Corporate Debt Maturity Matters For Monetary Policy*

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February 15, 2021

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Abstract

We provide novel empirical evidence that firms’ investment is more responsive to surprise changes in monetary policy when a higher fraction of their debt is due. In a heterogeneous firm New Keynesian model with financial frictions and endogenous debt maturity, two channels explain this finding: (1.) Firms with more maturing debt roll over more debt and are therefore more exposed to fluctuations in the real interest rate (roll-over risk). (2.) These firms have higher default risk and therefore react more strongly to fluctuations in the real burden of outstanding nominal debt (debt overhang). The aggregate effectiveness of monetary policy therefore depends on the joint distribution of debt maturity and default risk across firms.

Keywords: monetary policy, investment, corporate debt, debt maturity. JEL classifications: E32, E44, E52.

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“Suffice it here to note that over-indebtedness (...) is not a mere one-dimensional magnitude to be measured simply by the number of dollars owed. It must also take account of the distribution in time of the sums coming due. Debts due at once are more embarrassing than debts due years hence; (...) Thus debt embarrassment is great (...) for early maturities.’


1 Introduction

Corporate debt has reached historically high levels. But not all debt is created equal. A large part of debt is issued with long time to maturity and need not be repaid until several years in the future. Another part is due in the short-run – possibly with very different implications for firm behavior. Figure 1 provides a snapshot of the vast heterogeneity across listed U.S. firms with respect to debt maturity. For almost half of all firms, less than 10% of their total debt comes due within the next twelve months. At the other extreme of the distribution there are about 10% of firms for which more than 90% of debt matures within the next year. In this paper, we show that debt maturity shapes firms’ investment response to monetary policy.

Debt maturity matters for monetary policy for two reasons. The first reason is roll-over risk. Firms that borrow at shorter maturities need to roll over their debt more frequently and are therefore more exposed to surprise changes in interest rates. Long-term debt insures firms against this roll-over risk. The second reason is debt overhang. Surprise changes in inflation affect the real burden of outstanding nominal debt which matters for default risk and investment. This debt overhang channel of monetary policy is stronger if outstanding debt has longer remaining maturity (Gomes et al., 2016). Whether firms with shorter or longer debt maturity are more responsive to monetary policy shocks is therefore theoretically ambiguous.

To understand the role of debt maturity for monetary policy, this paper makes both an empirical and a theoretical contribution. Empirically, we show that firms react more strongly to monetary policy shocks in periods when a large fraction of their debt matures. We then develop a heterogeneous firm model to study the implications for the effectiveness of monetary policy.

In the empirical analysis, we combine balance sheet data of U.S. listed firms with bond-level information on debt principal and maturity. This allows us to construct a detailed empirical picture of the distribution of debt maturity across firms and time. To this we add high-frequency identified monetary policy shocks and estimate their effect on firm-level investment using panel local projections. The corporate bonds in our sample have an average maturity at issuance of about eight years and all bonds in our baseline specification are non-callable. Whether a particular bond matures shortly before or after a monetary policy shock

\[1\] In the U.S., the ratio of debt securities and loans of non-financial businesses over GDP surpassed a record high of 74% in 2018. At the onset of the ongoing pandemic-induced recession, this ratio increased further peaking at 90% in Q2 of 2020. Similar developments can be observed in several advanced and emerging economies (Bank for International Settlements, 2020).
has therefore typically been decided several years before a shock occurs.

We find that firms’ investment response to a monetary policy shock is larger if the ratio of maturing bonds to total firm debt is higher at the time of the shock. This result is statistically and economically significant. Eight quarters after a contractionary one-standard deviation monetary policy shock, the capital stock of a firm with maturing bonds at the time of the shock is about one percent lower than the capital stock of a firm without maturing bonds. Importantly, this result holds even if we control for permanent differences across firms and for various time-varying firm characteristics (e.g., size, leverage, asset liquidity). A placebo exercise shows that a large share of maturing bonds in the quarter before the shock has no statistically significant effect on firms investment responses. Overall, these results suggest an important role for roll-over risk.

