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Social Choice and Welfare

1. Exam Winter Term 2014/15

Important Instructions

1. There are 90 points on this 90 minutes exam.
2. You are not allowed to use any course material (books, slides, lecture notes etc.).
3. Please answer the questions only on the paper that is handed out to you.
4. Please write your name on each sheet of paper, number the pages and leave a margin (2.5cm) on the right of each page.
5. Please write legibly and make sure that your answers are coherent and complete.

Good Luck!

Problem 1 (28 Points)

1. What is a strategy-proof social choice function? Give a definition of this notion and explain all terms that you use in your definition.

   (10 Points)

2. Consider the social choice function derived from the plurality rule with a specific tie-breaking rule:

   Let there be m alternatives $x_1, \ldots, x_m$, and n individuals. Every individual i has a preference ordering over the set of alternatives $S = \{x_1, \ldots, x_m\}$ which is assumed to be strict, i.e. no individual is indifferent between any two alternatives. For every alternative x_k in S let $N(x_k)$ be the number of individuals for whom x_k is the top alternative, i.e. who prefer x_k over any other alternative in S. Call $N(x_k)$ the *support* of alternative x_k. If there exists an alternative x_k in S which has a larger support than any other alternative x_j in S, i.e. $N(x_k) > N(x_j)$ for all $j \neq k$, then x_k is chosen under the plurality rule. If there is a tie, i.e. if there are two or more alternatives which achieve maximal support, the plurality rule chooses the alternative with the lowest index among those receiving maximal support. For example, if $N(x_k) = N(x_l) > N(x_j)$ for all $j \neq k$ and $j \neq l$, then x_k is chosen if $k < \ell$ and x_l is chosen if $\ell < k$.

   For each of the following strict preference profiles of four individuals over the set of alternatives $S = \{x_1, x_2, x_3, x_4\}$, check whether the social choice function derived from the plurality rule is manipulable at the given preference profile. If your answer is 'yes', show how the social choice function can it be manipulated. If your answer is 'no', argue why it cannot be manipulated.

In the following tables the alternatives are listed top to bottom according to the individuals' preference orderings:

(a)

Preferences	Individuals 1	2	3	4
	x_1	x_2	x_3	x_4
	x_2	x_1	x_2	x_1
	x_3	x_4	x_1	x_2
	x_4	x_3	x_4	x_3

(b)

Preferences	Individuals 1	2	3	4
	x_4	x_2	x_2	x_3
	x_3	x_1	x_4	x_1
	x_2	x_3	x_3	x_2
	x_1	x_4	x_1	x_4

(18 Points)

Problem 2 (36 Points)

Consider the social aggregation rule which assigns to any profile of individual preference orderings $(R_1, \ldots, R_n)$ the social preference relation R which is defined as follows: Let x and y be two alternatives. Then,

$$xRy \iff xR_iy \quad \text{for at least one individual } i \in \{1, \ldots, n\}.$$

1. Is this aggregation rule an Arrovian social welfare function on an unrestricted domain of individual preferences if there are at least three alternatives? Argue why or why not.

   (12 Points)

2. For each of the conditions *Weak Pareto Principle*, *Independence of Irrelevant Alternatives*, *Non-Dictatorship* and *Positive Responsiveness* argue whether it is satisfied or not satisfied by this rule.

   (24 Points)

Problem 3 (26 Points)

Consider the following preference orderings of three voters over four alternatives x, y, w, z:

$$xP_1zP_1wP_1y$$
$$zP_2yP_2wP_2x$$
$$yP_3zP_3wP_3x$$

1. Are the preferences single-peaked over the set of alternatives $\{x, y, w, z\}$? Give a reason for your answer.

   (10 Points)

2. Are the preferences single-peaked over every triple of alternatives in $\{x, y, w, z\}$? Give a reason for your answer.

   (16 Points)