi_t - \pi_t^*) + \theta \phi_y (y_t / y_{ss} - 1) - \theta (i_t - i_t^*)) \mathrm{d}t + \sigma_i \mathrm{d}B_{i,t}) \\
-d_t V_a^{-2} V_{av} (\delta (1 - (\varepsilon - 1)(\pi_t - \chi \pi_t^*)/\delta)^{\frac{\varepsilon}{1-\varepsilon}} + (\varepsilon (\pi_t - \chi \pi_t^*) - \delta) v_t) \mathrm{d}t \\
-d_t V_a^{-2} V_{aA} (-\left(\rho_A \log A_t - \tfrac{1}{2} \sigma_A^2 \right) A_t \mathrm{d}t + \sigma_A A_t \mathrm{d}B_{A,t}) \\
+(V_a^{-1} - d_t V_a^{-2} V_{ad})(-\left(\rho_d \log d_t - \tfrac{1}{2} \sigma_d^2 \right) d_t \mathrm{d}t + \sigma_d d_t \mathrm{d}B_{d,t}) \\
-d_t V_a^{-2} V_{ag} (-\left(\rho_g \log s_{g,t} - \tfrac{1}{2} \sigma_g^2 \right) s_{g,t} \mathrm{d}t + \sigma_g s_{g,t} \mathrm{d}B_{g,t})
\end{gathered}
$$

which can be simplified to

$$
\begin{gathered}
-(\rho - i_t + \pi_t) V_a \mathrm{d}t = \\
-\left((i_t - \pi_t) a_t - c_t + w_t l_t + T_t + F_t \right) V_{aa} \mathrm{d}t \\
-(\theta \phi_\pi (\pi_t - \pi_t^*) + \theta \phi_y (y_t / y_{ss} - 1) - \theta (i_t - i_t^*)) V_{ai} \mathrm{d}t - \tfrac{1}{2} V_{aii} \sigma_i^2 \, \mathrm{d}t \\
-(\delta (1 - (\varepsilon - 1)(\pi_t - \chi \pi_t^*)/\delta)^{\frac{\varepsilon}{1-\varepsilon}} + (\varepsilon (\pi_t - \chi \pi_t^*) - \delta) v_t) V_{av} \mathrm{d}t \\
+\left(\rho_A \log A_t - \tfrac{1}{2} \sigma_A^2 \right) A_t V_{aA} \mathrm{d}t - \tfrac{1}{2} V_{aAA} (\sigma_A A_t)^2 \mathrm{d}t \\
+V_{ad} \left(\rho_d \log d_t - \tfrac{1}{2} \sigma_d^2 \right) d_t \mathrm{d}t - \tfrac{1}{2} V_{add} \sigma_d^2 d_t^2 \mathrm{d}t \\
+V_{ag} \left(\rho_g \log s_{g,t} - \tfrac{1}{2} \sigma_g^2 \right) s_{g,t} \mathrm{d}t - \tfrac{1}{2} V_{agg} (\sigma_g s_{g,t})^2 \mathrm{d}t
\end{gathered}
$$

such that (43) must hold as an identity.

C.5. Steady state values

Steady-state. Suppose that without shocks the economy moves towards its steady state. Setting the variance of shocks to zero yields the deterministic steady state values.

- Euler equation, the first-order conditions of the household, and budget constraint:

Equation 1

$$\pi_t^* \equiv \pi_{ss} = i_t^* - \rho \equiv i_{ss} - \rho$$

Equation 2

$$\psi l_{ss}^{\vartheta} c_{ss} = w_{ss}$$

Equation 3

$$d_{ss} c_{ss}^{-1} = \lambda_{ss}$$

- Profit maximization is given by:

Equation 4

$$\Pi_{ss}^* = \frac{\varepsilon}{\varepsilon - 1} \frac{x_{2,ss}}{x_{1,ss}}$$

Equation 5

$$0 = (\rho + \delta - (\varepsilon - 1)(1 - \chi)\pi_{ss}) x_{1,ss} - \lambda_{ss} y_{ss}$$

Equation 6

$$0 = (\rho + \delta - \varepsilon(1 - \chi)\pi_{ss})\, x_{2,ss} - \lambda_{ss} y_{ss} mc_{ss}$$

Equation 7

$$F_{ss} = (1 - mc_{ss} v_{ss}) y_{ss}$$

Equation 8

$$w_{ss} = A_{ss} mc_{ss}$$

- Government policy:

Equation 9

(This equation is an identity in the steady state.)

Equation 10

$$g_{ss} = s_g s_{g,ss} y_{ss}$$

- Inflation evolution and price dispersion:

Equation 11

$$(1 - \chi)\pi_{ss} = \frac{\delta}{1 - \varepsilon} \left((\Pi_{ss}^*)^{1-\varepsilon} - 1 \right)$$

Equation 12

$$0 = \delta \left(\Pi_{ss}^*\right)^{-\varepsilon} + (\varepsilon(1 - \chi)\pi_{ss} - \delta)\, v_{ss}$$

- Market clearing on goods and labor markets (one condition is redundant):

Equation 13

$$y_{ss} = c_{ss} + g_{ss} \quad \text{(expenditure)}$$

Equation 14

$$y_{ss} = \frac{A_{ss}}{v_{ss}} l_{ss} \quad \text{(production)}$$

(redundant)

$$y_{ss} = w_{ss} l_{ss} + F_{ss} \quad \text{(income)}$$

- Stochastic processes:

Equation 15

$$d_{ss} = 1$$

Equation 16

$$A_{ss} = 1$$

Equation 17

$$s_{g,ss} = 1$$

Given the level of steady-state inflation, around which the model often is linearized, we obtain the following steady-state values. Using **Equation 1**, we obtain:

$$i_t^* = \pi_t^* + \rho \quad \Leftrightarrow \quad i_{ss} = \pi_{ss} + \rho$$

Using **Equation 11**, we obtain the steady-state value for the price ratio:

$$\Pi_{ss}^* = (1 + (1 - \varepsilon)(1 - \chi)\pi_{ss}/\delta)^{\frac{1}{1-\varepsilon}}$$

From **Equation 12**, we obtain the steady-state value for price dispersion as:

$$v_{ss} = \frac{\delta\left(\Pi_{ss}^*\right)^{-\varepsilon}}{\delta - \varepsilon(1 - \chi)\pi_{ss}}$$

Using **Equations 5** and **6** we can solve for the steady-state value of the marginal cost:

$$mc_{ss} = \frac{\rho + \delta - \varepsilon(1 - \chi)\pi_{ss}}{\rho + \delta - (\varepsilon - 1)(1 - \chi)\pi_{ss}}(x_{2,ss}/x_{1,ss})$$

which by inserting **Equation 4** gives:

$$mc_{ss} = \frac{\rho + \delta - \varepsilon(1-\chi)\pi_{ss}}{\rho + \delta - (\varepsilon - 1)(1-\chi)\pi_{ss}} \frac{\varepsilon - 1}{\varepsilon} \Pi^*_{ss}$$

Hence, we obtain

$$x_{1,ss} = d_{ss}/((1 - s_g s_{g,ss})(\rho + \delta - (\varepsilon - 1)(1-\chi)\pi_{ss}))$$

and

$$x_{2,ss} = (1 - 1/\varepsilon) x_{1,ss} \Pi^*_{ss}$$

Using **Equation 8**, we obtain

$$w_{ss} = A_{ss} mc_{ss}$$

Using **Equation 14**, we obtain

$$y_{ss} = A_{ss} l_{ss}/v_{ss}$$

Using **Equation 13** and **Equation 10** yields

$$y_{ss} = c_{ss}/(1 - s_g s_{g,ss})$$

Combining the last two equations gives

$$A_{ss} l_{ss}/v_{ss} = c_{ss}/(1 - s_g s_{g,ss})$$

Using **Equation 2** we get

$$\psi l^{\vartheta}_{ss} c_{ss} = w_{ss}$$

hence we can collect terms to obtain

$$l_{ss} = \left(\frac{w_{ss} v_{ss}}{\psi (1 - s_g s_{g,ss}) A_{ss}} \right)^{\frac{1}{1+\vartheta}}$$

Using **Equation 7** and **Equation 14** we get

$$F_{ss} = (1 - mc_{ss} v_{ss}) A_{ss} l_{ss}/v_{ss}$$

C.6. Linear approximations

In order to analyze local dynamics, the traditional approach is to approximate the dynamic equilibrium system around steady-state values. We define we $\hat{x}_t \equiv (x_t - x_{ss})/x_{ss}$, where

x_{ss} is the steady-state value for the variable x_t. Thus, we can write $x_t = (1 + \hat{x}_t)x_{ss}$.[1]

- Euler equation, the first-order conditions of the household, and budget constraint:

Equation 1

$$\mathrm{d}(c_t/c_{ss} - 1) = -(\rho - i_t + \pi_t + \rho_d(d_t/d_{ss} - 1))\mathrm{d}t$$

Equation 2

$$c_t/c_{ss} + \vartheta(l_t/l_{ss} - 1) = w_t/w_{ss}$$

Equation 3

$$d_t/d_{ss} - c_t/c_{ss} = \lambda_t/\lambda_{ss} - 1$$
$$(1 + d_t/d_{ss} - \lambda_t/\lambda_{ss})c_{ss} = c_t$$

- Profit maximization is given by:

Equation 4

$$\hat{\Pi}^*_t = \hat{x}_{2,t} - \hat{x}_{1,t}$$

Equation 5

$$\begin{gathered}\mathrm{d}(x_{1,t}/x_{1,ss} - 1) = ((\rho + \delta + (1 - \varepsilon)(1 - \chi)\pi_{ss})(x_{1,t}/x_{1,ss} - 1) \\ -(\varepsilon - 1)(\pi_t - \chi\pi^*_t - (1 - \chi)\pi_{ss}))\,\mathrm{d}t \\ -y_{ss}(d_{ss}/c_{ss})\left((y_t/y_{ss} - 1) + (d_t/d_{ss} - 1) - (c_t/c_{ss} - 1)\right)/x_{1,ss}\,\mathrm{d}t\end{gathered}$$

Equation 6

$$\begin{gathered}\mathrm{d}(x_{2,t}/x_{2,ss} - 1) = ((\rho + \delta - \varepsilon(1 - \chi)\pi_{ss})(x_{2,t}/x_{2,ss} - 1) \\ -\varepsilon(\pi_t - \chi\pi^*_t - (1 - \chi)\pi_{ss}))\,\mathrm{d}t \\ -mc_{ss}y_{ss}(d_{ss}/c_{ss})\left((mc_t/mc_{ss} - 1) + (y_t/y_{ss} - 1) + (d_t/d_{ss} - 1) - (c_t/c_{ss} - 1)\right)/x_{2,ss}\,\mathrm{d}t\end{gathered}$$

Equation 7

$$F_t/F_{ss} = y_t/y_{ss} - \frac{mc_{ss}v_{ss}}{1 - mc_{ss}v_{ss}}(mc_t/mc_{ss} - 1 + v_t/v_{ss} - 1)$$

Equation 8

$$w_t/w_{ss} - 1 = A_t/A_{ss} + mc_t/mc_{ss}$$

[1] In what follows we (log-)linearize around non-stochastic steady-state values, in particular, we assume certainty equivalence (as an approximation), which amounts to setting $\sigma^2_d = \sigma^2_A = \sigma^2_d = \sigma^2_i = 0$.

- Government policy:

Equation 9

$$\mathrm{d}(i_t - i_t^*) = (\theta\phi_\pi(\pi_t - \pi_t^*) + \theta\phi_y(y_t/y_{ss} - 1) - \theta(i_t - i_t^*))\,\mathrm{d}t$$

Equation 10

$$g_t/g_{ss} = s_{g,t}/s_{g,ss} - 1 + y_t/y_{ss}$$

- Inflation and price dispersion:

Equation 11

$$\pi_t - \chi\pi_t^* - (1-\chi)\pi_{ss} = (\delta + (1-\varepsilon)(1-\chi)\pi_{ss})(x_{2,t}/x_{2,ss} - x_{1,t}/x_{1,ss})$$

Equation 12

$$\begin{aligned}\mathrm{d}(v_t/v_{ss} - 1) = &\frac{\varepsilon(1-\chi)\pi_{ss}}{\delta + (1-\varepsilon)(1-\chi)\pi_{ss}}(\pi_t - \chi\pi_t^* - (1-\chi)\pi_{ss})\mathrm{d}t \\ &+(\varepsilon(1-\chi)\pi_{ss} - \delta)(v_t/v_{ss} - 1)\mathrm{d}t\end{aligned}$$

- Market clearing on goods and labor markets:

Equation 13

$$y_t/y_{ss} = c_t/c_{ss} + s_g s_{g,ss}/(1 - s_g s_{g,ss})(s_{g,t}/s_{g,ss} - 1)$$

Equation 14

$$y_t/y_{ss} = A_t/A_{ss} + l_t/l_{ss} - v_t/v_{ss}$$

- Stochastic processes follow:

Equation 15

$$\mathrm{d}(d_t/d_{ss} - 1) = -\rho_d(d_t/d_{ss} - 1)\mathrm{d}t$$

Equation 16

$$\mathrm{d}(A_t/A_{ss} - 1) = -\rho_A(A_t/A_{ss} - 1)\mathrm{d}t$$

Equation 17

$$\mathrm{d}(s_{g,t}/s_{g,ss} - 1) = -\rho_g(s_{g,t}/s_{g,ss} - 1)\mathrm{d}t$$

Hence, we may summarize the local equilibrium dynamics around steady-state values as:

$$\begin{aligned}
\mathrm{d}i_t &= \theta(\phi_\pi a_2(\hat{x}_{2,t} - \hat{x}_{1,t}) + \phi_y(\hat{c}_t + s_g s_{g,ss}/(1 - s_g s_{g,ss})\hat{s}_{g,t}) - (i_t - i_t^*))\,\mathrm{d}t \\
\mathrm{d}\hat{v}_t &= \varepsilon(1-\chi)\pi_{ss}(\hat{x}_{2,t} - \hat{x}_{1,t})\mathrm{d}t + (\varepsilon(1-\chi)\pi_{ss} - \delta)\hat{v}_t\mathrm{d}t \\
\mathrm{d}\hat{d}_t &= -\rho_d\hat{d}_t\mathrm{d}t \\
\mathrm{d}\hat{A}_t &= -\rho_A\hat{A}_t\mathrm{d}t \\
\mathrm{d}\hat{s}_{g,t} &= -\rho_g\hat{s}_{g,t}\mathrm{d}t \\
\mathrm{d}\hat{x}_{1,t} &= ((\rho + \varepsilon a_2)\hat{x}_{1,t} - (\varepsilon - 1)a_2\hat{x}_{2,t} - y_{ss}(d_{ss}/c_{ss})(s_g s_{g,ss}/(1 - s_g s_{g,ss})\hat{s}_{g,t} + \hat{d}_t)/x_{1,ss})\mathrm{d}t \\
\mathrm{d}\hat{x}_{2,t} &= (a_1\hat{x}_{2,t} - \varepsilon a_2(\hat{x}_{2,t} - \hat{x}_{1,t}))\,\mathrm{d}t \\
&\quad -a_1((1+\vartheta)(s_g s_{g,ss}/(1 - s_g s_{g,ss})\hat{s}_{g,t} + \hat{c}_t - \hat{A}_t) + \vartheta\hat{v}_t + \hat{d}_t)\mathrm{d}t \\
\mathrm{d}\hat{c}_t &= (i_t - i_t^* - a_2(\hat{x}_{2,t} - \hat{x}_{1,t}) - \rho_d\hat{d}_t)\mathrm{d}t
\end{aligned}$$

in which we define percentage deviations $\hat{x}_t \equiv (x_t - x_{ss})/x_{ss}$ and used the definitions for $a_1 \equiv \rho + \delta - \varepsilon(1-\chi)\pi_{ss}$, and $a_2 \equiv \delta + (1-\varepsilon)(1-\chi)\pi_{ss}$ in the main text.

In order to analyze local dynamics around the non-stochastic steady state, we need to study the eigenvalues of the Jacobian matrix evaluated at the steady state:

$$\mathrm{d}\begin{bmatrix} i_t - i_{ss} \\ \hat{v}_t \\ \hat{d}_t \\ \hat{A}_t \\ \hat{s}_{g,t} \\ \hat{x}_{1,t} \\ \hat{x}_{2,t} \\ \hat{c}_t \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 & 0 & a_{15} & a_{16} & a_{17} & a_{18} \\ 0 & a_{22} & 0 & 0 & 0 & a_{26} & a_{27} & 0 \\ 0 & 0 & a_{33} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} & 0 & 0 & 0 \\ 0 & 0 & a_{63} & 0 & a_{65} & a_{66} & a_{67} & 0 \\ 0 & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77} & a_{78} \\ a_{81} & 0 & a_{83} & 0 & 0 & a_{86} & a_{87} & 0 \end{bmatrix} \begin{bmatrix} i_t - i_{ss} \\ \hat{v}_t \\ \hat{d}_t \\ \hat{A}_t \\ \hat{s}_{g,t} \\ \hat{x}_{1,t} \\ \hat{x}_{2,t} \\ \hat{c}_t \end{bmatrix} \mathrm{d}t$$

$$
\begin{aligned}
\text{where} \quad a_{11} &\equiv -\theta \\
a_{15} &\equiv \theta\phi_y s_g s_{g,ss}/(1 - s_g s_{g,ss}) \\
a_{16} &\equiv -\theta\phi_\pi a_2 \\
a_{17} &\equiv \theta\phi_\pi a_2 \\
a_{18} &\equiv \theta\phi_y \\
a_{22} &\equiv \varepsilon(1-\chi)\pi_{ss} - \delta \\
a_{26} &\equiv -\varepsilon(1-\chi)\pi_{ss} \\
a_{27} &\equiv \varepsilon(1-\chi)\pi_{ss} \\
a_{33} &\equiv -\rho_d \\
a_{44} &\equiv -\rho_A \\
a_{55} &\equiv -\rho_g \\
a_{63} &\equiv -y_{ss}(d_{ss}/c_{ss})/x_{1,ss} \\
a_{65} &\equiv -y_{ss}(d_{ss}/c_{ss})s_g s_{g,ss}/(1 - s_g s_{g,ss})/x_{1,ss} \\
a_{66} &\equiv \rho + \varepsilon a_2 \\
a_{67} &\equiv -(\varepsilon - 1)a_2 \\
a_{72} &\equiv -a_1\vartheta \\
a_{73} &\equiv -a_1 \\
a_{74} &\equiv a_1(1+\vartheta) \\
a_{75} &\equiv -a_1(1+\vartheta)s_g s_{g,ss}/(1 - s_g s_{g,ss}) \\
a_{76} &\equiv \varepsilon a_2 \\
a_{77} &\equiv a_1 - \varepsilon a_2 \\
a_{78} &\equiv -a_1(1+\vartheta) \\
a_{81} &\equiv 1 \\
a_{83} &\equiv -\rho_d \\
a_{86} &\equiv a_2 \\
a_{87} &\equiv -a_2
\end{aligned}
$$

C.7. Stochastic steady state

The deterministic values, however, do not necessarily correspond to the stationary points in the absence of shocks, i.e., the values at which the variables are expected to stay idle in the presence of risk. Hence, the stochastic steady state values are obtained from the conditional deterministic equations, setting the random shocks (not their variances) to zero. We may thus start with (11) and compute $\mathbb{E}(\,\mathrm{d}d_t) = 0$, or

$$0 \;=\; -\left(\rho_d \log d_t - \tfrac{1}{2}\sigma_d^2\right) d_t \mathrm{d}t \quad \Rightarrow \quad d_{ss} = \exp(\tfrac{1}{2}\sigma_d^2/\rho_d)$$

The stochastic steady state values do not necessarily reflect moments of the variables. For example, the preference shock implies:

$$\mathrm{d}\log d_t = -\rho_d \log d_t \mathrm{d}t + \sigma_d \mathrm{d}B_{d,t} \;\Leftrightarrow\; \log d_t = e^{-\rho_d t}\log d_0 + \sigma_d \int_0^t e^{\rho(s-t)}\,\mathrm{d}B_s,$$

which has a long-run (or stationary) Normal distribution $\log d_t \sim \mathcal{N}(0, \frac{1}{2}\sigma_d^2/\rho_d)$.[2] Hence, if $\log d_t$ is asymptotically normally distributed, $d_t \sim \mathcal{LN}(0, \frac{1}{2}\sigma_d^2/\rho_d)$ with

$$\mathbb{E}(d_t) = \exp(\tfrac{1}{4}\sigma_d^2/\rho_d).$$

It shows that both the unconditional mean value of the stationary distribution and the stochastic steady state increase in σ_d^2.

Similarly, we obtain the stochastic steady states for the remaining shocks

$$\begin{aligned} 0 \;&=\; -\left(\rho_A \log A_t - \tfrac{1}{2}\sigma_A^2\right) A_t \mathrm{d}t \quad \Rightarrow \quad A_{ss} = \exp(\tfrac{1}{2}\sigma_A^2/\rho_A) \\ 0 \;&=\; -\left(\rho_g \log s_{g,t} - \tfrac{1}{2}\sigma_g^2\right) s_{g,t} \mathrm{d}t \quad \Rightarrow \quad s_{g,ss} = \exp(\tfrac{1}{2}\sigma_g^2/\rho_g) \end{aligned}$$

Steady-state. In the presence of uncertainty, in case the dynamic variables approach a stochastic steady-state distribution (a stationary distribution). Analogous to the perfect foresight model, we define the *conditional* deterministic steady state values as the variables where the (conditional) deterministic system (44) stays idle. For given inflation targets

[2] The moments of the stationary distribution can be obtained from

$$\begin{aligned} \mathrm{d}(\log d_t)^2 \;&=\; 2\log d_t\, \mathrm{d}\log d_t + \sigma_d^2\, \mathrm{d}t \\ &=\; -\rho_d 2\log d_t \log d_t \mathrm{d}t + \sigma_d 2\log d_t \mathrm{d}B_{d,t} + \sigma_d^2\, \mathrm{d}t \end{aligned}$$

the expected value reads

$$\mathrm{d}\mathbb{E}(\log d_t) = -\rho_d \mathrm{d}\mathbb{E}(\log d_t)\mathrm{d}t \;\Leftrightarrow\; \mathbb{E}(\log d_t) = e^{-\rho_d t}\log d_0 \quad \Rightarrow \quad \lim_{t\to\infty} \mathbb{E}(\log d_t) = 0$$

such that

$$\mathbb{E}((\log d_t)^2) = \mathbb{V}ar((\log d_t)^2) = \tfrac{1}{2}\sigma_d^2/\rho_d$$

π_t^*, the Euler equation (44) determines the long-run values i_t^*.