To study the implications of these empirical results for the effectiveness of monetary policy, we develop a heterogeneous firm New Keynesian model with financial frictions and endogenous debt maturity. In the model, firms finance investment using equity and nominal debt. Debt has a tax advantage relative to equity but introduces the risk of costly default. Firms can choose between short-term debt and long-term debt. Long-term debt saves roll-over costs but creates a debt overhang problem which increases future leverage and default risk. The negative effects of debt overhang are particularly strong for firms with high default risk.

In our calibration, we target key empirical moments which characterize the investment and financing behavior of listed U.S. firms. The quantitative model generates rich cross-sectional heterogeneity in debt maturity. Because the effects of debt overhang are more distortive for small firms with high default risk, these firms choose to borrow at short maturities. Through this mechanism, the model replicates key characteristics of the empirical firm distribution: Smaller firms have higher default risk and lower shares of long-term debt. Their average share of maturing debt is therefore higher. As firms grow, their default risk
decreases, their share of long-term debt rises, and the maturing debt share falls.

Importantly, the model also replicates our empirical results on the role of debt maturity for firms’ investment response to monetary policy: Firms with a higher share of maturing debt react more strongly to monetary policy shocks. A surprising result of our quantitative model is that both theoretical channels introduced above contribute to this result: (1.) Firms with more maturing debt roll over more debt and are therefore more exposed to fluctuations in the real interest rate (roll-over risk). (2.) As described above, these firms also have higher default risk and therefore react more strongly to fluctuations in the real burden of outstanding nominal debt (debt overhang).

These results imply that the aggregate effects of monetary policy depend on the joint distribution of debt maturity and default risk in the economy. Feeding empirical moments of observed firm distributions into our model allows us to make predictions about the time-varying effectiveness of monetary policy. Aggregate firm investment reacts more strongly to monetary policy shocks when a higher share of firm debt matures and when average default risk is high.

Related literature This paper provides an empirical and theoretical analysis of the role of debt maturity for the transmission of monetary policy. It thereby contributes to three related strands of the literature. First, our work adds to empirical studies of the link between debt maturity and firm investment in response to aggregate shocks. Duchin et al. (2010) and Almeida et al. (2012) exploit cross-sectional differences in debt maturity structures before the Financial Crisis of 2007–2008 and find that firms with more maturing debt at the onset of the crisis reduced investment by more. Using data from the Great Depression 1929–1933 and from the 2010–2012 European sovereign debt crisis, Benmelech et al. (2019), Kalemli-Ozcan et al. (2018), and Buera and Karmakar (2019) also find stronger investment declines associated to higher shares of maturing debt. These event studies of the role of debt maturity during financial crises complement our findings on the systematic role of debt maturity for monetary policy.

The second related group of empirical papers studies the role of financing conditions in explaining heterogeneous effects of monetary policy across firms. Ippolito et al. (2018) find that firms with floating-interest rate loans adjust investment more strongly in response to monetary policy shocks relative to firms with fixed-rate debt. While this paper does not explicitly study debt maturity, the result is consistent with the roll-over risk channel of monetary policy. In addition, a large body of work documents the empirical role of size, age, leverage, or asset liquidity in shaping the effects of monetary policy (Gertler and Gilchrist, 1994; Cloyne et al., 2018; Jeenas, 2019; Ottonello and Winberry, 2020; Anderson and Cesabianchi, 2020). We add to this evidence by investigating the role of debt maturity.

