Prof. Dr. Anke Gerber

Game Theory

1. Exam Summer Term 2012

Important Instructions

1. There are 90 points on this 90 minutes exam.
2. You are not allowed to use any material (books, lecture notes etc.).
3. You are not allowed to use a calculator.
4. Please answer the questions only on the paper that is handed out to you.
5. Please write your name on each sheet of paper, number the pages and leave a margin (2.5cm) on each page.
6. Please write legibly and make sure that your answers are coherent and complete.

Good Luck!

Problem 1 **(10 Points)**

Determine all **pure and mixed strategy** Nash equilibria of the following 2-player strategic game:

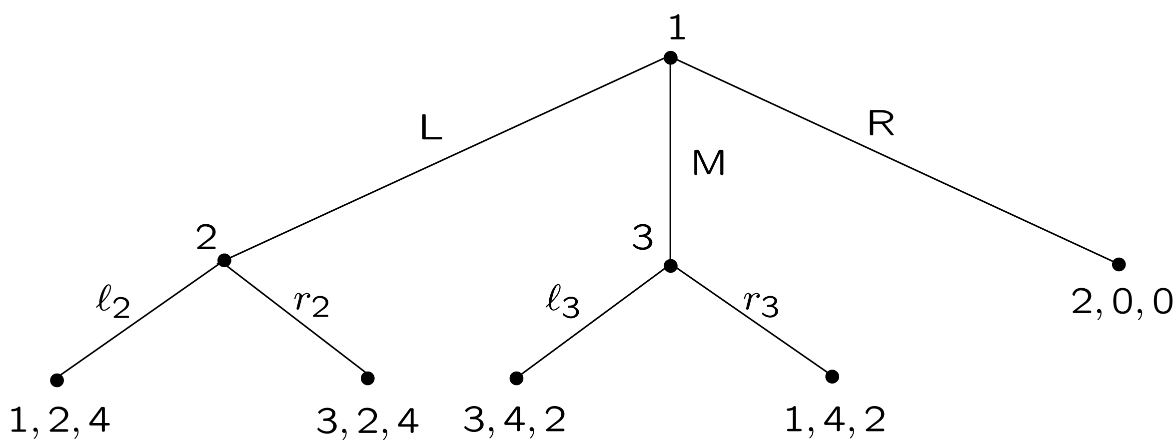
Player 2

Player 1		A	B
	A	$2, 2$	$3, 1$
	B	$1, 3$	$5, 5$

Give a brief reason why the strategy profiles you determined are indeed Nash equilibria of the game.

Problem 2 **(16 Points)**

Consider the following 3-player extensive game:



The first number at a terminal node denotes the utility payoff for player 1, the second number the utility payoff for player 2, and the third number the utility payoff for player 3.

1. Write down all possible **pure strategy** profiles, i.e. all possible combinations of pure strategies of the three players.

   (4 Points)

2. Which of the **pure strategy** profiles are subgame perfect equilibria? Give a reason for your answer.

   (12 Points)

Problem 3 **(40 Points)**

Consider the following strategic game:

Player 2

		A	**B**	**C**
	A	$1, 1$	$8, 0$	$0, 0$
Player 1	**B**	$0, 8$	$6, 6$	$0, 0$
	C	$0, 0$	$0, 0$	$4, 4$

1. Determine all Nash equilibria in pure strategies. Give a brief reason why the strategy profiles you determined are indeed Nash equilibria of the game.

   (6 Points)

2. Write down a mixed strategy for player 1 (or player 2), where only A and C are played with positive probability and which strictly dominates the pure strategy B.

   (4 Points)

3. Use 2. to argue that no player plays B in a mixed strategy Nash equilibrium and derive the unique properly mixed strategy Nash equilibrium of the game.

   (8 Points)

4. Suppose the game is repeated twice and the final payoff for a player is the sum of the payoffs obtained in each of the two repetitions of the game (no discounting). Show that there exists a subgame perfect equilibrium of the twice-repeated game, where the equilibrium outcome is (B, B) in period 1 and (C, C) in period 2. To show this, write down the strategies of the players in period 1 and in period 2 that support this equilibrium outcome. (12 Points)

5. Suppose the game is only played once but now player 1 moves first and player 2 observes the action chosen by player 1 (A, B, or C) before choosing his action. Write down the extensive form (game tree) and derive the subgame perfect equilibrium of the game (write down the complete strategy of both players). (10 Points)

Problem 4 **(24 Points)**

Consider the following 2-player game, where player 2 has one type only and player 1 is cooperative with probability p and selfish with probability $1 - p$. Both players simultaneously choose one of two possible actions, A or B. The payoffs for player 2 are

		Player 2: A	Player 2: B
Player 1	A	3	0
	B	0	1

If player 1 is cooperative, her payoffs are

		Player 2: A	Player 2: B
Cooperative Player 1	A	3	0
	B	0	1

If player 1 is selfish, her payoffs are

Player 2

Selfish Player 1		A	B
	A	0	3
	B	1	0

1. If player 2 plays A, what is the best-response of a cooperative and of a selfish player 1? Under which conditions on p is A in turn a best-response by player 2 to the best-response by player 1 you have just determined?

   (8 Points)

2. If player 2 plays B, what is the best-response of a cooperative and of a selfish player 1? Under which conditions on p is B in turn a best-response by player 2 to the best-response by player 1 you have just determined?

   (8 Points)

3. Use your answers to 1. and 2. in order to determine the **pure strategy** Nash equilibrium/equilibria of this Bayesian game depending on p.

   (8 Points)