- Euler equation, and the first-order conditions of the household:

Equation 1

$$i_t^* - \pi_t^* \equiv i_{ss} - \pi_{ss}$$
$$= \rho - \left(\tilde{c}_d^2\sigma_d^2 + \tilde{c}_A^2\sigma_A^2 + \tilde{c}_g^2\sigma_g^2 + \tilde{c}_i^2\sigma_i^2 - \frac{1}{2}\tilde{c}_{dd}\sigma_d^2 - \frac{1}{2}\tilde{c}_{AA}\sigma_A^2 - \frac{1}{2}\tilde{c}_{gg}\sigma_g^2 - \frac{1}{2}\tilde{c}_{ii}\sigma_i^2 - \tilde{c}_d\sigma_d^2\right)$$

Equation 2

$$\psi l_{ss}^{\vartheta} c_{ss} = w_{ss}$$

Equation 3

$$d_{ss} c_{ss}^{-1} = \lambda_{ss}$$

- Profit maximization is given by:

Equation 4

$$\Pi_{ss}^* = \frac{\varepsilon}{\varepsilon - 1}\frac{x_{2,ss}}{x_{1,ss}}$$

Equation 5

$$0 = (\rho + \delta - (\varepsilon - 1)(1 - \chi)\pi_{ss})x_{1,ss} - \lambda_{ss}y_{ss}$$

Equation 6

$$0 = \left(\rho + \delta - \varepsilon(1 - \chi)\pi_{ss}\right) x_{2,ss} - \lambda_{ss}y_{ss}mc_{ss}$$

Equation 7

$$F_{ss} = (1 - mc_{ss}v_{ss})y_{ss}$$

Equation 8

$$w_{ss} = A_{ss}mc_{ss}$$

- Government policy:

Equation 9

(This equation is an identity in the steady state.)

Equation 10

$$g_{ss} = s_g s_{g,ss} y_{ss}$$

- Inflation evolution and price dispersion:

Equation 11

$$(1-\chi)\pi_{ss} = \frac{\delta}{1-\varepsilon}\left((\Pi^*_{ss})^{1-\varepsilon} - 1\right)$$

Equation 12

$$0 = \delta\left(\Pi^*_{ss}\right)^{-\varepsilon} + \left(\varepsilon(1-\chi)\pi_{ss}) - \delta\right) v_{ss}$$

- Market clearing on goods and labor markets (one condition is redundant):

Equation 13

$$y_{ss} = c_{ss} + g_{ss} \quad \text{(expenditure)}$$

Equation 14

$$y_{ss} = \frac{A_{ss}}{v_{ss}} l_{ss} \quad \text{(production)}$$

(redundant)

$$y_{ss} = w_{ss} l_{ss} + F_{ss} \quad \text{(income)}$$

- Stochastic processes:

Equation 15

$$d_{ss} = \exp(\tfrac{1}{2}\sigma_d^2/\rho_d)$$

Equation 16

$$A_{ss} = \exp(\tfrac{1}{2}\sigma_A^2/\rho_A)$$

Equation 17

$$s_{g,ss} = \exp(\tfrac{1}{2}\sigma_g^2/\rho_g)$$

Using **Equation 11**, we obtain the steady-state value for the price ratio:

$$\Pi^*_{ss} = (1 + (1-\varepsilon)(1-\chi)\pi_{ss})/\delta)^{\frac{1}{1-\varepsilon}}$$

From **Equation 12**, we obtain the steady-state value for price dispersion as:

$$v_{ss} = \frac{\delta\left(\Pi^*_{ss}\right)^{-\varepsilon}}{\delta - \varepsilon(1-\chi)\pi_{ss}}$$

Using **Equations 5** and **6** we can solve for the steady-state value of the marginal cost:

$$mc_{ss} = \frac{\rho + \delta - \varepsilon(1-\chi)\pi_{ss}}{\rho + \delta - (\varepsilon - 1)(1-\chi)\pi_{ss}}(x_{2,ss}/x_{1,ss})$$

which by inserting **Equation 4** gives:

$$mc_{ss} = \frac{\rho + \delta - \varepsilon(1-\chi)\pi_{ss}}{\rho + \delta - (\varepsilon - 1)(1-\chi)\pi_{ss}} \frac{\varepsilon - 1}{\varepsilon} \Pi^*_{ss}$$

Hence, we obtain

$$x_{1,ss} = d_{ss}/((1 - s_g s_{g,ss})(\rho + \delta - (\varepsilon - 1)(1-\chi)\pi_{ss}))$$

and

$$x_{2,ss} = (1 - 1/\varepsilon) x_{1,ss} \Pi^*_{ss}$$

Using **Equation 8**, we obtain

$$w_{ss} = A_{ss} mc_{ss}$$

Using **Equation 14**, we obtain

$$y_{ss} = A_{ss} l_{ss} / v_{ss}$$

Using **Equation 13** and **Equation 10** yields

$$y_{ss} = c_{ss}/(1 - s_g s_{g,ss})$$

Combining the last two equations gives

$$A_{ss} l_{ss}/v_{ss} = c_{ss}/(1 - s_g s_{g,ss})$$

Using **Equation 2** we get

$$\psi l_{ss}^{\vartheta} c_{ss} = w_{ss}$$

hence we can collect terms to obtain

$$l_{ss} = \left(\frac{w_{ss} v_{ss}}{\psi(1 - s_g s_{g,ss}) A_{ss}} \right)^{\frac{1}{1+\vartheta}}$$

Using **Equation 7** and **Equation 14** we get

$$F_{ss} = (1 - mc_{ss} v_{ss}) A_{ss} l_{ss} / v_{ss}$$

C.8. Alternative Taylor principles and stability

We review insights related to positive trend inflation and determinacy in the NK model ($\chi = 0$). To study the stability properties of the dynamic system, the nonlinear system

$$dx_t \equiv f(x_t)dt$$

is approximated by the linear system

$$\frac{d}{dt}x_t = \frac{1}{dt}dx_t = A(x_t - x_{ss})$$

Equivalently, we may study (absolute) deviations from an equilibrium $x_t - x_{ss}$ by defining

$$\frac{d}{dt}(x_t - x_{ss}) = \frac{d}{dt}x_t = A(x_t - x_{ss})$$

such that the Jacobian matrix is identical, or define percentage deviations $\hat{x}_t \equiv x_t/x_{ss} - 1$ for each variable and use $x_t = (1 + \hat{x}_t)x_{ss}$ such that for each variable

$$\frac{d}{dt}\hat{x}_t = 1/x_{ss}\frac{d}{dt}x_t = A(x_t - x_{ss})/x_{ss} = A\hat{x}_t$$

with identical Jacobian matrix of the vector function $f(x_t)$.

For illustration, we show the linearized NK model with $s_g = 0$ (cf. Section C.6). We compare the feedback rule vs. partial adjustment. With partial adjustment, we have:

$$\begin{aligned} \mathrm{d}i_t &= (\theta\phi_\pi(\pi_t - \pi_t^*) + \theta\phi_y\hat{y}_t - \theta(i_t - i_t^*))\mathrm{d}t \\ \Leftrightarrow \mathrm{d}(i_t - i_{ss}) &= (\theta\phi_\pi(\pi_t - \pi_t^*) + \theta\phi_y\hat{y}_t - \theta(i_t - i_t^*))\mathrm{d}t \\ \Leftrightarrow \quad \mathrm{d}(e^{\theta t}(i_t - i_t^*))/\,\mathrm{d}t &= e^{\theta t}\theta\phi_\pi(\pi_t - \pi_t^*) + e^{\theta t}\theta\phi_y\hat{y}_t \\ \text{for} \;\; t_0 \to -\infty \quad \Rightarrow \quad i_t - i_t^* &= \theta\int_{-\infty}^{t} e^{-\theta(t-k)}(\phi_\pi(\pi_k - \pi_t^*) + \phi_y\hat{y}_t)\,\mathrm{d}k, \end{aligned}$$

which requires $\theta > 0$ or alternatively for the feedback rule model:

$$i_t - i_t^* = \phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1), \quad \phi_\pi > 1,\ \phi_y \geq 0.$$

C.8.1. Feedback rule

In the feedback rule in the simple NK model we have:

$$i_t - i_t^* = \phi(\pi_t - \pi_t^*), \quad \phi > 1$$

or more general, the feedback rule (used in the main text) with an output response:

$$i_t - i_t^* = \phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1), \quad \phi_\pi > 1,\ \phi_y \geq 0,$$

for example $\phi_\pi \approx 1.5$ and $\phi_y \approx 0.5$ for target rates $\pi_t^* \approx 0$ (see Woodford, 2001).

To study the properties of the equilibrium points, define $x_t \equiv (y_t, v_t, x_{1,t}, x_{2,t})$ such that

$$f(x_t) \equiv f(y_t, v_t, x_{1,t}, x_{2,t}) = \begin{bmatrix} -(\rho - i_t + \pi_t)\, y_t \\ \delta\,(1 + (1-\varepsilon)\pi_t/\delta)^{-\frac{\varepsilon}{1-\varepsilon}} + (\varepsilon\pi_t - \delta)v_t \\ (\rho + \delta - (\varepsilon - 1)\pi_t)x_{1,t} - 1 \\ (\rho + \delta - \varepsilon\pi_t)\, x_{2,t} - \psi v_t^{\vartheta} y_t^{1+\vartheta} \end{bmatrix}$$

Evaluating the Jacobian matrix at an equilibrium point $x_{ss} = (y_{ss}, v_{ss}, x_{1,ss}, x_{2,ss})$ yields

$$A_1 = \begin{bmatrix} \phi_y & 0 & (1-\phi_\pi)a_2 y_{ss}/x_{1,ss} & -(1-\phi_\pi)a_2 y_{ss}/x_{2,ss} \\ 0 & \varepsilon\pi_{ss} - \delta & -\varepsilon\pi_{ss}v_{ss}/x_{1,ss} & \varepsilon\pi_{ss}v_{ss}/x_{2,ss} \\ 0 & 0 & \rho + \varepsilon a_2 & -(\varepsilon-1)a_2 x_{1,ss}/x_{2,ss} \\ -(1+\vartheta)a_1 x_{2,ss}/y_{ss} & -\vartheta a_1 x_{2,ss}/v_{ss} & \varepsilon a_2 x_{2,ss}/x_{1,ss} & a_1 - \varepsilon a_2 \end{bmatrix}$$

where in this version $a_1 \equiv \rho + \delta - \varepsilon\pi_{ss}$, and $a_2 \equiv \delta + (1-\varepsilon)\pi_{ss}$.

Hence, we may approximate the equilibrium dynamics by

$$\begin{aligned} \mathrm{d}\hat{y}_t &= (i_t - \rho - \pi_t)\mathrm{d}t \\ \mathrm{d}\hat{v}_t &= ((\varepsilon\pi_{ss} - \delta)\hat{v}_t + \varepsilon\pi_{ss}/a_2(\pi_t - \pi_{ss}))\mathrm{d}t \\ \mathrm{d}\hat{x}_{1,t} &= ((\rho + a_2)\hat{x}_{1,t} + (1-\varepsilon)(\pi_t - \pi_{ss}))\,\mathrm{d}t \\ \mathrm{d}\hat{x}_{2,t} &= (a_1\hat{x}_{2,t} - \varepsilon(\pi_t - \pi_{ss}) - (1+\vartheta)a_1\hat{y}_t - \vartheta a_1\hat{v}_t)\mathrm{d}t \end{aligned}$$

where $\pi_t - \pi_{ss} = a_2(x_{2,t}/x_{2,ss} - x_{1,t}/x_{1,ss})$ and $i_t = \phi_y(y_t/y_{ss} - 1) + \phi_\pi(\pi_t - \pi_{ss}) + i_{ss}$ such that the inflation dynamics are:

$$\begin{aligned} \mathrm{d}\pi_t = {} & \rho(\pi_t - \pi_{ss})\,\mathrm{d}t - (\delta + (1-\varepsilon)\pi_{ss})\pi_{ss}(x_{2,t}/x_{2,ss} - 1)\mathrm{d}t \\ & -\kappa((y_t/y_{ss} - 1) + (v_t/v_{ss} - 1)\vartheta/(1+\vartheta))\mathrm{d}t \end{aligned}$$

in which $\kappa \equiv (\delta + (1-\varepsilon)\pi_{ss})(1+\vartheta)\,(\rho + \delta - \varepsilon\pi_{ss})$.

Around zero-inflation target $\pi_{ss} = 0$ and $i_{ss} = \rho$, the equilibrium dynamics are:

$$\begin{aligned} \mathrm{d}\hat{y}_t &= (i_t - \rho - \pi_t)\mathrm{d}t \\ \mathrm{d}\hat{v}_t &= -\delta\hat{v}_t\mathrm{d}t \\ \mathrm{d}\pi_t &= (\rho\pi_t - (1+\vartheta)(\rho+\delta)\delta\hat{y}_t - \vartheta(\rho+\delta)\delta\hat{v}_t)\mathrm{d}t \end{aligned}$$

In this first-order approximation, price dispersion is no longer affected by other variables, such that it will always converge. Analyzing equilibrium dynamics will be based on:

$$\begin{aligned} \mathrm{d}\hat{y}_t &= (i_t - \rho - \pi_t)\mathrm{d}t \\ \mathrm{d}\pi_t &= (\rho\pi_t - \kappa\hat{y}_t)\mathrm{d}t \end{aligned}$$

where $\kappa \equiv (1+\vartheta)(\rho+\delta)\delta$ and $i_t = i_{ss} + \phi_\pi \pi_t + \phi_y \hat{y}_t$. Sometimes the linearized model around zero inflation target is used to approximate the model around positive inflation targets, $\pi_{ss} > 0$ (e.g., Cochrane, 2017, eq. (4) with time-varying π_{ss} and ρ).

Based on the reduced system $x = (\hat{y}_t, \pi_t)$ for $\pi_{ss} = 0$, the 2×2 Jacobian matrix reads:

$$A_1 = \begin{bmatrix} \phi_y & \phi_\pi - 1 \\ -\kappa & \rho \end{bmatrix}$$

For a unique locally bounded equilibrium we need two positive eigenvalues, for the larger system $\pi_{ss} \neq 0$ we need three positive and one negative eigenvalue.

The Jacobian matrix has $\mathrm{tr}(A_1) = \lambda_1 + \lambda_2 = \phi_y + \rho > 0$ and $\det(A_1) = \rho\phi_y + (\phi_\pi - 1)\kappa$ is positive for $\phi_\pi > 1$, thus both eigenvalues have positive real parts, $\lambda_1\lambda_2 = \det(A_1)$,

$$\lambda^2 - (\phi_y + \rho)\lambda + \rho\phi_y + (\phi_\pi - 1)\kappa = 0$$

$$\lambda_{1,2} = \tfrac{1}{2}(\rho + \phi_y \pm \sqrt{(\phi_y + \rho)^2 - 4(\rho\phi_y + (\phi_\pi - 1)\kappa)})$$

So the unique locally bounded solution is $\hat{y}_t = 0$ and $\pi_t = \pi_{ss}$ such that $i_t = i_{ss}$.

C.8.2. Partial adjustment

For the partial adjustment model, we need to add the dynamics of the interest rate:

$$\mathrm{d}(i_t - i_t^*) = (\theta\phi_\pi(\pi_t - \pi_t^*) + \theta\phi_y\hat{y}_t - \theta(i_t - i_t^*))\mathrm{d}t$$

It relates to Graeve, Emiris, and Wouters (2009), where the Taylor rule has lagged interest rates and response to the output gap (percentage deviations).

To study the properties of the two equilibrium points, define $x_t \equiv (y_t, v_t, x_{1,t}, x_{2,t}, i_t)$ such that

$$f(x_t) \equiv f(y_t, v_t, x_{1,t}, x_{2,t}, i_t) = \begin{bmatrix} -(\rho - i_t + \pi_t)\,y_t \\ \delta\,(1 + (1-\varepsilon)\pi_t/\delta)^{-\frac{\varepsilon}{1-\varepsilon}} + (\varepsilon\pi_t - \delta)v_t \\ (\rho + \delta - (\varepsilon - 1)\pi_t)x_{1,t} - 1 \\ (\rho + \delta - \varepsilon\pi_t)\,x_{2,t} - \psi v_t^{\vartheta} y_t^{1+\vartheta} \\ \theta\phi_\pi(\pi_t - \pi_{ss}) + \theta\phi_y(y_t/y_{ss} - 1) - \theta(i_t - i_{ss}) \end{bmatrix}$$

Evaluating the Jacobian matrix at equilibrium point $x_{ss} = (y_{ss}, v_{ss}, x_{1,ss}, x_{2,ss}, i_{ss})$ yields

$$A_2 = \begin{bmatrix} 0 & 0 & a_2 y_{ss}/x_{1,ss} & -a_2 y_{ss}/x_{2,ss} & y_{ss} \\ 0 & \varepsilon\pi_{ss} - \delta & -\varepsilon\pi_{ss} v_{ss}/x_{1,ss} & \varepsilon\pi_{ss} v_{ss}/x_{2,ss} & 0 \\ 0 & 0 & \rho + \varepsilon a_2 & -(\varepsilon - 1) a_2 x_{1,ss}/x_{2,ss} & 0 \\ -(1+\vartheta) a_1 x_{2,ss}/y_{ss} & -\vartheta a_1 x_{2,ss}/v_{ss} & \varepsilon a_2 x_{2,ss}/x_{1,ss} & a_1 - \varepsilon a_2 & 0 \\ \theta\phi_y/y_{ss} & 0 & -\theta\phi_\pi a_2/x_{1,ss} & \theta\phi_\pi a_2/x_{2,ss} & -\theta \end{bmatrix}$$

where $a_1 \equiv \rho + \delta - \varepsilon\pi_{ss}$, and $a_2 \equiv \delta + (1-\varepsilon)\pi_{ss}$.

Hence, we may approximate the equilibrium dynamics by

$$\begin{aligned} \mathrm{d}\hat{y}_t &= (i_t - \rho - \pi_t)\,\mathrm{d}t \\ \mathrm{d}\hat{v}_t &= ((\varepsilon\pi_{ss} - \delta)\hat{v}_t + \varepsilon\pi_{ss}/a_2(\pi_t - \pi_{ss}))\,\mathrm{d}t \\ \mathrm{d}\hat{x}_{1,t} &= ((\rho + a_2)\hat{x}_{1,t} + (1-\varepsilon)(\pi_t - \pi_{ss}))\,\mathrm{d}t \\ \mathrm{d}\hat{x}_{2,t} &= (a_1\hat{x}_{2,t} - \varepsilon(\pi_t - \pi_{ss}) - (1+\vartheta)a_1\hat{y}_t - \vartheta a_1\hat{v}_t)\mathrm{d}t \\ \mathrm{d}i_t &= (\theta\phi_\pi(\pi_t - \pi_{ss}) + \theta\phi_y\hat{y}_t - \theta(i_t - i_{ss}))\,\mathrm{d}t \end{aligned}$$

where $\pi_t - \pi_{ss} = a_2(x_{2,t}/x_{2,ss} - x_{1,t}/x_{1,ss})$ such that the inflation dynamics are:

$$\mathrm{d}\pi_t = (\rho(\pi_t - \pi_{ss}) - a_2\pi_{ss}\hat{x}_{2,t} - \kappa\hat{y}_t - \vartheta a_1 a_2 \hat{v}_t)\mathrm{d}t$$

in which $\kappa \equiv (1+\vartheta)(\rho + \delta - \varepsilon\pi_{ss})(\delta + (1-\varepsilon)\pi_{ss})$.

Around zero-inflation target $\pi_{ss} = 0$ and $i_{ss} = \rho$, the equilibrium dynamics are:

$$\begin{aligned} \mathrm{d}\hat{y}_t &= (i_t - \rho - \pi_t)\,\mathrm{d}t \\ \mathrm{d}\hat{v}_t &= -\delta\hat{v}_t\,\mathrm{d}t \\ \mathrm{d}\pi_t &= (\rho\pi_t - (1+\vartheta)\delta(\rho+\delta)\hat{y}_t - \vartheta\delta(\rho+\delta)\hat{v}_t)\mathrm{d}t \\ \mathrm{d}i_t &= (\theta\phi_\pi\pi_t + \theta\phi_y\hat{y}_t - \theta(i_t - i_{ss}))\,\mathrm{d}t \end{aligned}$$

In this first-order approximation, price dispersion is no longer affected by other variables, such that it will always converge. Analyzing equilibrium dynamics will be based on:

$$\begin{aligned} \mathrm{d}\hat{y}_t &= (i_t - \rho - \pi_t)\,\mathrm{d}t \\ \mathrm{d}\pi_t &= (\rho\pi_t - \kappa\hat{y}_t)\mathrm{d}t \\ \mathrm{d}i_t &= (\theta\phi_\pi\pi_t + \theta\phi_y\hat{y}_t - \theta(i_t - \rho))\,\mathrm{d}t \end{aligned}$$

where $\kappa \equiv (1+\vartheta)(\rho+\delta)\delta$. Based on the reduced system $x_t = (\hat{y}_t, \pi_t, i_t)$ for $\pi_{ss} = 0$, the

3×3 Jacobian matrix reads:

$$A_2 = \begin{bmatrix} 0 & -1 & 1 \\ -\kappa & \rho & 0 \\ \theta\phi_y & \theta\phi_\pi & -\theta \end{bmatrix}$$

For a unique locally bounded equilibrium we need two positive and one negative eigenvalue, for the larger system $\pi_{ss} \neq 0$ we need three positive and two negative eigenvalues.

C.9. Local determinacy

In this section we study local determinacy of the minimal NK model. We illustrate how the results depend on the inflation target $\pi_t^* > 0$, and how the Taylor rule can be extended to allow for larger regions of determinacy. For comparison with the simple NK modelwe assume throughout the section $s_g = 0$, $\chi = 0$, and $\|(\sigma_d, \sigma_A, \sigma_g, \sigma_i)\| = 0$, such that $r_t = \rho$.