The theoretical contribution of this paper is to develop a heterogeneous firm New Keynesian model with financial frictions and endogenous debt maturity. Existing quantitative models of monetary policy do not allow for differences in debt maturity across firms. We build on Gomes et al. (2016) by introducing persistent firm heterogeneity and endogenous debt maturity along the lines of Jungherr and Schott (2020a). Our paper contributes to a large literature using heterogeneous firm models with financial frictions to study cross-sectional differences in firm-level responses to aggregate shocks (e.g. Bernanke et al., 1999; Cooley
and Quadrini, 2006; Covas and Den Haan, 2012; Khan and Thomas, 2013; Gilchrist et al., 2014; Khan et al., 2016; Begenaü and Salomao, 2018; Crouzet, 2018; Arellano et al., 2019; Ottonello and Winberry, 2020; Arellano et al., 2020). Because firms issue only one-period debt in these models, all firms have identical exposure to roll-over risk and no significant exposure to debt overhang.²

Firm heterogeneity is the focus of all papers described above. A parallel literature investigates the role of household heterogeneity for the transmission of monetary policy (e.g. Gornemann et al., 2016; Kaplan et al., 2018; Bayer et al., 2019). Debt maturity matters for households as well, as shown by Auclert (2019) who identifies cross-sectional differences in exposure to roll-over risk and Fisherian debt deflation as sources of heterogeneous effects of monetary policy.

The paper is organized as follows. In Section 2, we describe the data set, the estimation strategy, and the empirical results. Section 3 develops a heterogeneous firm New Keynesian model with financial frictions and endogenous debt maturity. We characterize equilibrium firm behavior in Section 4 highlighting the role of roll-over risk and debt overhang for firms’ investment response to monetary policy. Section 5 presents results from the quantitative model and compares them to the data. Concluding remarks follow.

## 2 Empirical Evidence

In this section, we show that firms’ investment (growth of fixed assets) respond more strongly to monetary policy shocks in periods when a large fraction of their debt matures.

### 2.1 Data

**Bond-level data.** We obtain comprehensive and detailed bond-level information from the Mergent Fixed Income Securities Database (FISD). FISD contains key characteristics of publicly-offered U.S. bonds such as issue date, maturity date, amount issued, principal, coupon, and whether the bond is callable. It also records ex-post reductions in the amount of outstanding bonds as well as the reason for the reduction, e.g., a call, reorganization, or default. The focus of our analysis is on fixed-coupon, non-callable, non-convertible, non-exchangeable bonds denominated in USD. In general, mixing different types of debt contracts may lead to misleading empirical results. More specifically, we expect maturing variable-coupon bonds to render firms less sensitive to changes in monetary policy than maturing fixed-coupon bonds. Callable bonds, which under some conditions can be repurchased before maturity, and similarly convertible and exchangeable bonds, are subject to a survivorship bias. Firms that decide not to repay their bonds before maturity may differ in their default

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²Net worth is the only endogenous state variable in one-period debt models. If firms are allowed to issue long-term debt, the existing stock of previously issued debt enters the firm problem as additional state variable. For quantitative models which explore the implications of long-term debt for firm financing and investment, see also Crouzet (2017), Caggese et al. (2019), Gomes and Schmid (2020), Karabarbounis and Macnamara (2020), Poeschl (2020), or Jungherr and Schott (2020b). For continuous-time approaches to modeling debt maturity in corporate finance, see DeMarzo and He (2020) or Dangl and Zechn (2020). None of these models studies the role of debt maturity for monetary policy.
risk, which may also affect their response to monetary policy shocks.\(^3\) Appendix A provides further details on the data treatment.

**Bond-firm linkage.** To link bond-level data to firm balance sheet data from Compustat, we use the nine-digit bond cusip identifier in FISD. The last three digits identify the bond, while the first six digits are the firm cusip identifier. The firm cusip encoded in the bond cusip is the historical firm cusip at the time of bond issuance, while in Compustat we only observe the current six-digit cusip identifier (via the CRSP-Compustat linking table). Current and historic identifiers often do not coincide. For example, the current firm cusip may change because the firm changes its name or trading symbol. An additional complication arises when M&A changes the ownership of bonds. We use Thomson–Reuters’ M&A Database and focus on events in which the target firm is fully acquired by the buying firm. Eventually, we obtain a bond panel which maps the bond in every quarter over its lifetime to a unique firm. We provide a more detailed explanation of linkage procedure in Appendix A.

