

Prof. Dr. Anke Gerber

## Social Choice and Welfare

2. Exam Winter Term 2013/14

### Important Instructions

1. There are 90 points on this 90 minutes exam.
2. You are not allowed to use any course material (books, slides, lecture notes etc.).
3. Please answer the questions only on the paper that is handed out to you.
4. Please write your name on each sheet of paper, number the pages and leave a margin (2.5cm) on the right of each page.
5. Please write legibly and make sure that your answers are coherent and complete.

Good Luck!

**Problem 1**

**(18 Points)**

A university ranks applicants for a master's program in climate science according to three criteria: the applicant's grade point average (GPA) of the bachelor degree, the applicant's grade in physics and the applicant's score in an English test. Assume that the GPA, the physics grade and the test score are each measured on a scale, where higher numbers are better than lower numbers.

Let  $A$  and  $B$  be two applicants. Then the university weakly prefers  $A$  over  $B$  ( $ARB$ ), if according to at least two out of the three criteria (GPA, physics grade, test score)  $A$  is at least as good as  $B$ .

1. Is the university's weak preference relation over applicants complete? Give a reason for your answer.

(8 Points)

2. Is the university's weak preference relation over applicants transitive? Give a reason for your answer.

(10 Points)

**Problem 2****(10 Points)**

Consider the set of alternatives  $X = \{x, y, z, w\}$  and let  $C$  be a choice function on  $X$ . Suppose

$$C(\{x, y, z\}) = \{x, y\}.$$

Determine  $C(\{x, y, z, w\})$  for the case where  $C$  satisfies contraction and expansion consistency. Write down all possible choice sets  $C(\{x, y, z, w\})$  such that contraction and expansion consistency is satisfied. Give a reason for your answer.

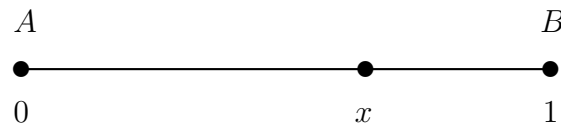
**Problem 3****(30 Points)**

Suppose there are only two distinct alternatives and at least two individuals. Show that Arrow's impossibility result does not hold in this case by providing an example of a social welfare function which satisfies all conditions in Arrow's impossibility theorem if there are only two alternatives.

Give a rigorous reason for why the rule in your example is indeed a social welfare function and why it satisfies all conditions in Arrow's impossibility theorem.

**Problem 4****(16 Points)**

An electrical tower has to be located in a city which extends on a line between 0 and 1. Suppose there are only two citizens,  $A$  and  $B$ , who live at the left, respectively right border of the city. The house of  $A$  is located in 0 and the house of  $B$  is located in 1:



Both  $A$  and  $B$  would like the electrical tower to be as far away from their house as possible. If the electrical tower is located in point  $x$ , where  $0 \leq x \leq 1$ , then  $A$ 's utility is

$$u(x, A) = 2x,$$

and  $B$ 's utility is

$$u(x, B) = 1 - x.$$

1. Determine the most preferred location  $x$  of the electrical tower according to the maximin rule.

(10 Points)

2. Determine the most preferred location  $x$  of the electrical tower according to the utilitarian rule.

(6 Points)

**Problem 5**

**(16 Points)**

Consider a two-person bargaining problem  $(S, d)$ . The set  $S$  contains all utility allocations  $(u_1, u_2)$  with  $u_1 \geq 0, u_2 \geq 0$ , and  $u_1 + u_2 \leq 12$ . The disagreement point is given by  $d = (0, 0)$ .

1. Determine the Nash solution of this bargaining problem.

(10 Points)

2. Suppose now that player 2's utility is restricted to be at most  $\bar{u}_2 = 7$ , while it still holds that  $u_1 \geq 0, u_2 \geq 0, u_1 + u_2 \leq 12$ , and  $d = (0, 0)$ . Determine the Nash solution to this new bargaining problem.

(6 Points)