While the simple NK model with a *feedback rule* has no state variables, the NK model with no shocks (henceforth minimal NK model) with $\pi_t^* > 0$ introduces price dispersion v_t as a relevant state variable, and a unique locally bounded solution requires three positive eigenvalues of the Jacobian matrix (cf. Appendix C.8.1)[3]

$$A_1 = \begin{bmatrix} \phi_y & 0 & (1-\phi_\pi)a_2 y_{ss}/x_{1,ss} & (\phi_\pi - 1)a_2 y_{ss}/x_{2,ss} \\ 0 & \varepsilon\pi_{ss} - \delta & -\varepsilon\pi_{ss} v_{ss}/x_{1,ss} & \varepsilon\pi_{ss} v_{ss}/x_{2,ss} \\ 0 & 0 & \rho + \varepsilon a_2 & -(\varepsilon - 1)a_2 x_{1,ss}/x_{2,ss} \\ -(1+\vartheta)a_1 x_{2,ss}/y_{ss} & -\vartheta a_1 x_{2,ss}/v_{ss} & \varepsilon a_2 x_{2,ss}/x_{1,ss} & a_1 - \varepsilon a_2 \end{bmatrix}$$

where

$$a_1 \equiv \rho + \delta - \varepsilon\pi_{ss}, \quad a_2 \equiv \delta + (1-\varepsilon)\pi_{ss}, \tag{C.12}$$

such that the (linearized) inflation dynamics are

$$\begin{aligned} \mathrm{d}\pi_t &= \rho(\pi_t - \pi_{ss})\,\mathrm{d}t - (\delta + (1-\varepsilon)\pi_{ss})\pi_{ss}(x_{2,t}/x_{2,ss} - 1)\mathrm{d}t \\ &\quad -\kappa((y_t/y_{ss} - 1) + (v_t/v_{ss} - 1)\vartheta/(1+\vartheta))\mathrm{d}t. \end{aligned} \tag{C.13}$$

So we define

$$\kappa \equiv (\delta + (1-\varepsilon)\pi_{ss})(1+\vartheta)\left(\rho + \delta - \varepsilon\pi_{ss}\right). \tag{C.14}$$

[3] We impose the parametric restriction $\delta > \varepsilon\pi_{ss}$ to ensure non-negative price dispersion, which in the frictionless case $\delta \to \infty$ the condition is fulfilled. For $\pi_{ss} = 0$ the system can be reduced to

$$A_1 = \begin{bmatrix} \phi_y & \phi_\pi - 1 \\ -\kappa & \rho \end{bmatrix},$$

which shows that the output response would not introduce different conclusions regarding stability in the simple NK model: A necessary (and sufficient) condition for local determinacy still would be $\phi_\pi > 1$.

For a unique locally bounded equilibrium we need three positive and one negative eigenvalue. In the NK model with *partial adjustment*, the two relevant state variables are the interest rate and the level of price dispersion, so a unique locally bounded solution requires three positive eigenvalues of the Jacobian matrix (cf. Appendix C.8.2)

$$A_2 = \begin{bmatrix} 0 & 0 & a_2 y_{ss}/x_{1,ss} & -a_2 y_{ss}/x_{2,ss} & y_{ss} \\ 0 & \varepsilon\pi_{ss} - \delta & -\varepsilon\pi_{ss} v_{ss}/x_{1,ss} & \varepsilon\pi_{ss} v_{ss}/x_{2,ss} & 0 \\ 0 & 0 & \rho + \varepsilon a_2 & (1-\varepsilon) a_2 x_{1,ss}/x_{2,ss} & 0 \\ -(1+\vartheta) a_1 x_{2,ss}/y_{ss} & -\vartheta a_1 x_{2,ss}/v_{ss} & \varepsilon a_2 x_{2,ss}/x_{1,ss} & a_1 - \varepsilon a_2 & 0 \\ \theta\phi_y/y_{ss} & 0 & -\theta\phi_\pi a_2/x_{1,ss} & \theta\phi_\pi a_2/x_{2,ss} & -\theta \end{bmatrix}$$

whereas for $\pi_{ss} = 0$ it collapses to the 3×3 matrix of the simple model. Note that the (linearized) inflation dynamics are not affected by the specification of the Taylor rule.

For a unique locally bounded equilibrium we need three positive and two negative eigenvalues. The determinacy regions are shown in the accompanying web appendix.

Apart from the effects of risk, the policy instruments are the same as before. The more general Taylor rules (24a) and (24b) introduce an output response ϕ_y, in addition to the inflation response ϕ_π as a new policy parameter.

Summarizing, the choice of the Taylor rule in the (continuous-time) NK model can be decisive for the answer whether higher interest rates raise or (temporarily) lower inflation. While the feedback rule postulates that higher interest rates necessarily correspond to higher inflation rates (varying the relevant state variables/shocks), the partial adjustment model supports both a negative and a positive link as in the simple model. Our results indicate that the policy experiments imply qualitatively the same responses for interest rates at near zero values compared to normal times about the long-run equilibrium.

We replicate the findings in Coibion and Gorodnichenko (2011), showing that the conclusion about determinacy in the NK model is different in models with positive trend inflation (no indexation). Similarly we find that the output response helps to obtain determinacy in the feedback model, whereas the partial adjustment model seems to be more robust to positive inflation target because of the interest smoothing component.

C.10. The dynamic system under the risk-neutral probability measure

Consider the system of stochastic processes, i.e., 5 endogenous processes for the auxiliary variables $x_{1,t}$, $x_{2,t}$, price dispersion v_t, the Taylor rule i_t, and the Euler equation c_t, and 3 exogenous processes for $s_{g,t}, d_t, A_t$, which summarize equilibrium dynamics:

$$\begin{aligned}
\mathrm{d}c_t &= -(\rho - i_t + \pi_t)c_t\mathrm{d}t + \sigma_d^2\frac{d_t^2}{c_t}c_d^2\mathrm{d}t + \sigma_A^2\frac{A_t^2}{c_t}c_A^2\mathrm{d}t + \sigma_g^2\frac{s_{g,t}^2}{c_t}c_g^2\mathrm{d}t + \sigma_i^2\frac{1}{c_t}c_i^2\mathrm{d}t \\
&\quad +\sigma_d c_d d_t \mathrm{d}B_{d,t} + \sigma_A A_t c_A \mathrm{d}B_{A,t} + \sigma_g s_{g,t} c_g \mathrm{d}B_{g,t} + \sigma_i c_i \mathrm{d}B_{i,t} \\
&\quad -c_t\rho_d \log d_t \mathrm{d}t + \tfrac{1}{2}c_t\sigma_d^2\mathrm{d}t - c_d d_t \sigma_d^2 \mathrm{d}t \\
\mathrm{d}x_{1,t} &= ((\rho+\delta-(\varepsilon-1)(\pi_t-\chi\pi_t^*))x_{1,t} - d_t/(1-s_g s_{g,t}))\,\mathrm{d}t \\
\mathrm{d}x_{2,t} &= ((\rho+\delta-\varepsilon(\pi_t-\chi\pi_t^*))\,x_{2,t} - mc_t d_t/(1-s_g s_{g,t}))\,\mathrm{d}t \\
\mathrm{d}i_t &= \theta(\phi_\pi(\pi_t-\pi_t^*) + \phi_y(y_t/y_{ss}-1) - (i_t - i_t^*))\mathrm{d}t + \sigma_i \mathrm{d}B_{i,t} \\
\mathrm{d}v_t &= (\delta(1-(\varepsilon-1)(\pi_t-\chi\pi_t^*)/\delta)^{-\frac{\varepsilon}{1-\varepsilon}} + (\varepsilon(\pi_t-\chi\pi_t^*)-\delta)v_t)\mathrm{d}t \\
\mathrm{d}d_t &= -\left(\rho_d \log d_t - \tfrac{1}{2}\sigma_d^2\right) d_t \mathrm{d}t + \sigma_d d_t \mathrm{d}B_{d,t} \\
\mathrm{d}A_t &= -(\rho_A \log A_t - \tfrac{1}{2}\sigma_A^2)A_t\,\mathrm{d}t + \sigma_A A_t \mathrm{d}B_{A,t} \\
\mathrm{d}s_{g,t} &= -(\rho_g \log s_{g,t} - \tfrac{1}{2}\sigma_g^2)s_{g,t}\,\mathrm{d}t + \sigma_g s_{g,t}\mathrm{d}B_{g,t}
\end{aligned}$$

Suppose that $B_t = (B_{i,t}, B_{d,t}, B_{A,t}, B_{g,t})^\top$ is the k-vector of Brownian motions under the physical probability measure $\mathbb{P}$, we define $B_t^{\mathbb{Q}} = (B_{i,t}^{\mathbb{Q}}, B_{d,t}^{\mathbb{Q}}, B_{A,t}^{\mathbb{Q}}, B_{g,t}^{\mathbb{Q}})^\top$ as the equivalent k-vector of Brownian motions under the risk-neutral probability measure, such that

$$\mathrm{d}\begin{bmatrix} B_{i,t}^{\mathbb{Q}} \\ B_{d,t}^{\mathbb{Q}} \\ B_{A,t}^{\mathbb{Q}} \\ B_{g,t}^{\mathbb{Q}} \end{bmatrix} = \mathrm{d}\begin{bmatrix} B_{i,t} \\ B_{d,t} \\ B_{A,t} \\ B_{g,t} \end{bmatrix} - \begin{bmatrix} \sigma_i V_{ai} V_a^{-1} \\ \sigma_d d_t V_{ad} V_a^{-1} \\ \sigma_A A_t V_{aA} V_a^{-1} \\ \sigma_g s_{g,t} V_{ag} V_a^{-1} \end{bmatrix} \mathrm{d}t$$

Hence, we may write the equilibrium dynamics under the risk-neutral measure $\mathbb{Q}$ as

$$
\begin{aligned}
\mathrm{d}c_t &= -(\rho - i_t + \pi_t)c_t\mathrm{d}t + \sigma_d^2 \frac{d_t^2}{c_t} c_d^2 \mathrm{d}t + \sigma_A^2 \frac{A_t^2}{c_t} c_A^2 \mathrm{d}t + \sigma_g^2 \frac{s_{g,t}^2}{c_t} c_g^2 \mathrm{d}t + \sigma_i^2 \frac{1}{c_t} c_i^2 \mathrm{d}t \\
&\quad + \sigma_d^2 d_t^2 V_{ad} V_a^{-1} c_d \mathrm{d}t + \sigma_A^2 A_t^2 V_{aA} V_a^{-1} c_A \mathrm{d}t + \sigma_g^2 s_{g,t}^2 V_{ag} V_a^{-1} c_g \mathrm{d}t + \sigma_i^2 V_{ai} V_a^{-1} c_i \mathrm{d}t \\
&\quad + \sigma_d c_d d_t \mathrm{d}B_{d,t}^{\mathbb{Q}} + \sigma_A A_t c_A \mathrm{d}B_{A,t}^{\mathbb{Q}} + \sigma_g s_{g,t} c_g \mathrm{d}B_{g,t}^{\mathbb{Q}} + \sigma_i c_i \mathrm{d}B_{i,t}^{\mathbb{Q}} \\
&\quad - c_t \rho_d \log d_t \mathrm{d}t + \tfrac{1}{2} c_t \sigma_d^2 \mathrm{d}t - c_d d_t \sigma_d^2 \mathrm{d}t \\
\mathrm{d}x_{1,t} &= \left((\rho + \delta - (\varepsilon - 1)(\pi_t - \chi \pi_t^*)) x_{1,t} - d_t/(1 - s_g s_{g,t}) \right) \mathrm{d}t \\
\mathrm{d}x_{2,t} &= \left((\rho + \delta - \varepsilon(\pi_t - \chi \pi_t^*)) \, x_{2,t} - mc_t d_t/(1 - s_g s_{g,t}) \right) \mathrm{d}t \\
\mathrm{d}i_t &= \theta(\phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1) - (i_t - i_t^*))\mathrm{d}t + \sigma_i^2 V_{ai} V_a^{-1} \mathrm{d}t + \sigma_i \mathrm{d}B_{i,t}^{\mathbb{Q}} \\
\mathrm{d}v_t &= (\delta(1 - (\varepsilon - 1)(\pi_t - \chi\pi_t^*)/\delta)^{-\frac{\varepsilon}{1-\varepsilon}} + (\varepsilon(\pi_t - \chi\pi_t^*) - \delta) v_t)\mathrm{d}t \\
\mathrm{d}d_t &= -\left(\rho_d \log d_t - \tfrac{1}{2}\sigma_d^2\right) d_t \mathrm{d}t + \sigma_d^2 d_t^2 V_{ad} V_a^{-1} \mathrm{d}t + \sigma_d d_t \mathrm{d}B_{d,t}^{\mathbb{Q}} \\
\mathrm{d}A_t &= -(\rho_A \log A_t - \tfrac{1}{2}\sigma_A^2) A_t \, \mathrm{d}t + \sigma_A^2 A_t^2 V_{aA} V_a^{-1} \mathrm{d}t + \sigma_A A_t \mathrm{d}B_{A,t}^{\mathbb{Q}} \\
\mathrm{d}s_{g,t} &= -(\rho_g \log s_{g,t} - \tfrac{1}{2}\sigma_g^2) s_{g,t} \, \mathrm{d}t + \sigma_g^2 s_{g,t}^2 V_{ag} V_a^{-1} \mathrm{d}t + \sigma_g s_{g,t} \mathrm{d}B_{g,t}^{\mathbb{Q}}
\end{aligned}
$$

D. Figures

D.1. Data and implied dynamics

Figure D.1: US federal funds rate, output gap, cyclical components
In this figure we show time series plots of the US Effective Federal Funds Rate (Fed Funds), and different estimates of the Output gap based on potential output from the Congressional Budget Office (CBO), the Hodrick-Prescott (HP) filter, and the Beveridge-Nelson (BN) trend-cycle decomposition, and the same filter with dynamic mean adjustment (DMA). All series are obtained from the Federal Reserve Bank of St. Louis Economic Dataset (FRED). The sample runs from January, 1990, through June, 2017.

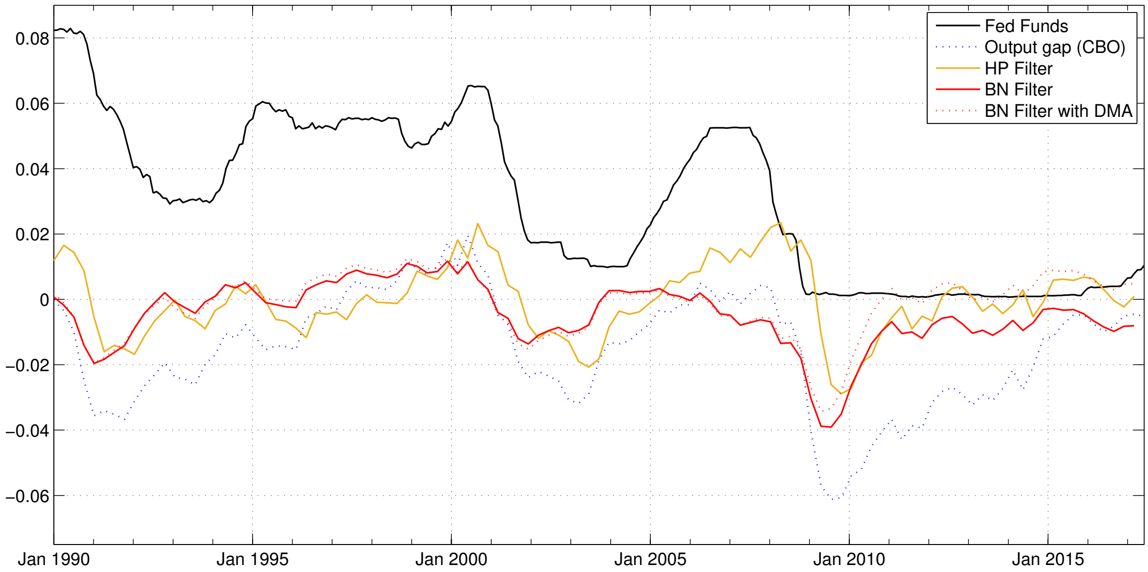


Figure D.2: Implied natural rate
In this figure we show time series plots of the model-implied 'natural rate' using the simple NK model, allowing for temporary and permanent shocks to the natural rate, when matching the observed US Effective Federal Funds Rate (Fed Funds), the 10-Year Treasury Constant Maturity Rate (10Y Govt), and the Consumer Price Index (Core CPI), seasonally adjusted, at the monthly frequency, and the same series together with the Output gap (HP Filter) at the quarterly frequency. The sample runs from January, 1990, through June, 2017.

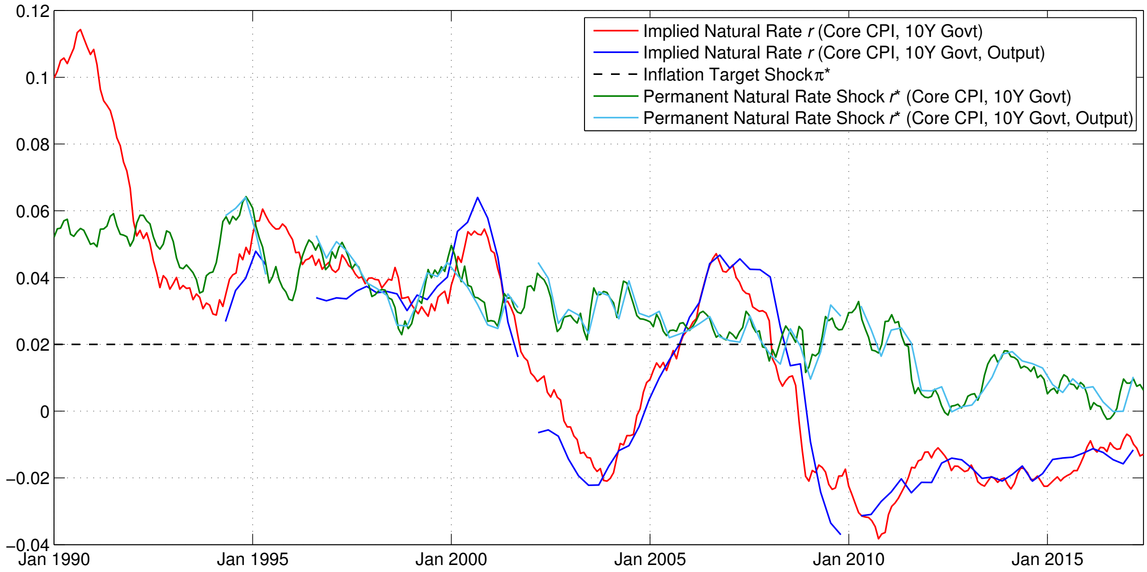


Figure D.3: Implied inflation rates and 10-year treasury rates

In this figure we show time series plots of the model-implied inflation and the 10-year treasury rates using the simple NK model, allowing for temporary and permanent shocks to the natural rate, when matching the observed US Effective Federal Funds Rate (Fed Funds), the 10-Year Treasury Constant Maturity Rate (10Y Govt), and the Consumer Price Index (Core CPI), seasonally adjusted, at the monthly frequency. The sample runs from January, 1990, through June, 2017.

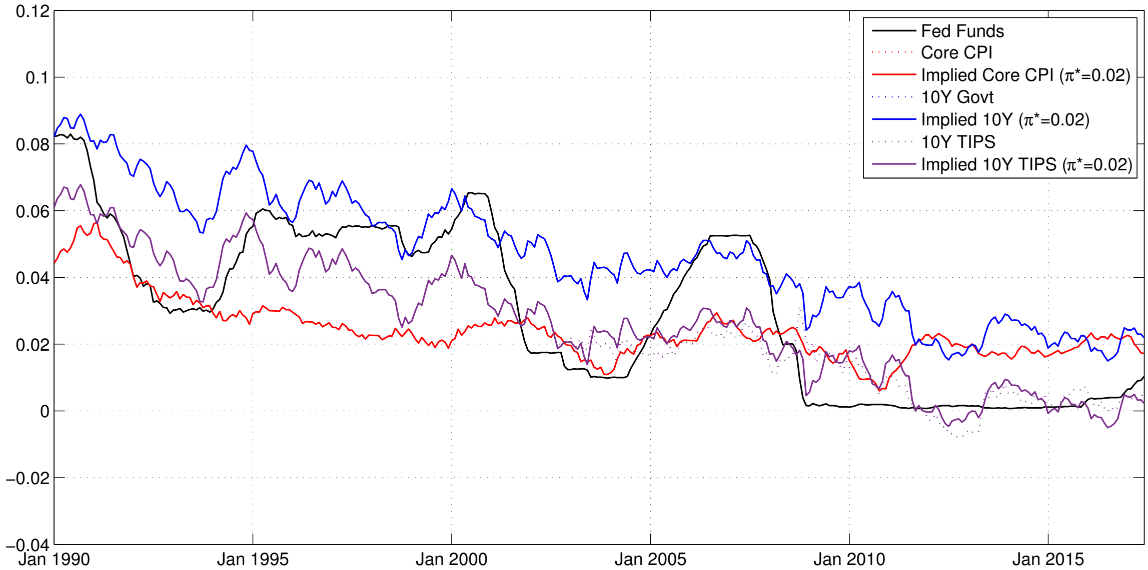
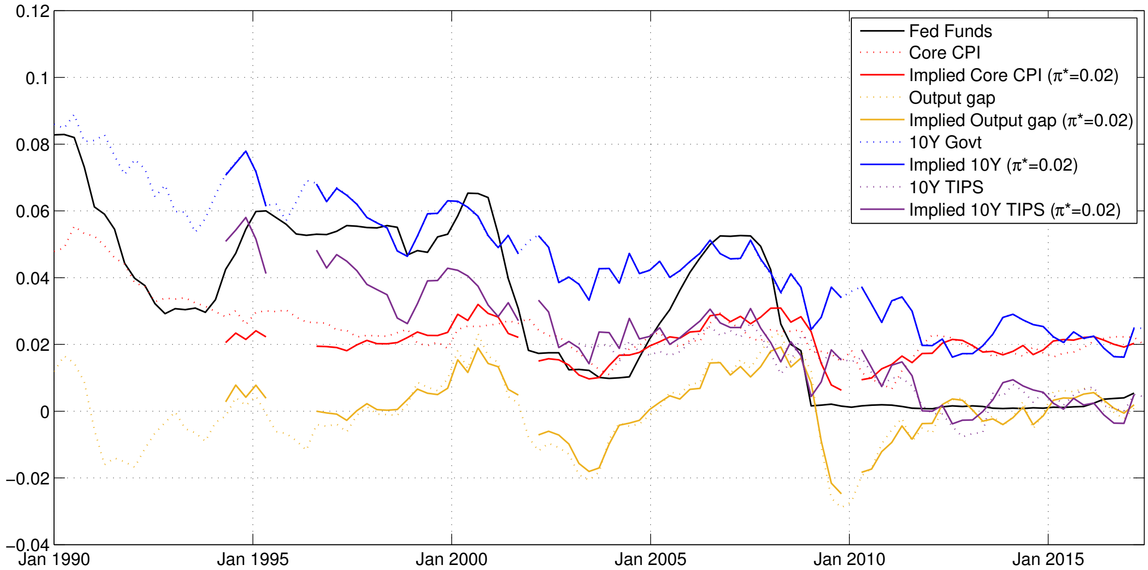


Figure D.4: Implied inflation rates, 10-year treasury rates and output gap

In this figure we show time series plots of the model-implied inflation, 10-year treasury rates, and the output gap using the simple NK model, allowing for temporary and permanent shocks to the natural rate, when matching the observed US Effective Federal Funds Rate (Fed Funds), the 10-Year Treasury Constant Maturity Rate (10Y Govt), and the Consumer Price Index (Core CPI), seasonally adjusted, and the Output gap (HP Filter) at the quarterly frequency (1990Q1-2017Q2).