**Firm-level data.** We use quarterly firm balance sheet data from Compustat. We exclude firms in sectors public administration, finance, insurance, real estate, or utilities. After merging firm-level and bond-level data, we exclude firm-quarters in which no bond is maturing or outstanding. The remaining observations account for 70.0% of total sales and 72.9% of total fixed assets. We compute the firm-quarter specific dollar value of maturing and outstanding bonds and the average bond maturity weighted by the outstanding value of bonds. The value of maturing bonds only includes bonds held until maturity. We deflate outstanding bonds and maturing bonds by the CPI. A central object of our empirical analysis is the maturing bond share, which we compute as the value of maturing bonds relative to total debt in the preceding quarter:

\[
M_{it} = \frac{\text{maturing bonds}_{it}}{\text{total debt}_{it-1}} \times 100,
\]  

(2.1)

where \(i\) denotes a firm and \(t\) a quarter. We use fixed assets in the balance sheet data to construct the capital stock by applying a perpetual inventory method.\(^5\) We further use total assets, total debt, leverage, distance to default, liquidity, and real sales growth. We provide details on these variables in Appendix A.

**Monetary policy shocks.** We use high-frequency price changes of federal fund futures around FOMC meetings to identify monetary policy shocks. Our baseline shocks are based on the three-months ahead federal funds future and 30-minute event windows, as in Gertler and Karadi (2015). We exclude unscheduled meetings and conference calls, which helps to mitigate the problem that monetary surprises may contain private central bank information about the state of the economy (Meier and Reinelt, 2020). We further follow Jarociński and

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\(^3\) At the end of this section, we discuss separate results for callable and variable-coupon bonds.

\(^4\) Since the quarterly series of firm debt (in Compustat: dlcq+dlttq) displays large transitory volatility, we compute firm debt as average debt over the current and the preceding three quarters. Our main results are dampened and less significant with the ‘raw’ debt series as we discuss toward the end of this section.

\(^5\) Our results hold robust when instead using deflated fixed assets.
Karadi (2020) by using sign restrictions to separate information effect from conventional monetary policy shocks. Finally, the daily shocks are aggregated to quarterly frequency. Daily shocks are assigned fully to the current quarter if they occur on the first day of the quarter. If they occur within the quarter, they are partially assigned to the current and subsequent quarter.

2.2 Precursory analysis

Descriptive statistics. Our sample covers 1995Q1 through 2017Q4. Table 1 reports descriptive statistics of the variables used in our empirical analysis. The first column provides means and standard deviations for the full sample. This sample includes all firm-quarters with positive outstanding or maturing bonds. In addition, it excludes firms for which we observe fixed assets and the maturing bonds share for less than ten years, similar to Ottonello and Winberry (2020). The other columns report descriptive statistics for sub-samples: the sample of firms with maturing bonds in some period (column 2), the sample of firm-quarters without maturing bonds (column 3), and with maturing bonds (column 4). The average quarterly share of maturing bonds is 0.2%, which increases to 0.4% when only considering firms with maturing bonds at least once in the sample. Conditional on any bonds maturing, the average quarterly share of maturing bonds is about 8%. On average, firms with some maturing bonds within our sample have more assets and debt than firms without maturing bonds. This may reflect different stages of the firms’ life-cycle. Differences in financial ratios are less striking and overall the distributions overlap widely for assets, debt, and financial ratios across the different samples. We control for potentially systematic differences between firms with and without maturing bonds in the subsequent empirical analysis by focusing on within-firm variation and by controlling for time-varying firm characteristics such as size, liquidity, leverage, and distance to default. Note that for the firms in our sample, bonds are the primary source of finance. On average, outstanding bonds account for more than 60% of total debt.

