Prof. Dr. Anke Gerber

# Game Theory

## 2. Exam Summer Term 2012

# Important Instructions

1. There are 90 points on this 90 minutes exam.

2. You are not allowed to use any material (books, lecture notes etc.).

3. You are not allowed to use a calculator.

4. Please answer the questions only on the paper that is handed out to you.

5. Please write your name on each sheet of paper, number the pages and leave a margin (2.5cm) on each page.

6. Please write legibly and make sure that your answers are coherent and complete.

Good Luck!

**Problem 1** **(20 Points)**

Determine all **pure and mixed strategy** Nash equilibria of the following 2-player strategic game:

**Player 2**

| | | A | B |
|---|---|---|---|
| **Player 1** | A | 2, 2 | 4, 2 |
| | B | 2, 4 | 0, 0 |

Give a brief reason why the strategy profiles you determined are indeed Nash equilibria of the game.

## Problem 2 (34 Points)

Two firms compete on the same product market. They can either sell their product at a high price $(H)$ or a low price $(L)$. The firms simultaneously choose their prices. If both firms choose a high price, every firm makes a profit of 9. If both choose a low price, every firm makes a profit of 1. If one firm chooses a high price and the other firm chooses a low price, all consumers buy at the low price firm. In this case, the low price firm makes a profit of 21, while the high price firm makes zero profits.

1. Represent the firms' price competition as a strategic game in matrix form.

   (8 Points)

2. Determine the Nash equilibrium of the game. Give a brief reason for your answer.

   (4 Points)

3. Suppose the firms not only compete once, but every period for infinitely many periods. Both firms discount future profits with the discount factor $\delta = \dfrac{1}{1+r}$, where $r$ is the riskless interest rate.

   (a) In order to sustain high prices the firms consider playing the "grim-trigger strategy" in the infinitely repeated game. Give a definition of the grim-trigger strategy, i.e. write down under which conditions a firm chooses a high or low price in a given period of the infinitely repeated game if it plays grim-trigger. Determine the discounted average profit of a firm if both firms play the grim-trigger strategy.
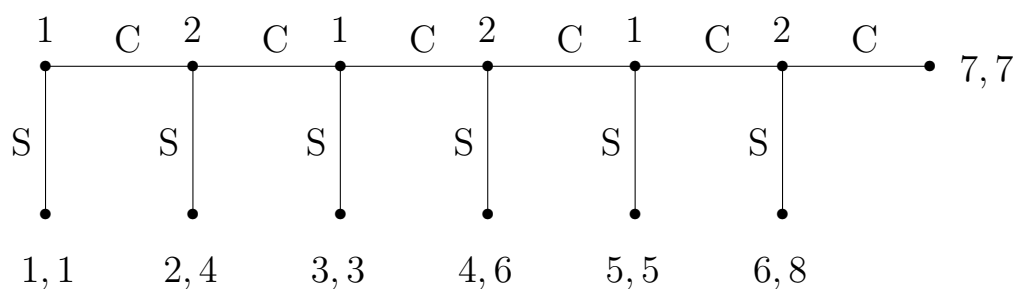
       (10 Points)

   (b) Determine the range of values of $\delta$ (or $r$), for which both firms playing grim-trigger is a Nash equilibrium of the infinitely repeated game.

       (12 Points)

## Problem 3 (10 Points)

Consider the following version of the centipede game:



In this game, player 1 moves first and either stops the game (S) or continues (C), in which case player 2 moves and either stops or continues and so on until the final move of player 2, where both actions, S and C, lead to a terminal node.

The first number at a terminal node always denotes the utility payoff for player 1, the second number the utility payoff for player 2.

Determine the subgame perfect Nash equilibrium of this game.

## Problem 4                                                (26 Points)

Consider the following 3-player extensive game with imperfect information:
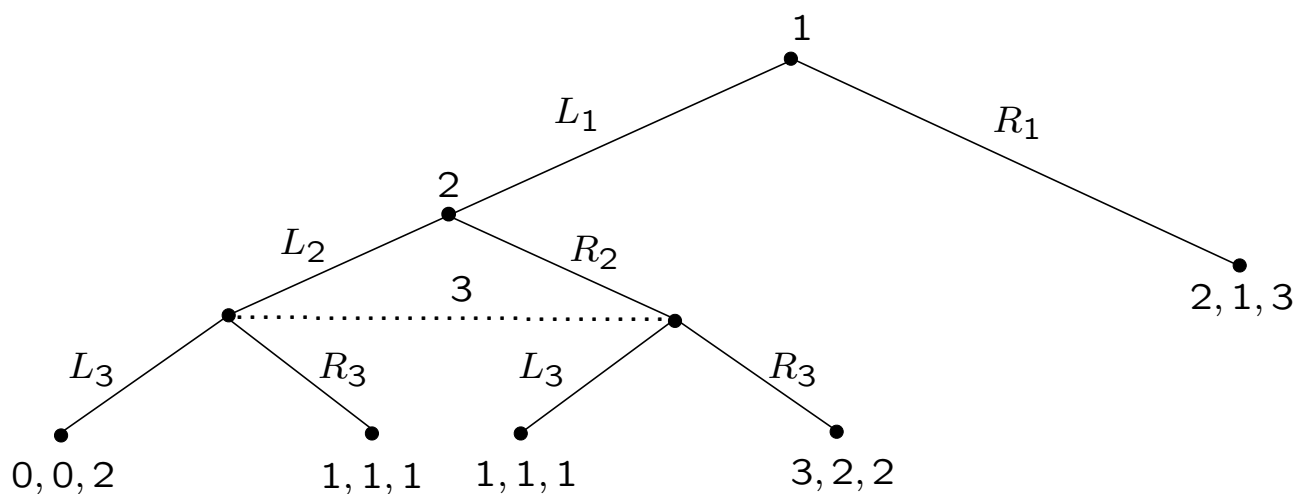


The first number at a terminal node denotes the utility payoff for player 1, the second number the utility payoff for player 2, and the third number the utility payoff for player 3. The dotted line indicates that player 3 does not observe the action chosen by player 2.

1. Determine all pure strategy Nash equilibria of the game.

   (14 Points)

2. For all pure strategy Nash equilibria determine the belief of player 3 at his information set, so that the Nash equilibrium strategy profile together with the belief of player 3 is a weak perfect Bayesian equilibrium.

   (12 Points)