D.2. Policy functions

Figure D.5: Solution of the linearized NK model with partial adjustment

In this figure we show (from left to right) the output gap, and the inflation rate as a function of the (initial) interest rate for a parameterization $(\rho, \kappa, \phi, \theta, \pi_{ss}) = (0.03, 0.8842, 4, 0.5, 0.02)$.

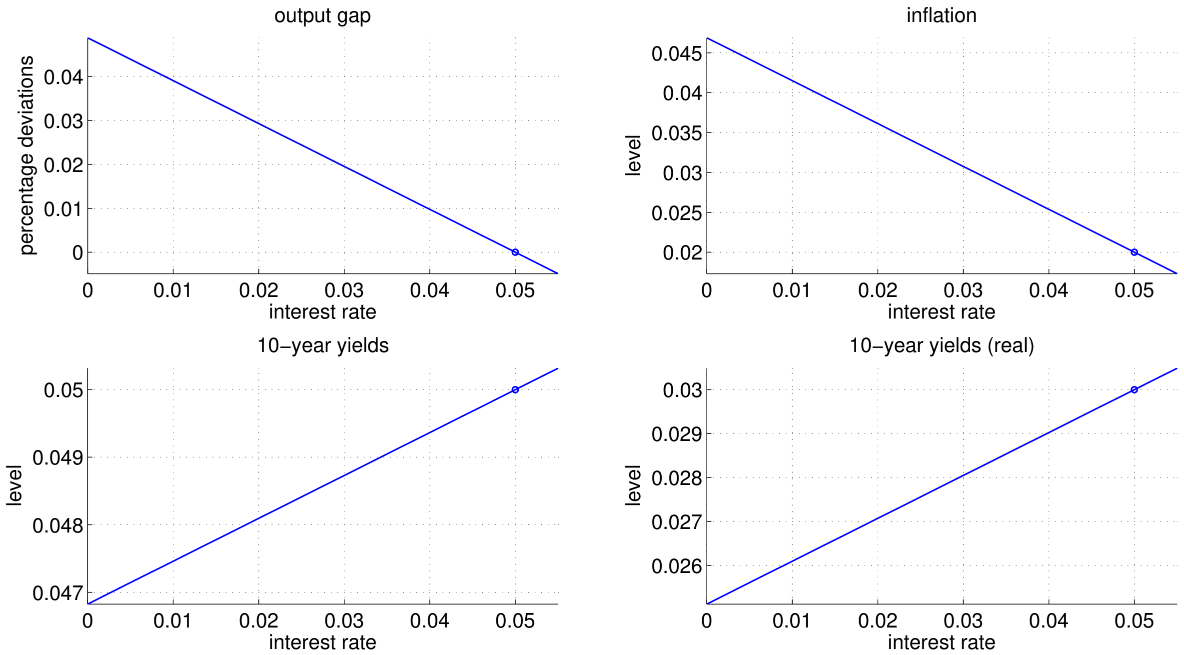


Figure D.6: Solution of the minimal NK model with partial adjustment

In this figure we show (from left to right) the output gap, and the inflation rate as a function of the (initial) interest rate in the minimal model (blue solid), in the linearized model (dashed) with full indexation at trend inflation, for a parameterization $(\rho, \kappa, \phi_\pi, \phi_y, \theta, \pi_{ss}, \chi) = (0.03, 0.8842, 4, 0, 0.5, 0.02, 1)$.

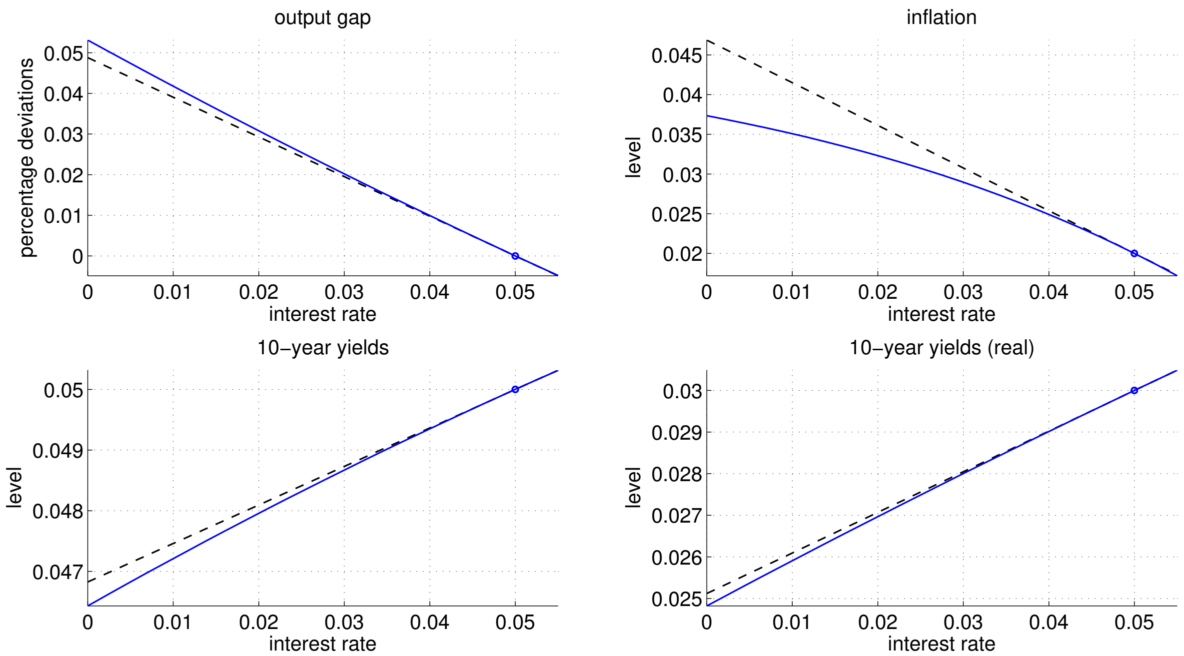


Figure D.7: Solution of the stochastic NK model

In this figure we show (from left to right, top to bottom) the optimal consumption, Euler equation errors, optimal hours, value function, output gap, auxiliary variable x_1, marginal cost, and auxiliary variable x_2 as a function of the interest rate. A blue solid line shows the solution of the stochastic model with partial adjustment, the black dotted line indicates the solution of the deterministic model.

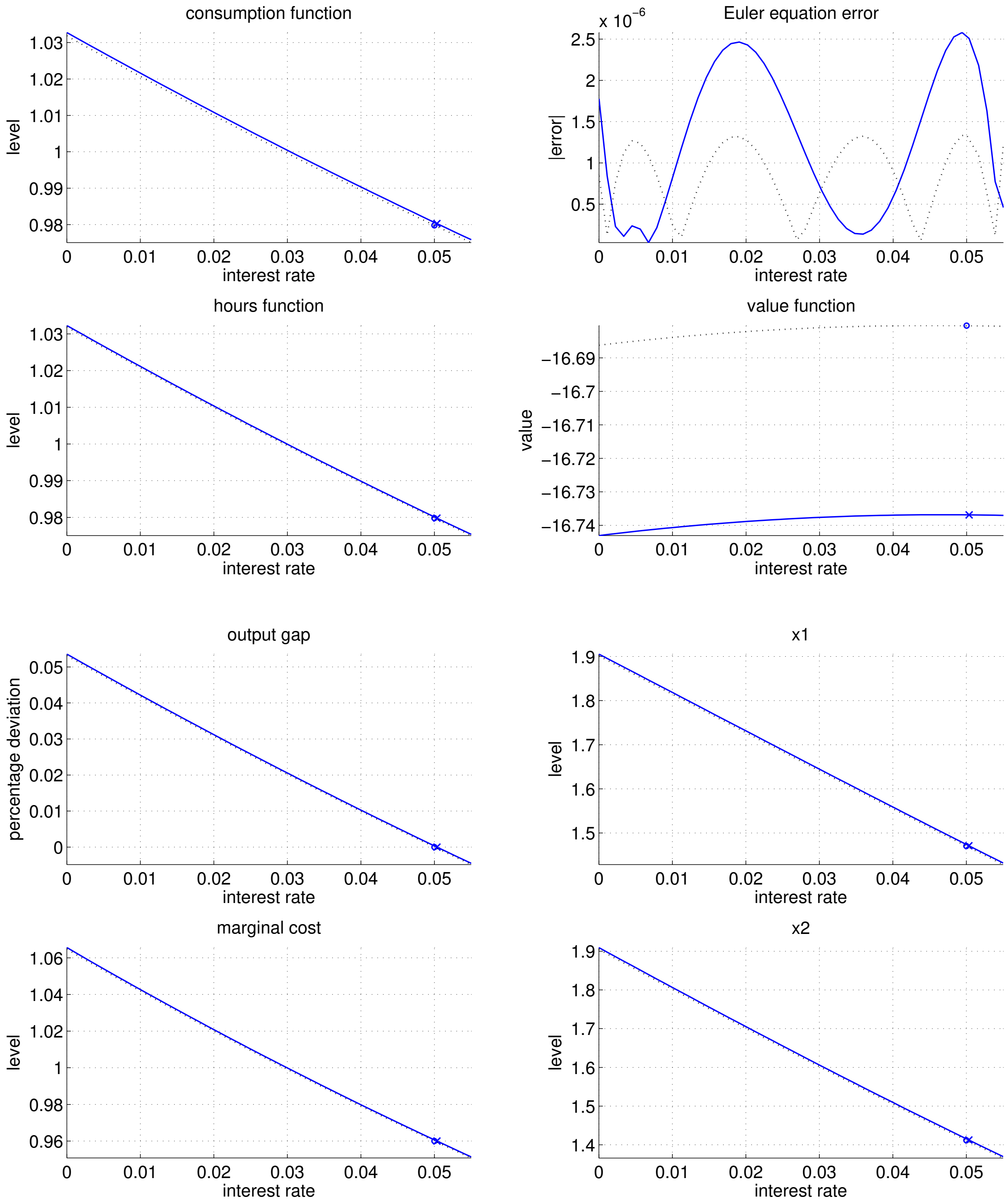


Figure D.8: Solution of the stochastic NK model

In this figure we show (from left to right, top to bottom) the real interest rate, natural rate, inflation, slope of the yield curve, interest rate, 1-year yields, 5-year yields, and 10-year yields as a function of the interest rate. A blue solid line shows the solution of the stochastic model with partial adjustment, the black dotted line indicates the solution of the deterministic model.

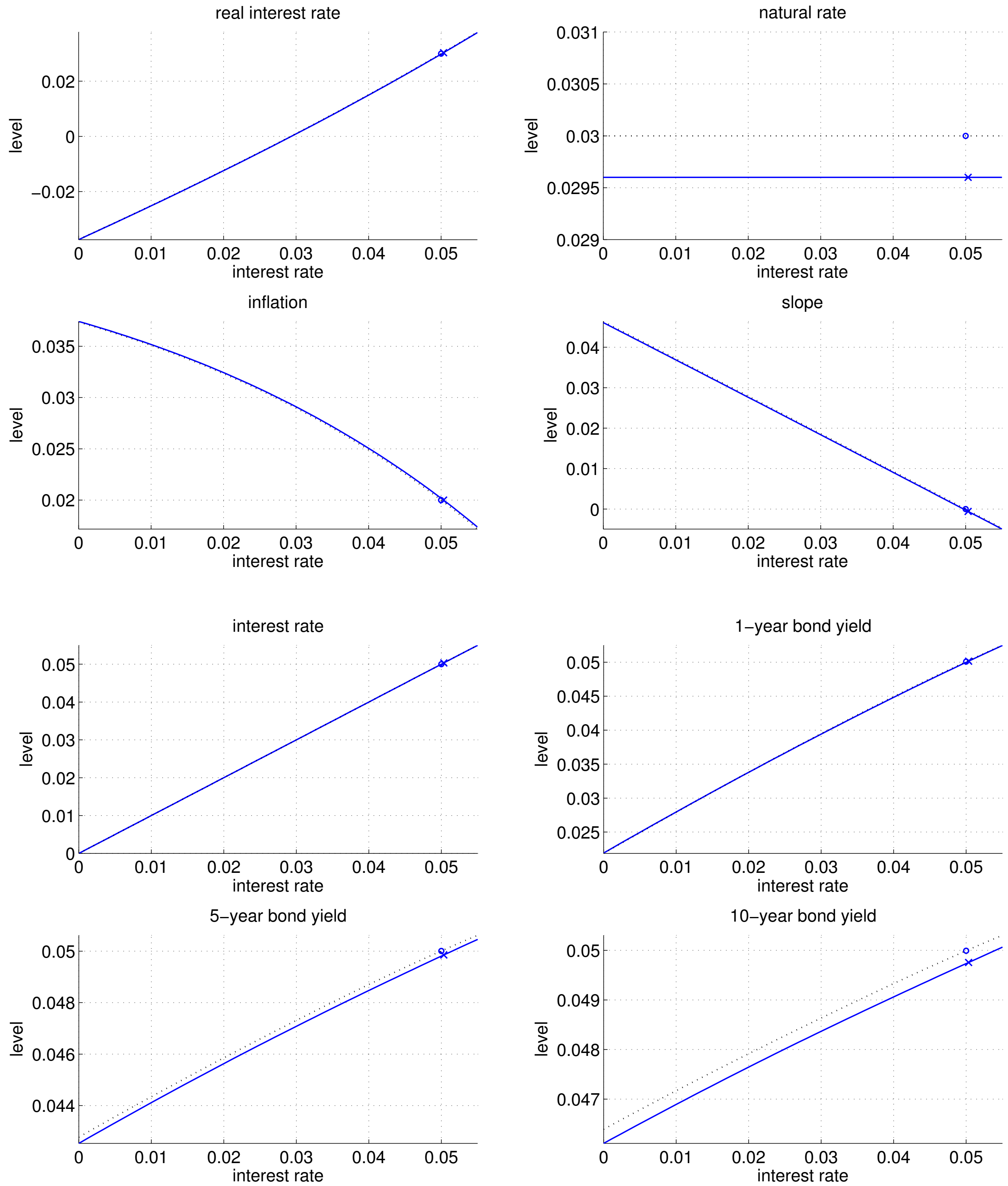


Figure D.9: Solution of the stochastic NK model

In this figure we show (from left to right, top to bottom) the optimal consumption, Euler equation errors, optimal hours, value function, output gap, auxiliary variable x_1, marginal cost, and auxiliary variable x_2 as a function of the preference shock. A blue solid line shows the solution of the stochastic model with partial adjustment, a red solid line shows the solution of the stochastic model with a feedback rule, the black dotted lines indicate the solutions of the deterministic models.

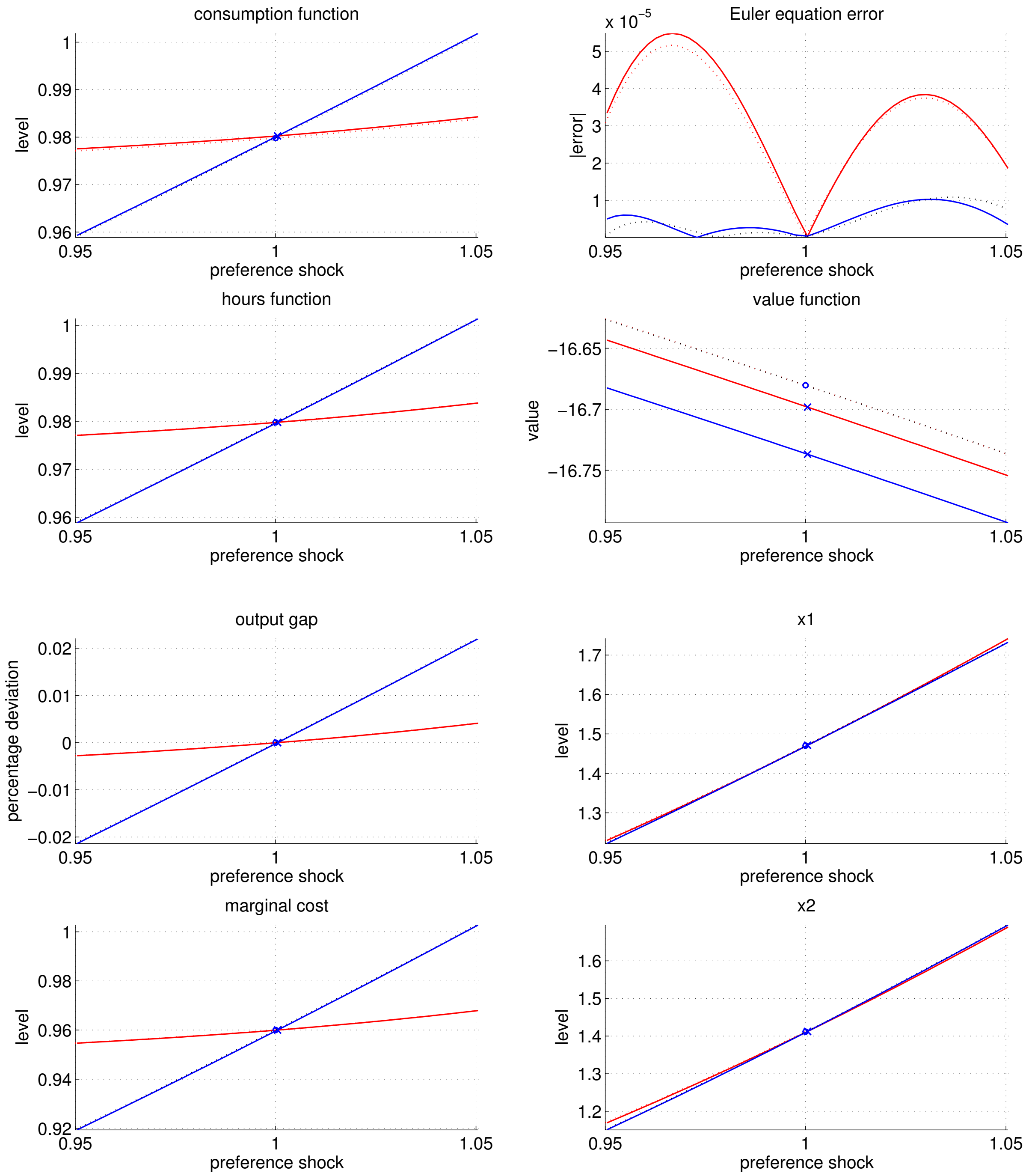


Figure D.10: Solution of the stochastic NK model

In this figure we show (from left to right, top to bottom) the real interest rate, natural rate, inflation, slope of the yield curve, interest rate, 1-year yields, 5-year yields, and 10-year yields as a function of the interest rate. A blue solid line shows the solution of the stochastic model with partial adjustment, a red solid line shows the solution of the stochastic model with a feedback rule, the black dotted lines indicate the solutions of the deterministic models.