Bond rollover. When a bond matures, the firm needs to repay the bond’s principal. The maturing bond is fully rolled over if the principal is fully refinanced by new debt. To investigate the average debt roll-over, we regress changes in total debt on the value of maturing bonds (both in levels). Table 2 summarizes six of these regressions, which differ in sample selection and the type of included fixed effects. Overall, we find that about 50% of the maturing bond is refinanced on average. The point estimates are fairly similar and highly significant whether or not an industry-time fixed effect is included and whether or not we consider the total sample or only the sample of firms with at least one quarter of maturing bonds. The point estimates are similar in the sample of firm-quarters with strictly positive maturing bond shares, albeit the standard errors are substantially larger. Because for many firms in our sample we only observe few instances of bonds maturing, we have not considered firm fixed effects in the sample of firm-quarters with maturing bonds.
Table 1: Descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>Full sample</th>
<th>Only $\mathcal{M}_i &gt; 0$</th>
<th>Only $\mathcal{M}_{it} = 0$</th>
<th>Only $\mathcal{M}_{it} &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fixed assets (in mln.)</strong></td>
<td>4,262.79</td>
<td>6,512.17</td>
<td>4,098.72</td>
<td>10,486.65</td>
</tr>
<tr>
<td></td>
<td>(9,946.04)</td>
<td>(11,409.52)</td>
<td>(9,691.56)</td>
<td>(15,780.55)</td>
</tr>
<tr>
<td><strong>Total assets (in mln.)</strong></td>
<td>14,414.49</td>
<td>22,203.75</td>
<td>13,942.62</td>
<td>32,317.89</td>
</tr>
<tr>
<td></td>
<td>(39,210.92)</td>
<td>(51,238.58)</td>
<td>(38,720.33)</td>
<td>(51,595.81)</td>
</tr>
<tr>
<td><strong>Total debt (in mln.)</strong></td>
<td>4,439.59</td>
<td>6,951.47</td>
<td>4,318.75</td>
<td>9,020.35</td>
</tr>
<tr>
<td></td>
<td>(17,677.55)</td>
<td>(24,309.39)</td>
<td>(17,665.89)</td>
<td>(17,519.50)</td>
</tr>
<tr>
<td><strong>Leverage (debt/assets in %)</strong></td>
<td>34.02</td>
<td>30.90</td>
<td>34.11</td>
<td>30.37</td>
</tr>
<tr>
<td></td>
<td>(18.59)</td>
<td>(15.22)</td>
<td>(18.71)</td>
<td>(12.96)</td>
</tr>
<tr>
<td><strong>Distance to default</strong></td>
<td>7.35</td>
<td>8.31</td>
<td>7.33</td>
<td>8.02</td>
</tr>
<tr>
<td></td>
<td>(4.97)</td>
<td>(5.05)</td>
<td>(4.97)</td>
<td>(4.88)</td>
</tr>
<tr>
<td><strong>Liquidity (cash/assets in %)</strong></td>
<td>7.44</td>
<td>6.91</td>
<td>7.47</td>
<td>6.20</td>
</tr>
<tr>
<td></td>
<td>(8.14)</td>
<td>(7.37)</td>
<td>(8.17)</td>
<td>(6.54)</td>
</tr>
<tr>
<td><strong>Sales growth (ann. %)</strong></td>
<td>2.96</td>
<td>1.99</td>
<td>2.97</td>
<td>2.61</td>
</tr>
<tr>
<td></td>
<td>(79.84)</td>
<td>(64.65)</td>
<td>(80.00)</td>
<td>(73.48)</td>
</tr>
<tr>
<td><strong>Maturing bonds share (in %)</strong></td>
<td>0.20</td>
<td>0.40</td>
<td>0.00</td>
<td>7.67</td>
</tr>
<tr>
<td></td>
<td>(1.81)</td>
<td>(2.55)</td>
<td>(0.00)</td>
<td>(8.38)</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>33,892</td>
<td>16,831</td>
<td>33,022</td>
<td>870</td>
</tr>
</tbody>
</table>