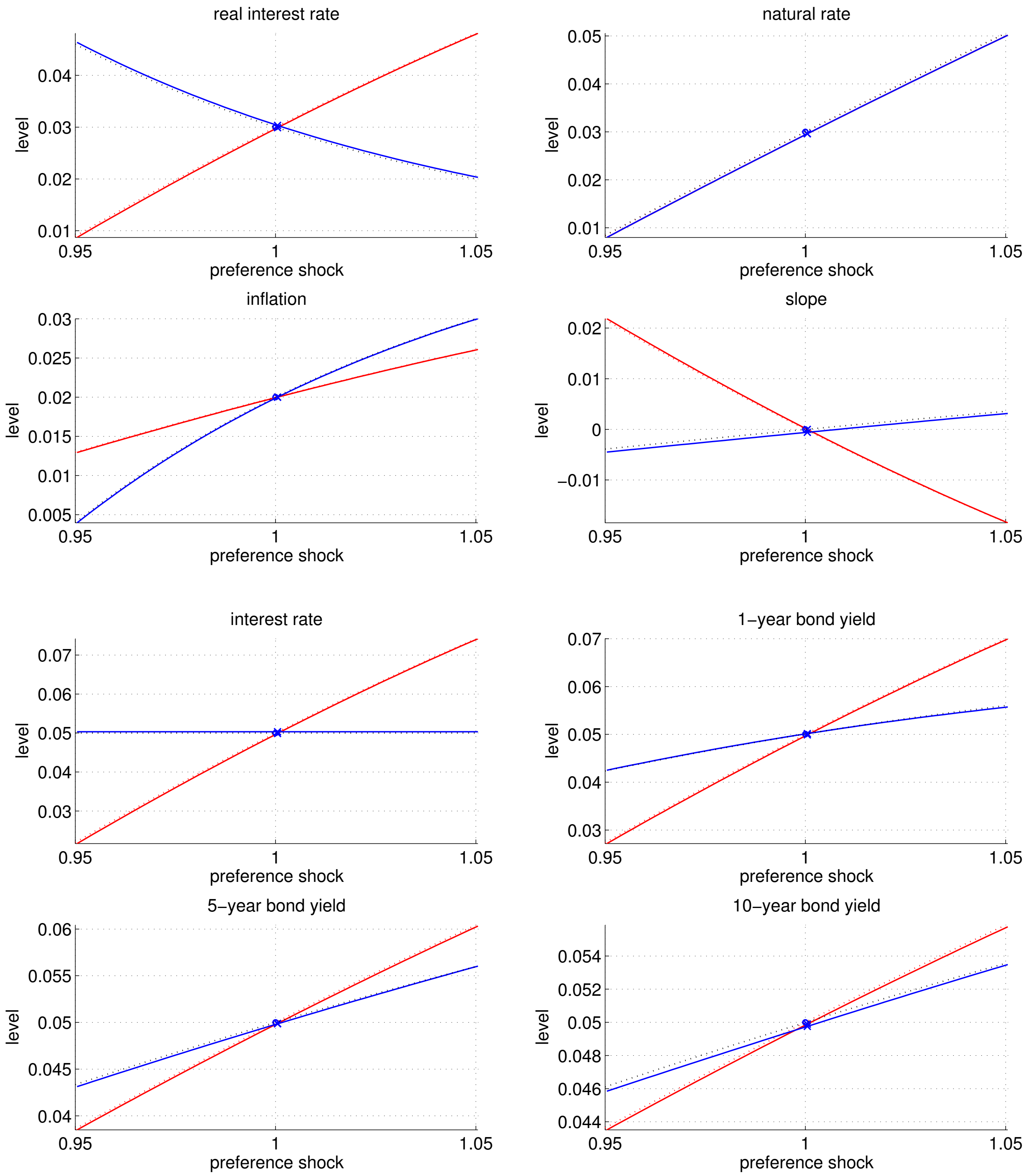


Figure D.11: Solution of the stochastic NK model

In this figure we show (from left to right, top to bottom) the optimal consumption, Euler equation errors, optimal hours, value function, output gap, auxiliary variable x_1, marginal cost, and auxiliary variable x_2 as a function of the technology shock. A blue solid line shows the solution of the stochastic model with partial adjustment, a red solid line shows the solution of the stochastic model with a feedback rule, the black dotted lines indicate the solutions of the deterministic models.

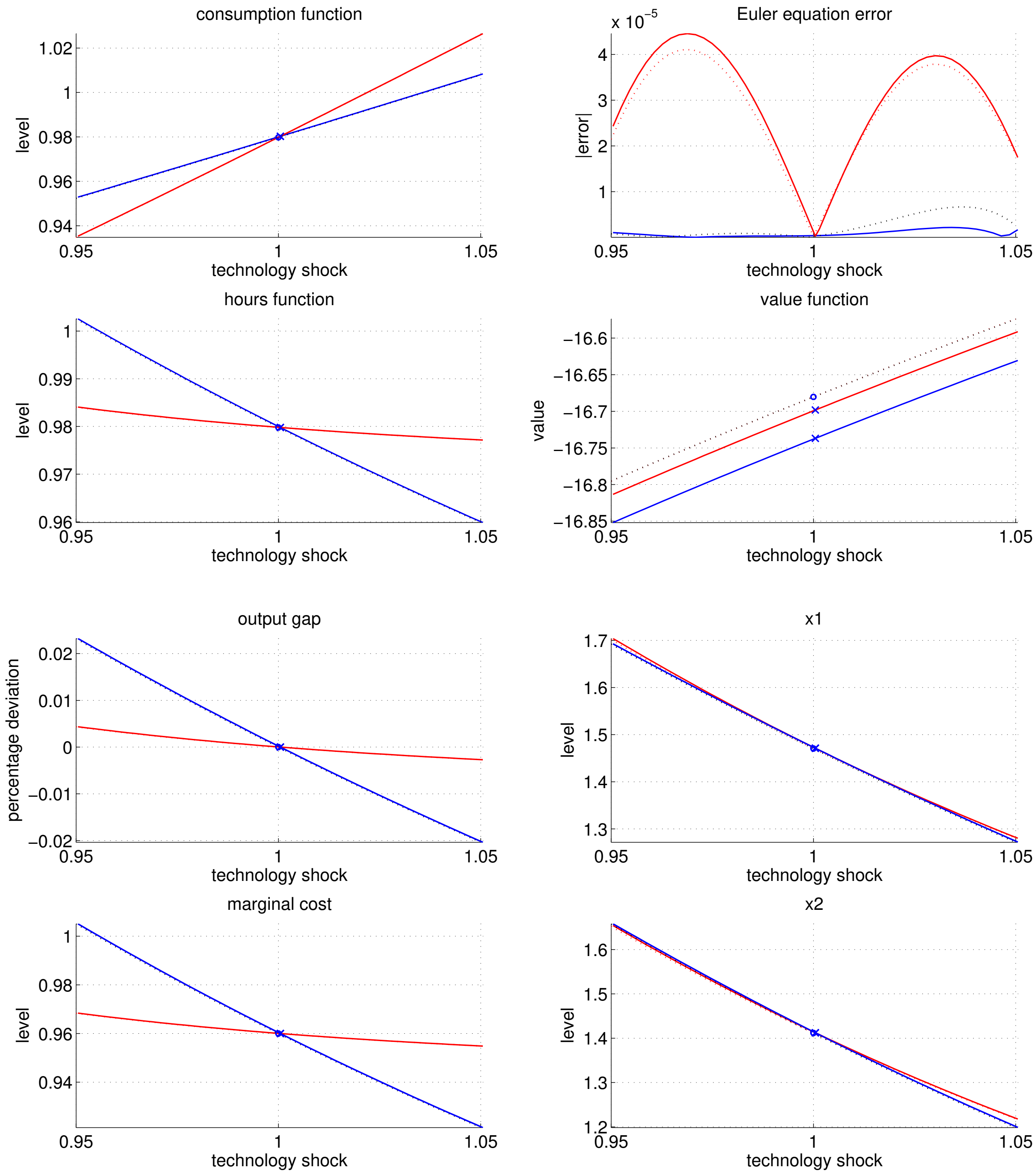
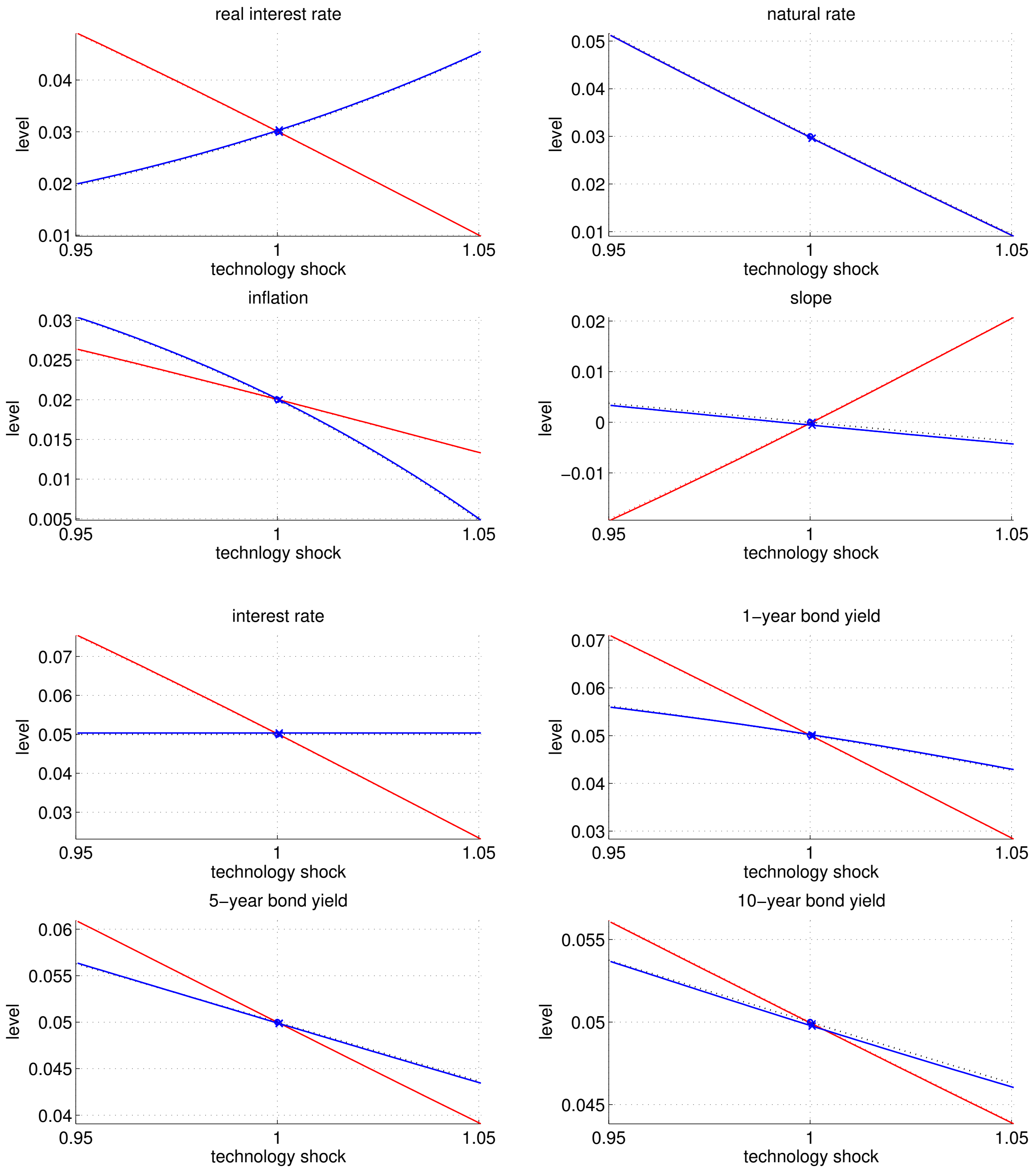


Figure D.12: Solution of the stochastic NK model

In this figure we show (from left to right, top to bottom) the real interest rate, natural rate, inflation, slope of the yield curve, interest rate, 1-year yields, 5-year yields, and 10-year yields as a function of the interest rate. A blue solid line shows the solution of the stochastic model with partial adjustment, a red solid line shows the solution of the stochastic model with a feedback rule, the black dotted lines indicate the solutions of the deterministic models.



D.3. Impulse responses

Figure D.13: Responses to monetary policy shocks (temporary and permanent)
In this figure we show (from left to right, top to bottom) the simulated responses to unexpected monetary policy shocks to both the (initial) interest rate (−0.025) and the inflation target rate (−0.0075), with effects for the output gap, the inflation rate, and the level and slope of the interest rate in the minimal model (blue solid), in the linearized version (dashed), and in the simple NK model(around $\pi_{ss} = 0$, dotted).

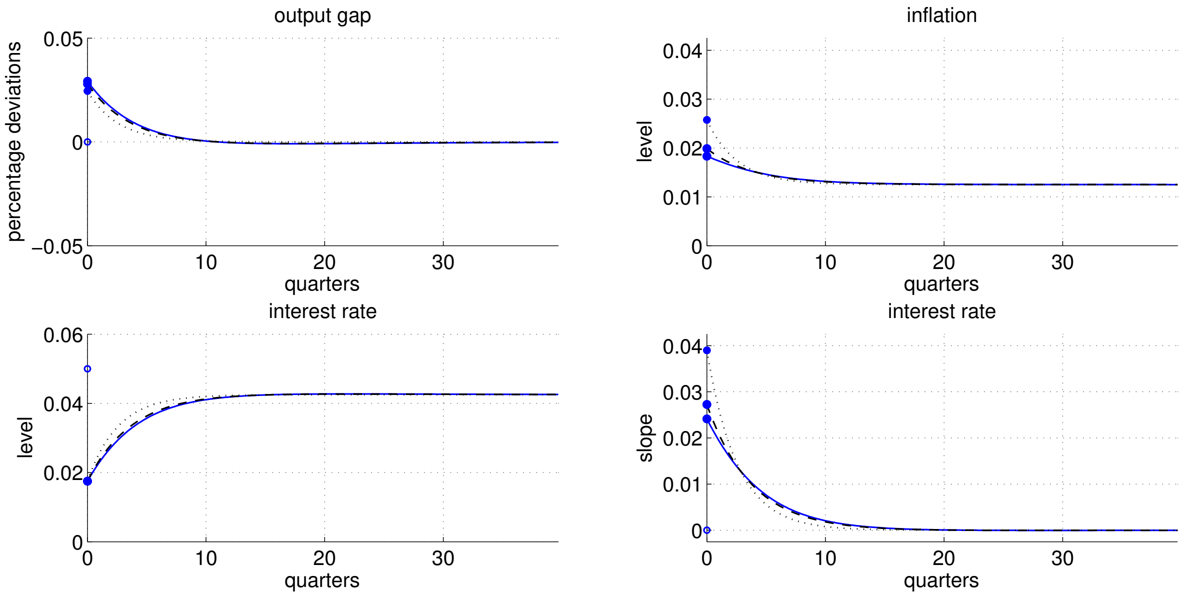


Figure D.14: Responses to monetary policy shocks at near zero interest rates
In this figure we show (from left to right, top to bottom) the simulated responses to unexpected monetary policy shocks to both the (initial) interest rate (−0.025) and the inflation target rate (−0.0075), with effects for the output gap, the inflation rate, and the level and slope of the interest rate in the minimal model (blue solid), in the linearized version (dashed), and in the simple NK model(around $\pi_{ss} = 0$, dotted).

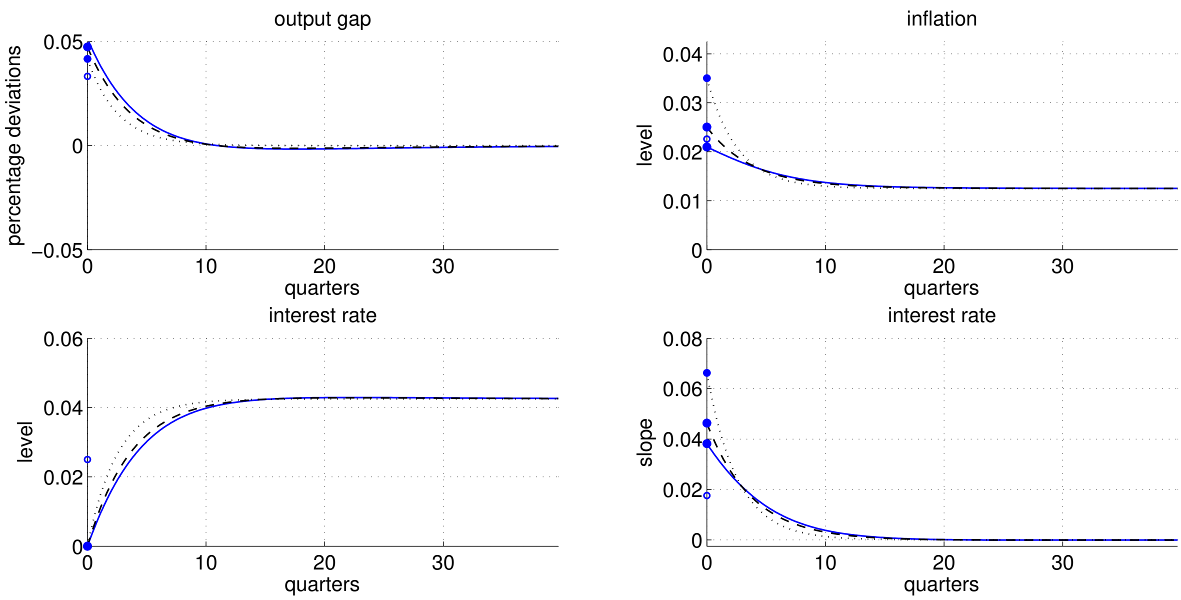


Figure D.15: Responses to monetary policy shocks (temporary and permanent)
In this figure we show (from left to right, top to bottom) the simulated responses to unexpected monetary policy shocks (0.01) either permanent (or target shock, left) or temporary (or initial interest rate, right), with effects for the interest rate (red dashed) and inflation (blue solid), and output in the minimal model (cf. Uribe, 2017, Figure 3). Effects for the simple NK modelare similar (not shown)

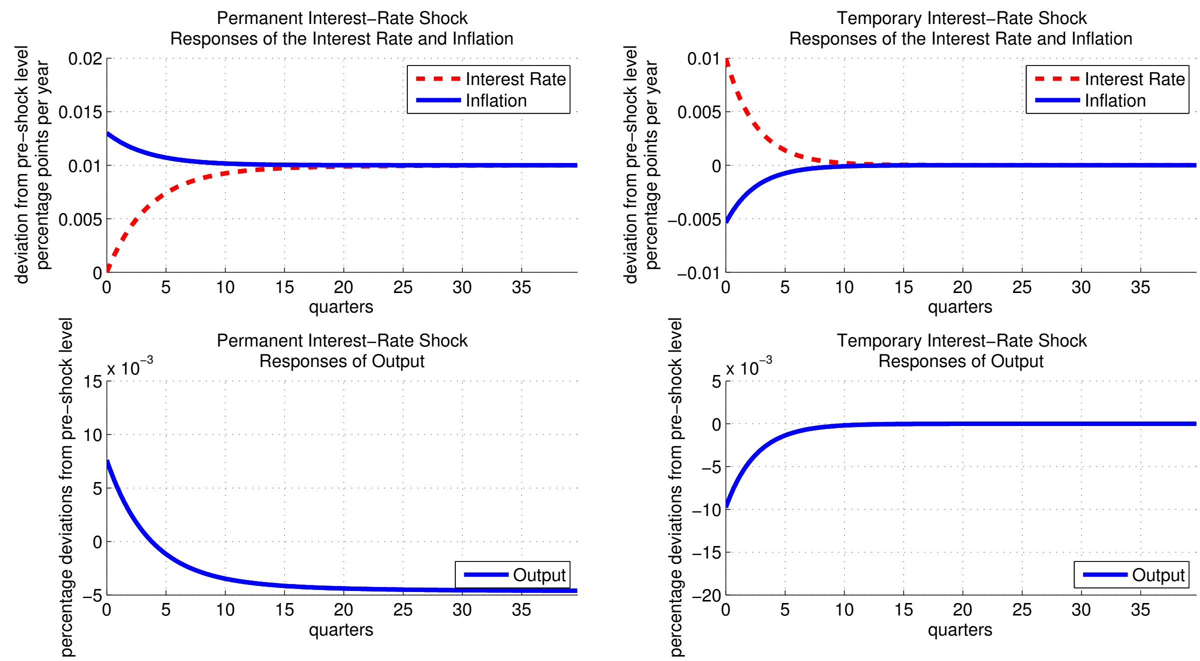
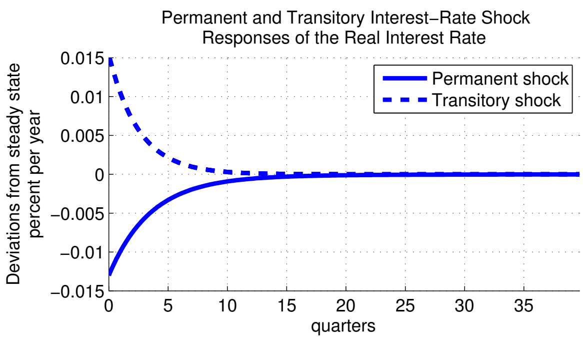


Figure D.16: Responses to monetary policy shocks (temporary and permanent)
In this figure we show (from left to right, top to bottom) the simulated responses to unexpected monetary policy shocks (0.01) either permanent (or target shock, left) or temporary (or initial interest rate, right), with effects for the real interest rate in the minimal model (cf. Uribe, 2017, Figure 4).



D.4. Simulated shocks

Figure D.17: Responses to monetary policy shocks (temporary and permanent)
In this figure we show (from left to right, top to bottom) the simulated responses for unexpected shocks to the (initial) interest rate (−0.025), and the inflation target rate (−0.005), with effects for the output gap, the inflation rate, and the level and slope of the interest rate (blue solid), and the no-target rate shock scenario in the simple NK model(black dashed, $\pi_{ss} = 0.02, \chi = 0$).

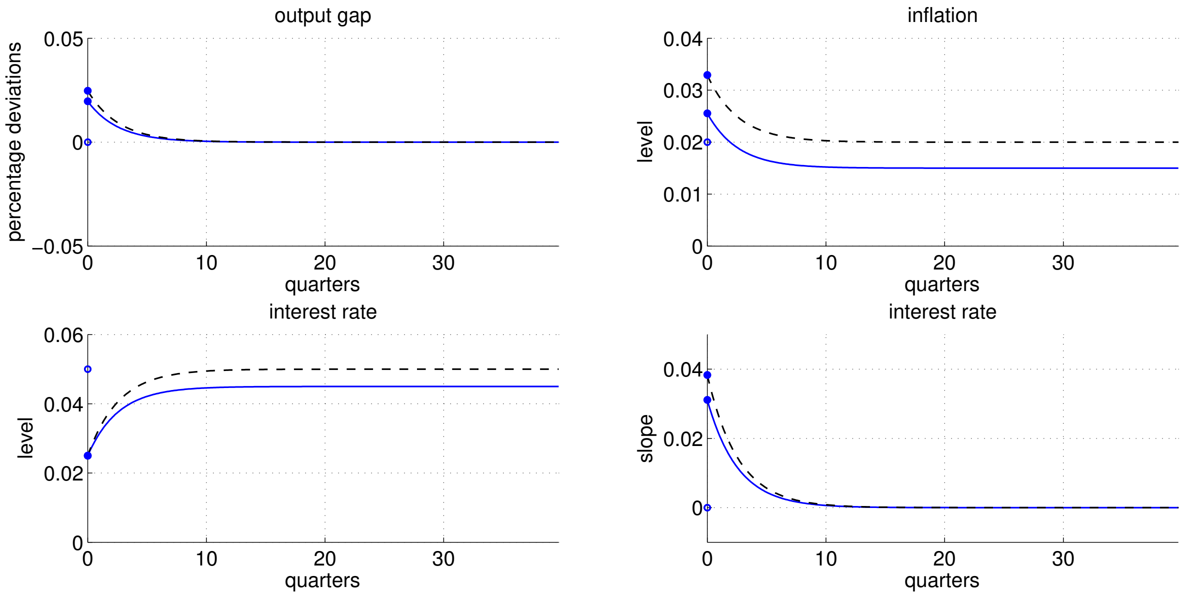


Figure D.18: Simulated shocks to interest rate and target rate (2001-2003)
In this figure we show (from left to right, top to bottom) the simulated responses to unexpected shocks to the (initial) interest rate (−0.05) and the inflation target rate (−0.015), with effects for the output gap, the inflation rate, and the level and slope of the interest rate (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0.02$), and the pre-shock scenario (dotted).

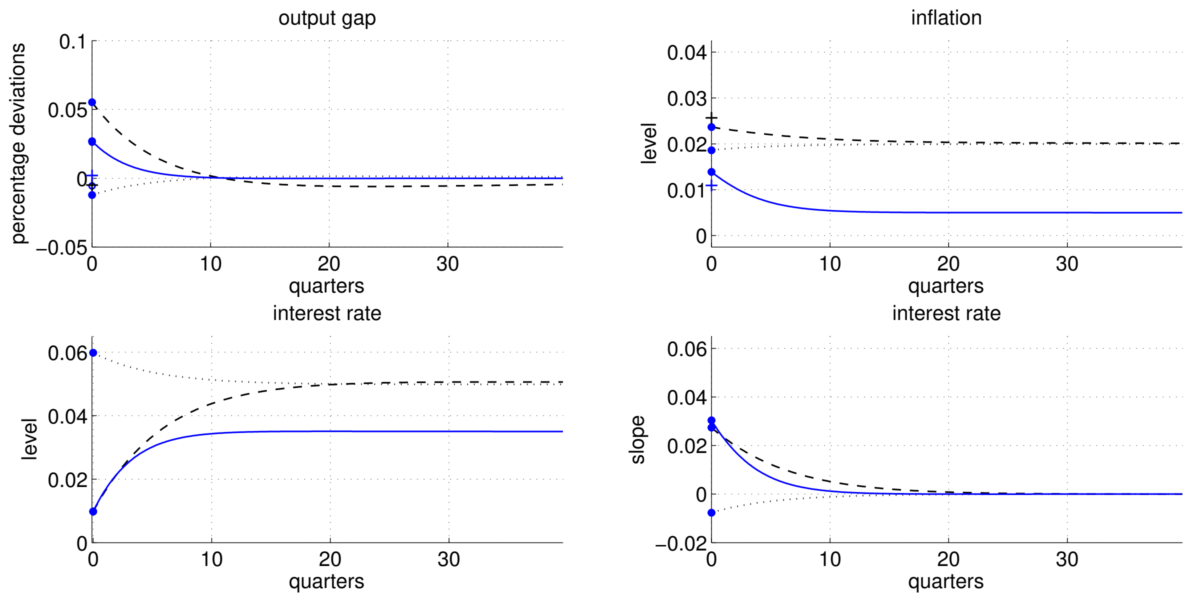


Figure D.19: Simulated shocks to interest rate and target rate (2001-2003), yields
In this figure we show the yield curve response to unexpected shocks to the (initial) interest rate (−0.05) and the inflation target rate (−0.015), with effects for the nominal and real yields (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0.02$), and the pre-shock scenario (dotted).

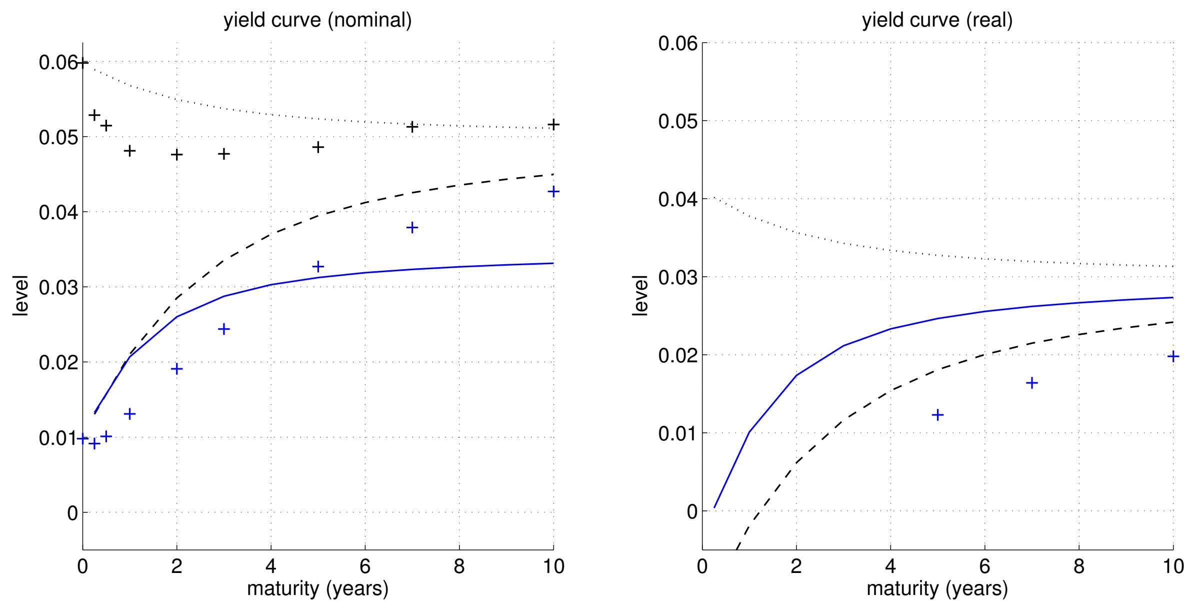


Figure D.20: Simulated shocks (2001-2003) with simple NK model around $\pi_{ss} \geq 0$
In this figure we show (from left to right, top to bottom) the simulated responses to unexpected shocks to the (initial) interest rate (-0.05) and the inflation target rate (-0.015), with effects for the output gap, the inflation rate, and the level and slope of the interest rate (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0.02$), and the pre-shock scenario (dotted).

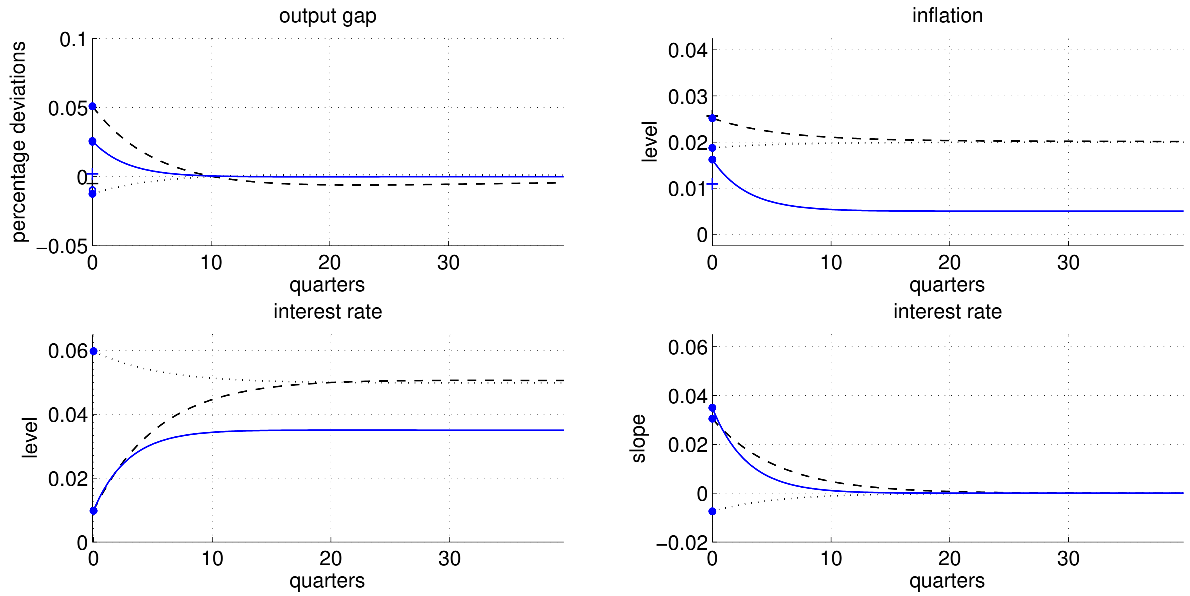


Figure D.21: Simulated shock (2001-2003) with simple NK model around $\pi_{ss} \geq 0$, yields
In this figure we show the yield curve response to unexpected shocks to the (initial) interest rate (-0.05) and the inflation target rate (-0.015), with effects for the nominal and real yields (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0.02$), and the pre-shock scenario (dotted).

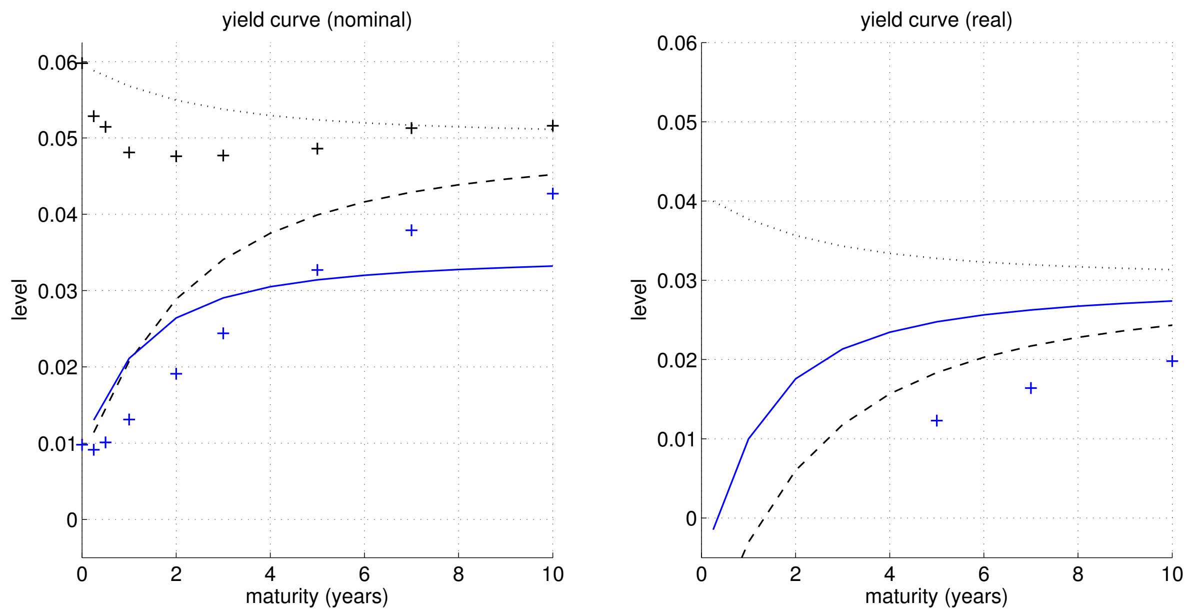


Figure D.22: Simulated shock to interest rate and target rate (2003-2007)
In this figure we show (from left to right, top to bottom) the simulated responses to unexpected shocks to the (initial) interest rate (0.04) and the inflation target rate (0.015) at price dispersion $v_0 = 1$, and its effect on the output gap, the inflation rate, and the level and slope of the interest rate (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0.005$), and the pre-shock scenario (dotted).

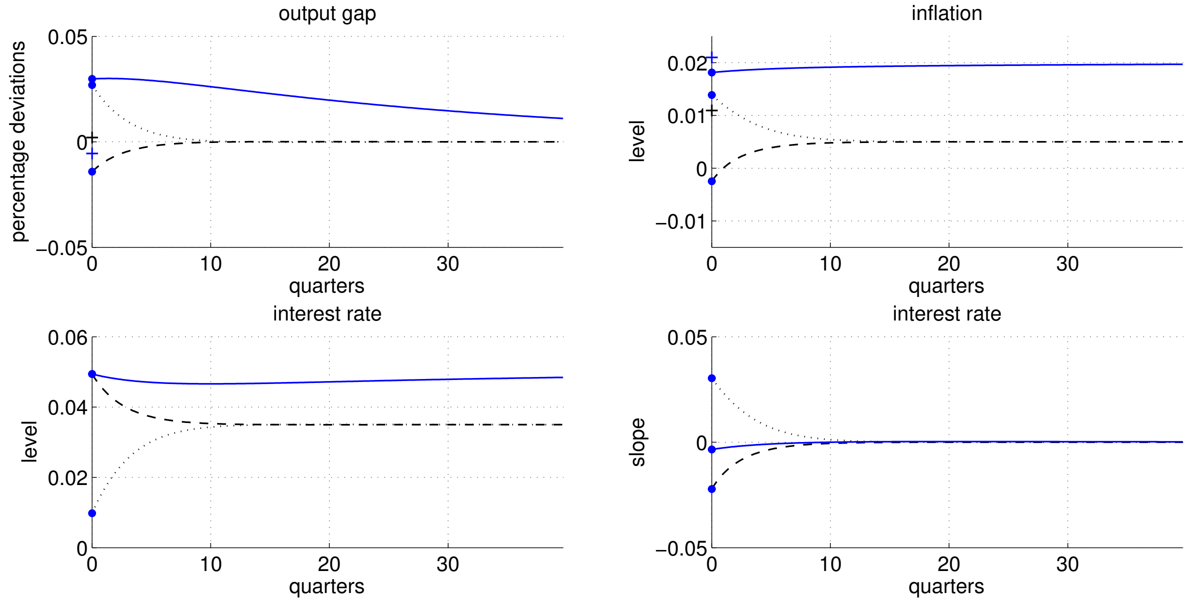


Figure D.23: Simulated shock to interest rate and target rate (2003-2007), yields
In this figure we show the yield curve response to unexpected shocks to the (initial) interest rate (0.04) and the inflation target rate (0.015) for $v_0 = 1$, with effects for the nominal and real yields (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0.005$), and the pre-shock scenario (dotted).

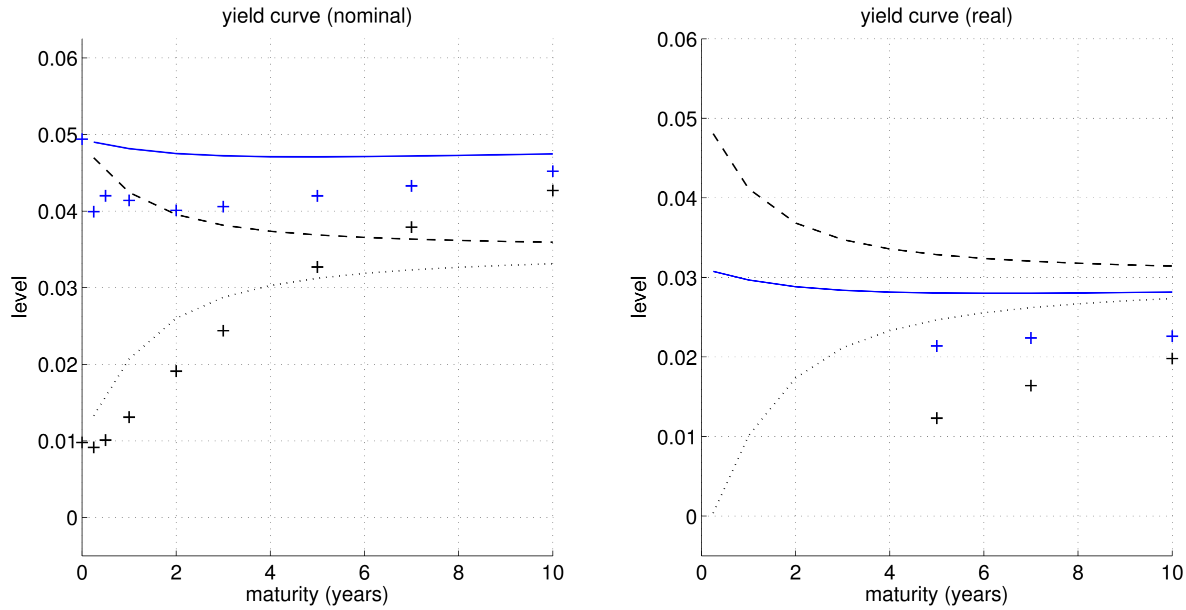


Figure D.24: Simulated shock (2003-2007) with simple NK model around $\pi_{ss} \geq 0$

In this figure we show (from left to right, top to bottom) the simulated responses to unexpected shocks to the (initial) interest rate (0.04) and the inflation target rate (0.015) at price dispersion $v_0 = 1$, and its effect on the output gap, the inflation rate, and the level and slope of the interest rate (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0.005$), and the pre-shock scenario (dotted).

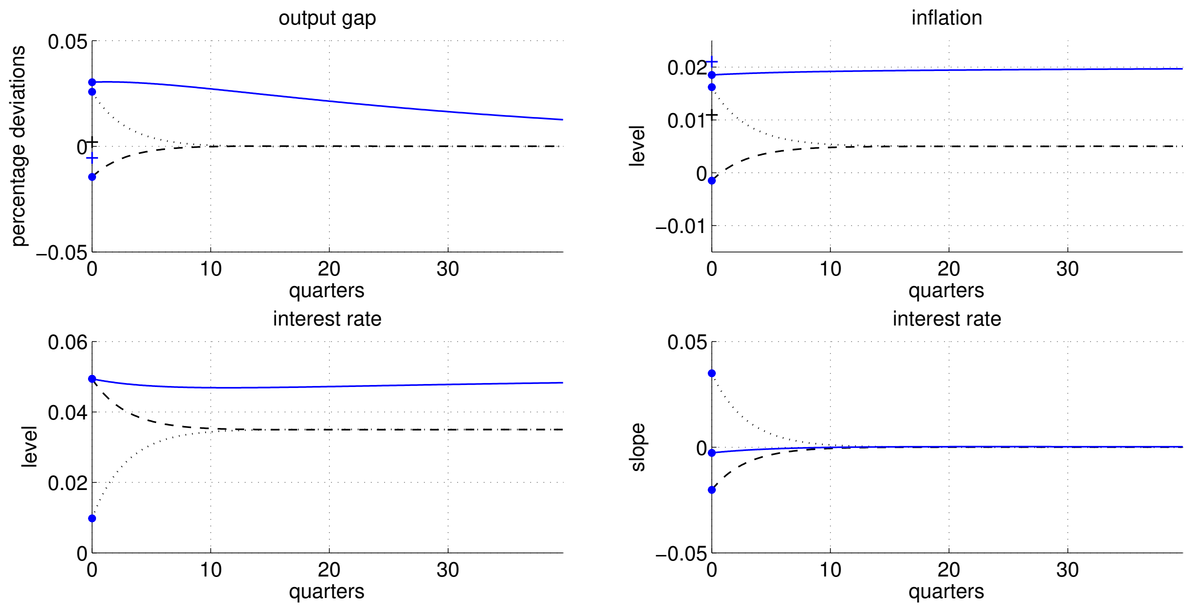


Figure D.25: Simulated shock (2003-2007) with simple NK model around $\pi_{ss} \geq 0$, yields

In this figure we show the yield curve response to unexpected shocks to the (initial) interest rate (0.04) and the inflation target rate (0.015) for $v_0 = 1$, with effects for the nominal and real yields (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0.005$), and the pre-shock scenario (dotted).

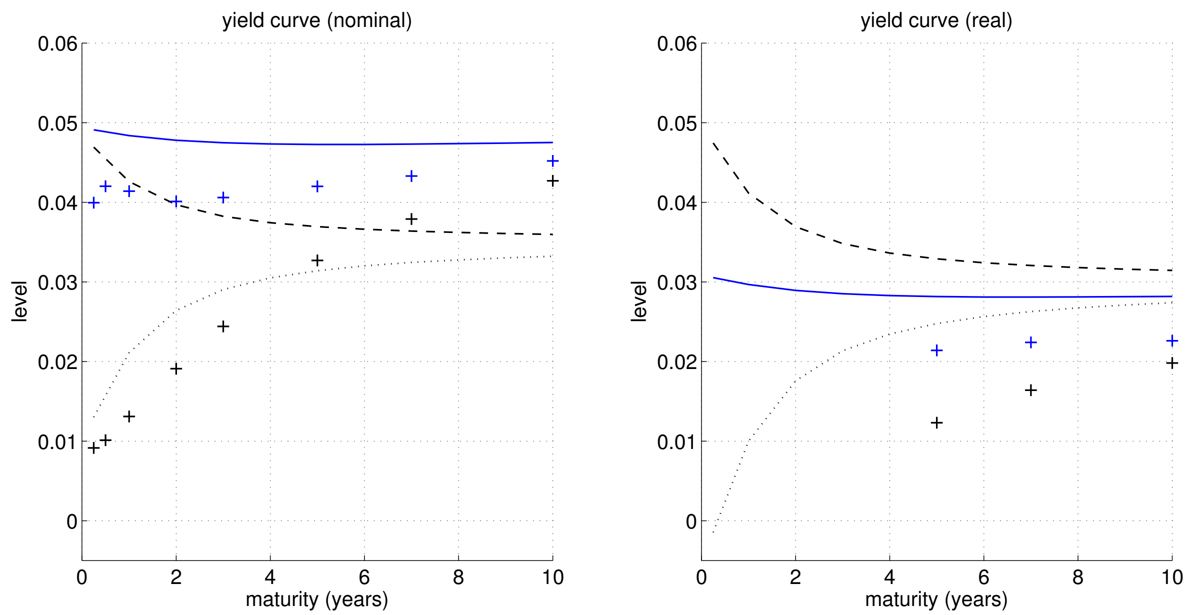


Figure D.26: Simulated shock to interest rate and target rate (2007-2009)
In this figure we show (from left to right, top to bottom) the simulated responses to unexpected shocks to the (initial) interest rate (−0.0475) and the inflation target rate (−0.02) and its effect on the output gap, the inflation rate, and the level and slope of the interest rate (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0.02$), and the pre-shock scenario (dotted).

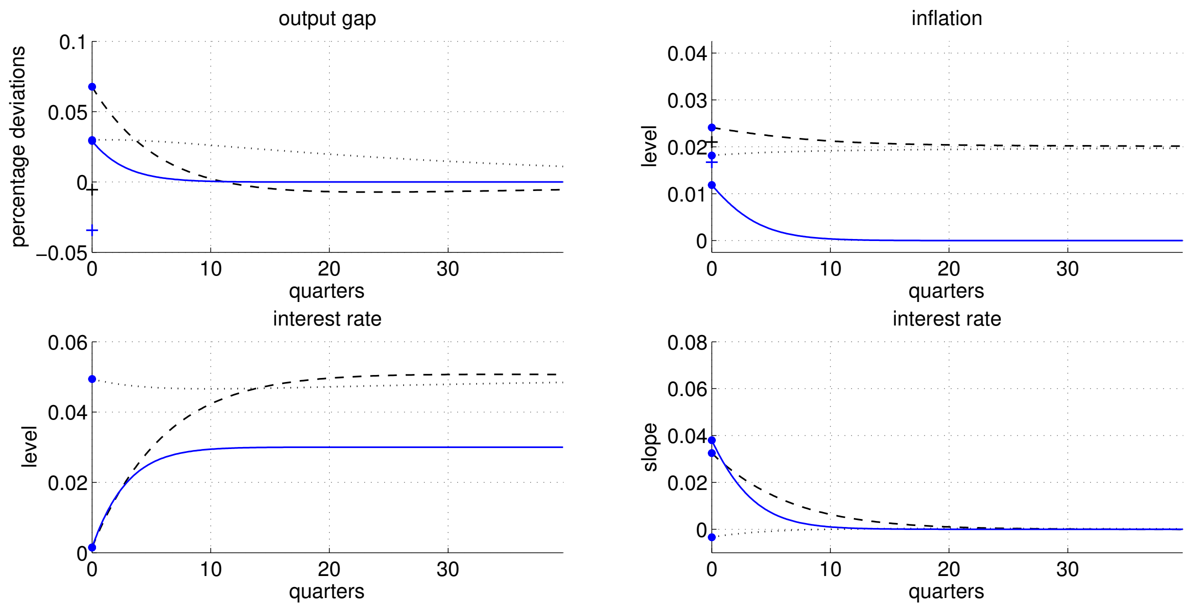


Figure D.27: Simulated shock to interest rate and target rate (2007-2009), yields
In this figure we show the yield curve response to unexpected shocks to the (initial) interest rate (−0.0475) and the inflation target rate (−0.02), with effects for the nominal and real yields (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0.02$), and the pre-shock scenario (dotted).

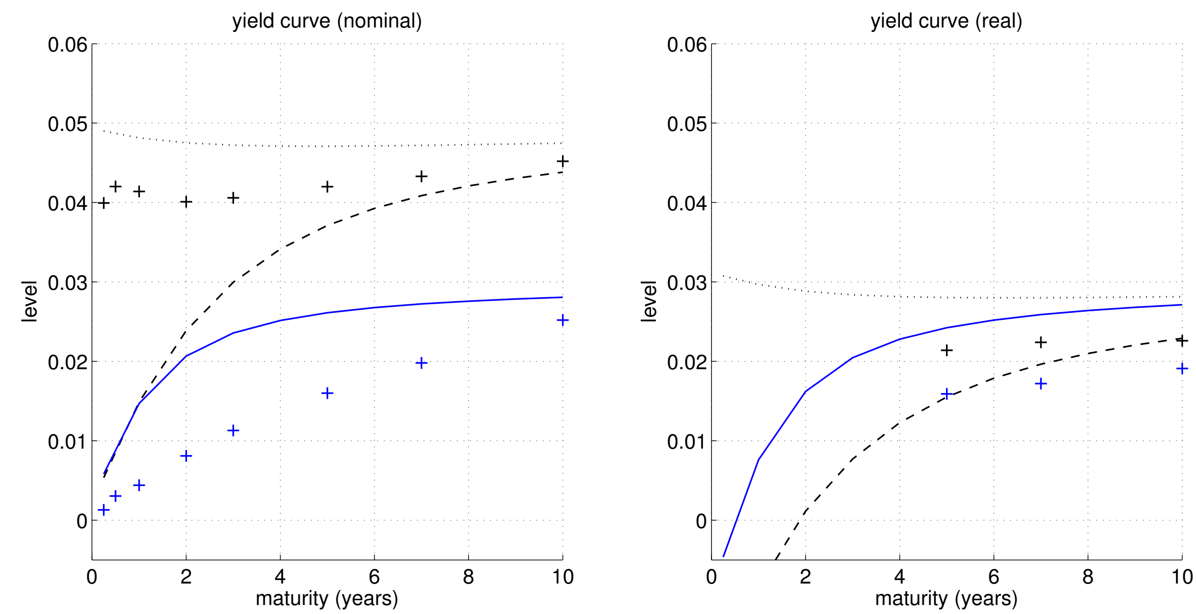


Figure D.28: Simulated shock (2007-2009) with simple NK model around $\pi_{ss} \geq 0$
In this figure we show (from left to right, top to bottom) the simulated responses to unexpected shocks to the (initial) interest rate (-0.0475) and the inflation target rate (-0.02) and its effect on the output gap, the inflation rate, and the level and slope of the interest rate (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0.02$), and the pre-shock scenario (dotted).

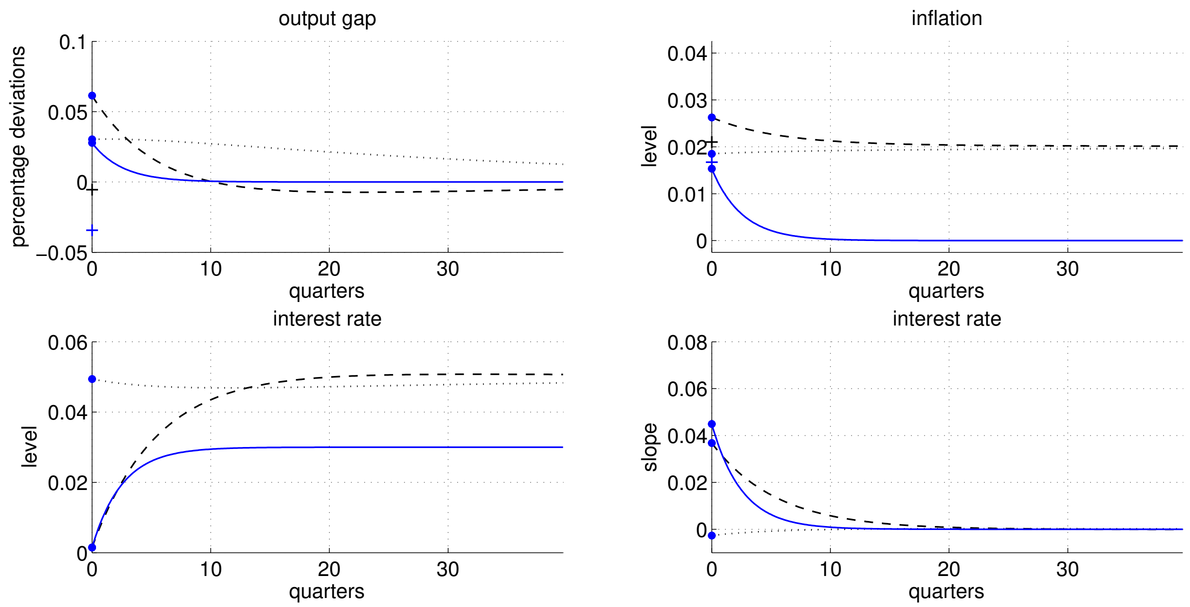


Figure D.29: Simulated shock (2007-2009) with simple NK model around $\pi_{ss} \geq 0$, yields
In this figure we show the yield curve response to unexpected shocks to the (initial) interest rate (-0.0475) and the inflation target rate (-0.02), with effects for the nominal and real yields (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0.02$), and the pre-shock scenario (dotted).

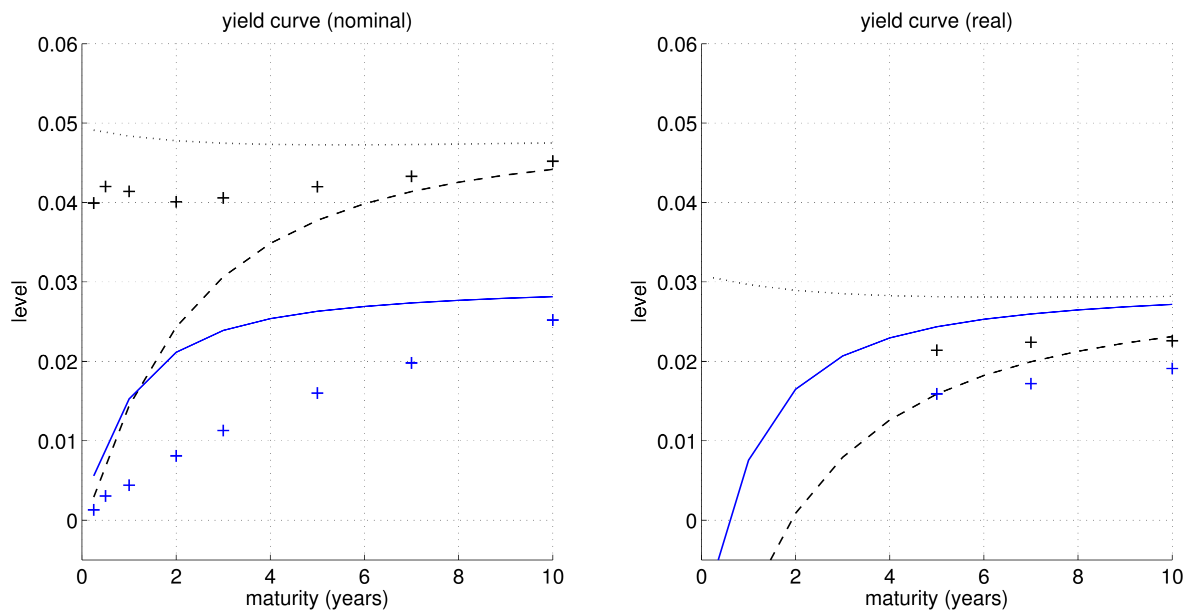


Figure D.30: Simulated shock to interest rate and target rate (2007-2010)
In this figure we show (from left to right, top to bottom) the simulated responses to unexpected shocks to the (initial) interest rate (-0.0475), the inflation target rate (-0.02), and preferences (-0.1), and its effect on the output gap, the inflation rate, and the level and slope of the interest rate (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0.02$), and the pre-shock scenario (dotted).

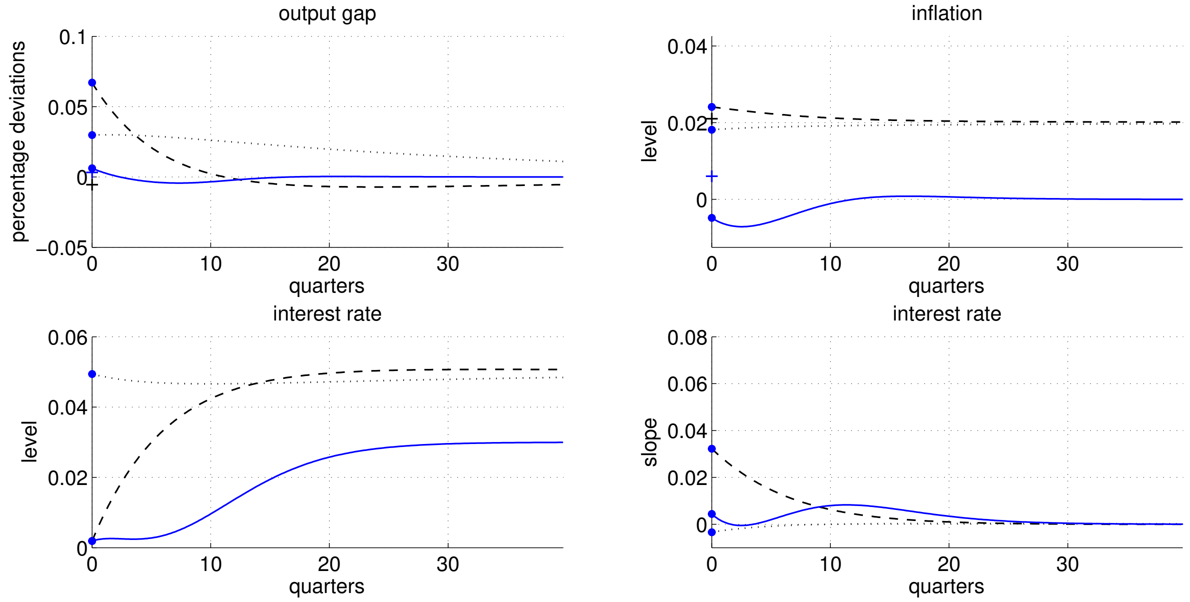


Figure D.31: Simulated shock to interest rate and target rate (2007-2010), yields
In this figure we show the yield curve response to unexpected shocks to the (initial) interest rate (-0.0475), the inflation target rate (-0.02), and preferences (-0.1), with effects for the nominal and real yields (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0.02$), and the pre-shock scenario (dotted).

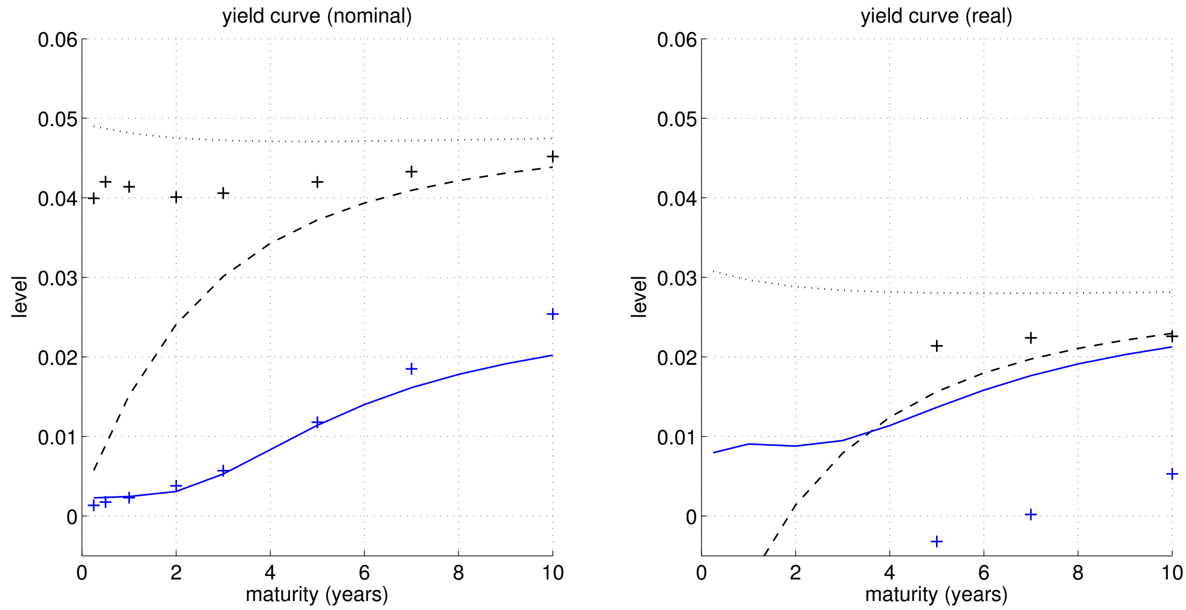


Figure D.32: Simulated shock (2007-2010) with simple NK model around $\pi_{ss} \geq 0$

In this figure we show (from left to right, top to bottom) the simulated responses to unexpected shocks to the (initial) interest rate (-0.0475), the inflation target rate (-0.02), and preferences (-0.1), and its effect on the output gap, the inflation rate, and the level and slope of the interest rate (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0.02$), and the pre-shock scenario (dotted).

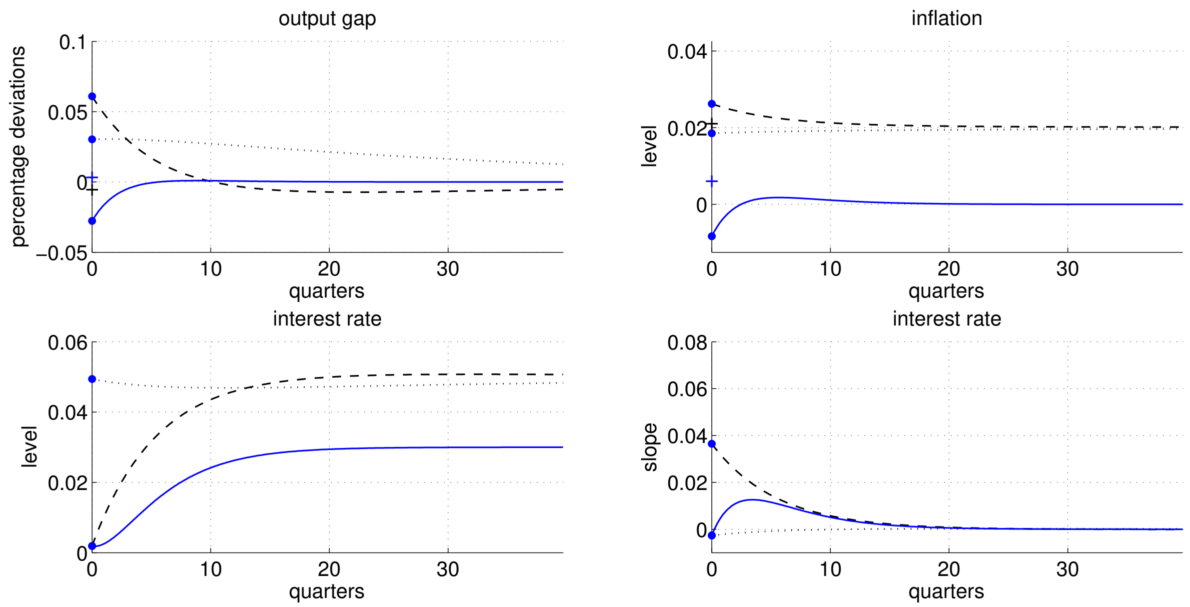


Figure D.33: Simulated shock (2007-2010) with simple NK model around $\pi_{ss} \geq 0$, yields

In this figure we show the yield curve response to unexpected shocks to the (initial) interest rate (-0.0475), the inflation target rate (-0.02), and preferences (-0.1), with effects for the nominal and real yields (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0.02$), and the pre-shock scenario (dotted).

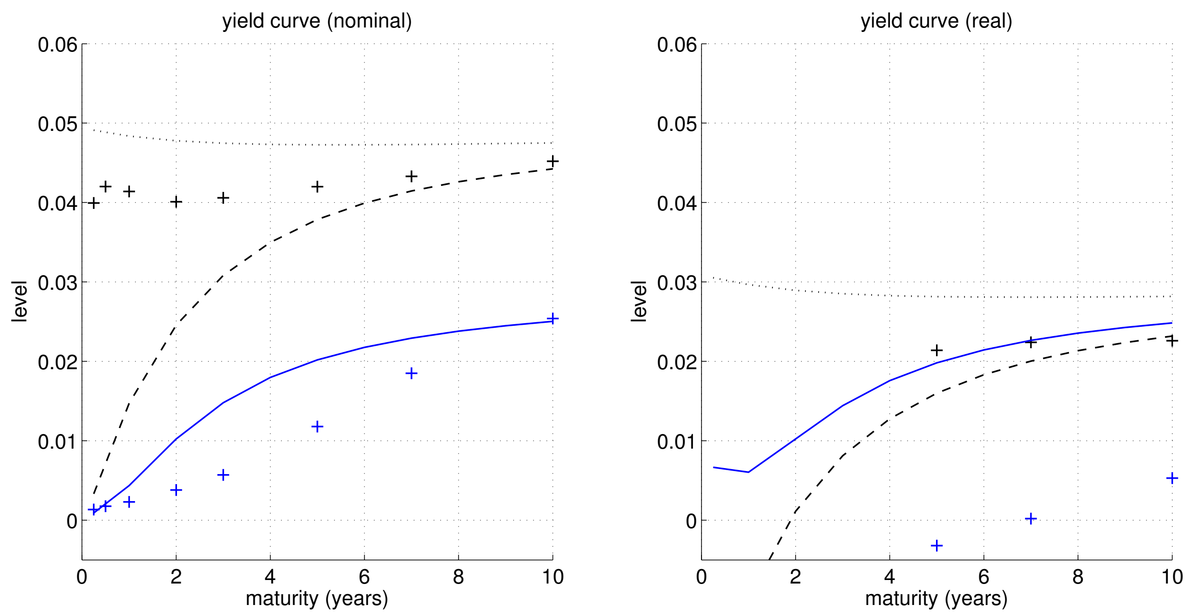


Figure D.34: Simulated shock to target rate (2010-2011)

In this figure we show (from left to right, top to bottom) the simulated responses to unexpected shocks to the inflation target rate (0.02), and preferences (−0.15), and its effect on the output gap, the inflation rate, and the level and slope of the interest rate (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0$, preferences −0.10), and the no-shock scenario (dotted, $\pi_{ss} = 0$).

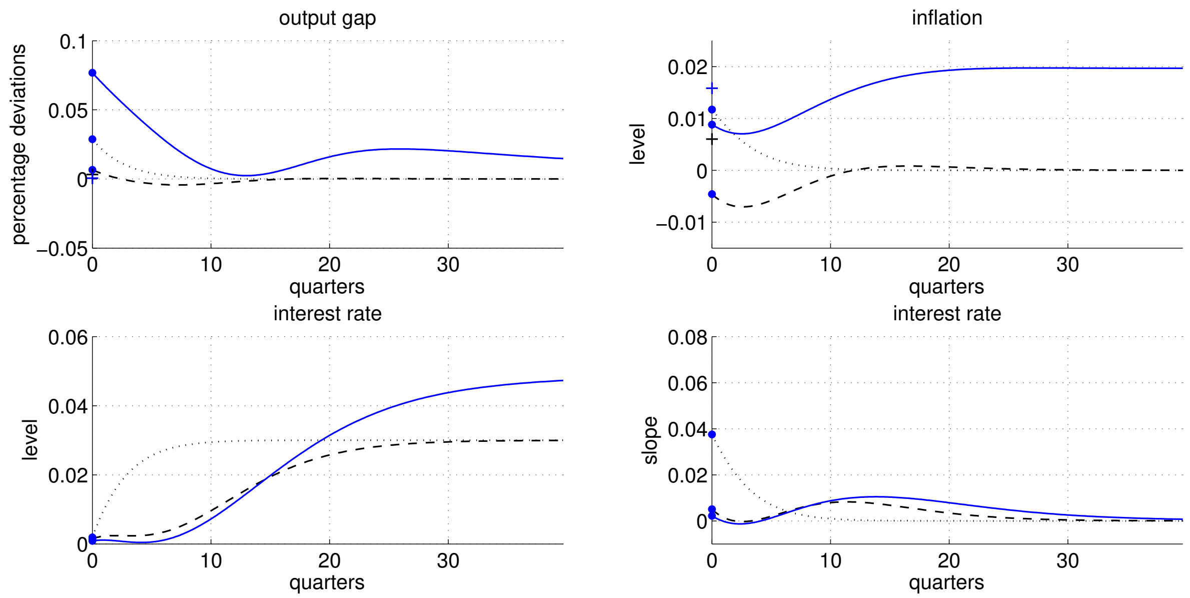


Figure D.35: Simulated shock to target rate (2010-2011), yields

In this figure we show the yield curve response to unexpected shocks to the inflation target rate (0.02), and preferences (−0.15), with effects for the nominal and real yields (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0$, preferences −0.10), and the no-shock scenario (dotted, $\pi_{ss} = 0$).

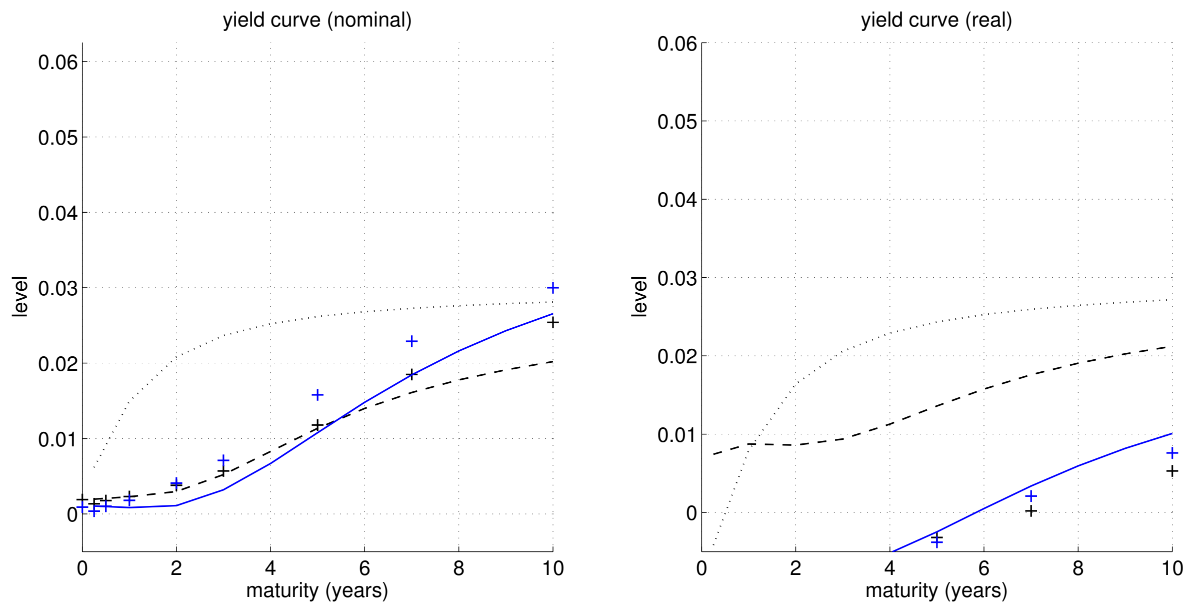


Figure D.36: Simulated shock (2010-2011) with simple NK model around $\pi_{ss} \geq 0$
In this figure we show (from left to right, top to bottom) the simulated responses to unexpected shocks to the inflation target rate (0.02), and preferences (−0.15), and its effect on the output gap, the inflation rate, and the level and slope of the interest rate (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0$, preferences −0.10), and the no-shock scenario (dotted, $\pi_{ss} = 0$).

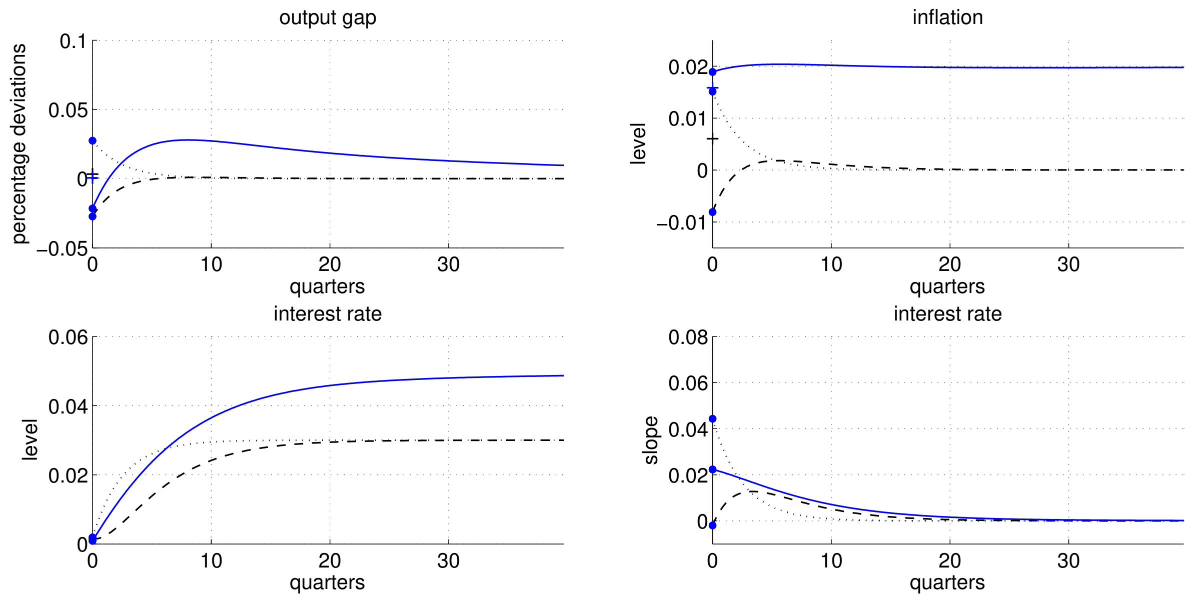


Figure D.37: Simulated shock (2010-2011) with simple NK model around $\pi_{ss} \geq 0$, yields
In this figure we show the yield curve response to unexpected shocks to the inflation target rate (0.02), and preferences (−0.15), with effects for the nominal and real yields (blue solid), the no-target rate shock scenario (black dashed, $\pi_{ss} = 0$, preferences −0.10), and the no-shock scenario (dotted, $\pi_{ss} = 0$).

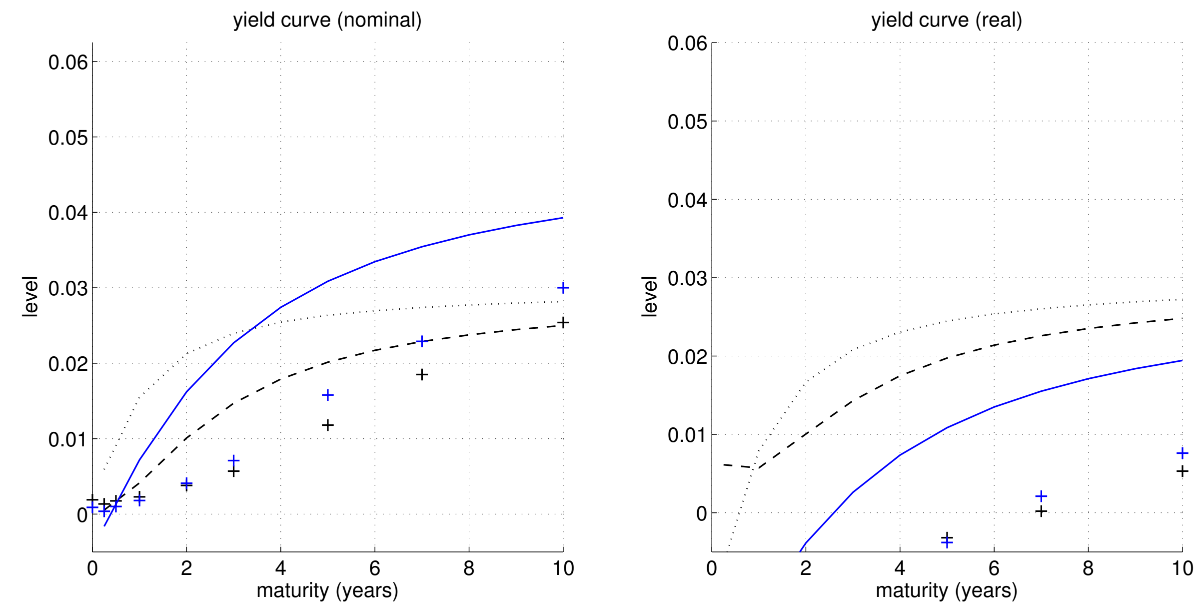


Figure D.38: Simulated shock to interest rate (2004-2005)

In this figure we show (from left to right, top to bottom) the simulated responses to unexpected shocks to the (initial) interest rate (0.015), and preferences (−0.1), and its effect on the output gap, the inflation rate, and the level and slope of the interest rate (blue solid), and the pre-shock scenario (dotted).

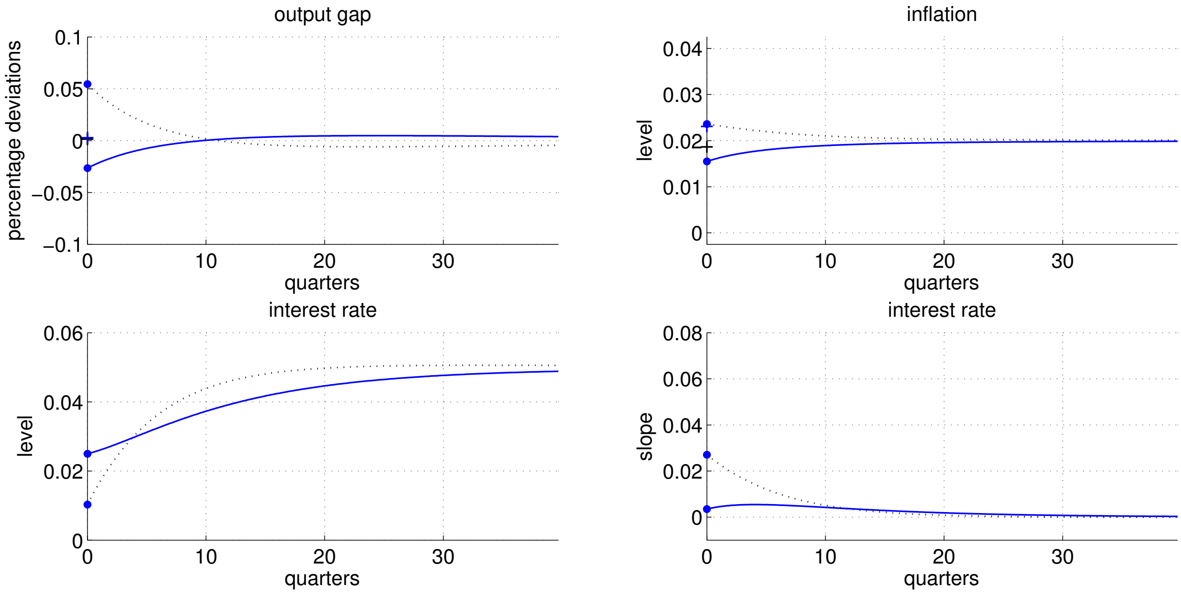


Figure D.39: Simulated shock to interest rate (2004-2005), yields

In this figure we show the yield curve response to unexpected shocks to the (initial) interest rate (0.015), and preferences (−0.1), with effects for the nominal and real yields (blue solid), and the pre-shock scenario (dotted).

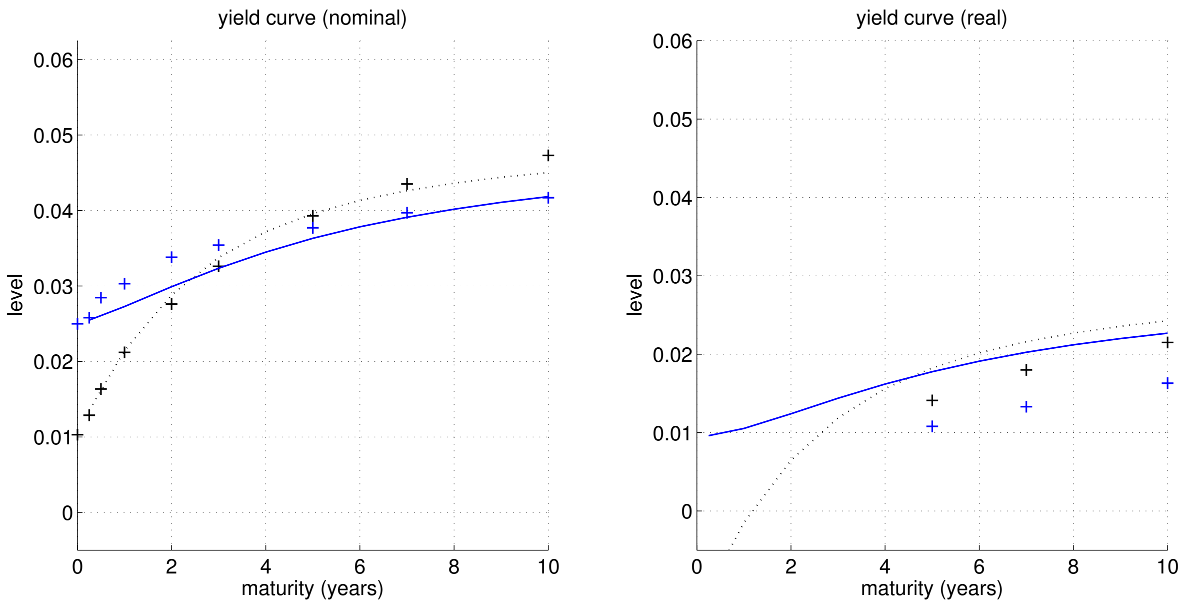


Figure D.40: Simulated shock (2004-2005) with simple NK model around $\pi_{ss} \geq 0$
In this figure we show (from left to right, top to bottom) the simulated responses to unexpected shocks to the (initial) interest rate (0.015), and preferences (-0.1), and its effect on the output gap, the inflation rate, and the level and slope of the interest rate (blue solid), and the pre-shock scenario (dotted).

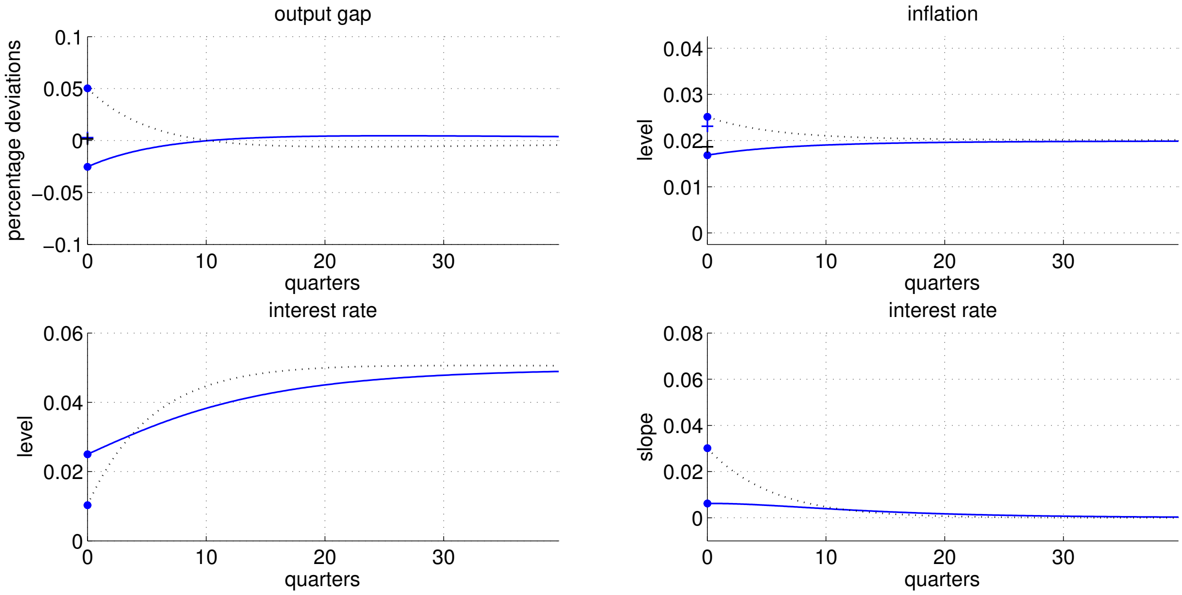
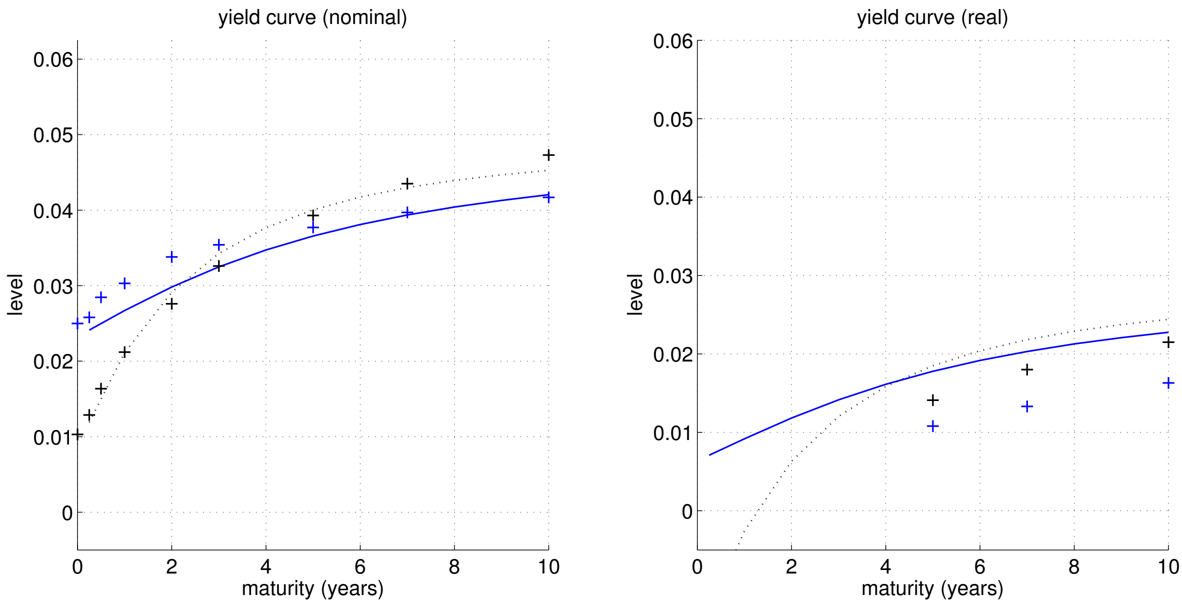


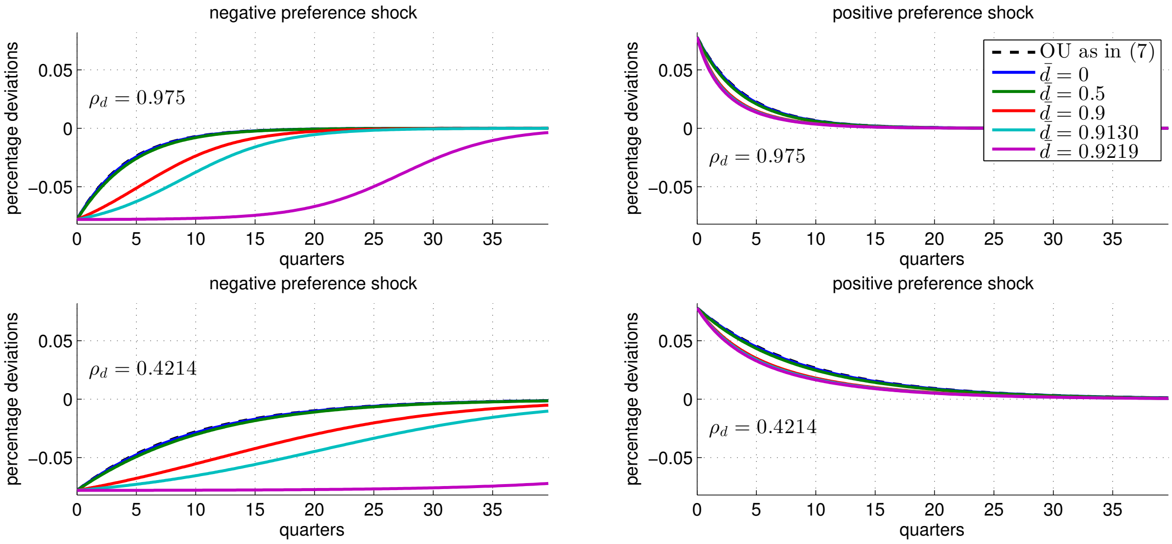
Figure D.41: Simulated shock (2004-2005) with simple NK model around $\pi_{ss} \geq 0$, yields
In this figure we show the yield curve response to unexpected shocks to the (initial) interest rate (0.015), and preferences (-0.1), with effects for the nominal and real yields (blue solid), and the pre-shock scenario (dotted).



D.5. Alternative shock dynamics

Figure D.42: Generalized logistic preference shock

In this figure we plot the dynamics of the logistic process, $\mathrm{d}d_t = \rho_d(d_t - \bar{d})\,(1 - d_t)\,/(1 - \bar{d})\,\mathrm{d}t$, and the Ornstein-Uhlenbeck (OU) process, $\mathrm{d}\log d_t = -\rho_d \log d_t \mathrm{d}t$, for different parameterizations of ρ_d and $\bar{d}$. It shows that the dynamics are similar if the lower bound $\bar{d}$ is sufficiently far away from $d_0 > \bar{d}$. For $\bar{d} = 0$ we obtain the (standard) logistic growth model $\mathrm{d}d_t = \rho_d d_t\,(1 - d_t)\,\mathrm{d}t$ (cf. Section A.2).



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