*Note:* The table provides sample averages (and standard deviations in parentheses) of various firm-level measures for 1995Q1 through 2017Q4. Our full sample (first column) includes all firm-quarters with positive outstanding or maturing bonds. The second column is based on firms with maturing bonds at least in one quarter ($\mathcal{M}_i$ denotes the firm-specific average share of maturing bonds). The third (fourth) column is based on all firm-quarters with a zero (strictly positive) value of maturing bonds. Assets and debt are expressed in mln. 2005 USD.

Table 2: Rollover regressions

<table>
<thead>
<tr>
<th></th>
<th>Change in debt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full sample</td>
</tr>
<tr>
<td><strong>Value of maturing bonds</strong></td>
<td>-0.581</td>
</tr>
<tr>
<td></td>
<td>(0.0933)</td>
</tr>
<tr>
<td><strong>Industry-time FE</strong></td>
<td>No</td>
</tr>
<tr>
<td><strong>Firm FE</strong></td>
<td>Yes</td>
</tr>
<tr>
<td><strong>$R^2$</strong></td>
<td>0.024</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>32,422</td>
</tr>
</tbody>
</table>

*Note:* The table shows the results when regressing the change in total debt on the value of maturing bonds. The six columns are based on different samples, analogously to Table 1, and differ by the inclusion of a one-digit industry-quarter fixed effect. Standard errors (in parentheses) are clustered by firm and quarter. Columns 4–6 contain less observations than columns 1–3 because we require at least two observations per industry-quarter pair.
2.3 Investment responses to monetary policy shocks

**Empirical specifications.** We estimate panel local projections of firm-level log changes in the capital stock on the interaction of monetary policy shocks and the share of bonds maturing. Formally, we estimate

\[
\log K_{it+h} - \log K_{it-1} = \alpha_i^h + \alpha_{it}^h + B^h \varepsilon_{it}^{MP} + \Gamma_1^h \Delta_{it}^{\Delta \text{gdp}} + \Gamma_2^h Z_{it} + \nu_{it}^h, \tag{2.2}
\]

for \( h = 0, \ldots, 16 \) quarters. \( K_{it} \) denotes the real capital stock of firm \( i \) in quarter \( t \), \( \varepsilon_{it}^{MP} \) is the monetary policy shock, \( \Delta_{it}^{\Delta \text{gdp}} \) quarterly real GDP growth, and \( Z_{it} \) a vector of firm-specific variables including the share of bonds maturing \( \mathcal{M}_{it} \). We include interactions with lagged GDP growth in order to control for differences in cyclical sensitivities across firms. The regressions include firm and sector-quarter fixed effects. We consider three alternative specifications of \( Z_{it} \): (1) We only consider the share of bonds maturing, \( Z_{it} = \mathcal{M}_{it} \). (2) We consider \( Z_{it} = \mathcal{M}_{it} - \overline{\mathcal{M}}_{it} \), which implies that our estimates only reflect within-firm variation, in contrast to variation across firms. This may be important because systematic differences across firms (e.g., the duration of investment projects and their cash-flow volatility) may simultaneously explain different average maturing bond shares and different response to monetary policy shocks. (3) We focus on within-firm variation and add time-varying firm-level controls: \( Z_{it} \) includes \( \mathcal{M}_{it} \), leverage, distance to default, liquidity, average maturity of all outstanding bonds (weighted by bond volume), real sales growth, and log real total assets; all in deviation from their respective firm-specific averages. This specification aims to control account for time-varying differences in the firm-specific responses, possibly linked to time-varying financial constraints. Our selection of controls is motivated by Ottonello and Winberry (2020), who study the transmission of monetary policy shocks through financial constraints. We are primarily interested in the interaction of monetary policy shock and the share of bonds maturing.

**Main empirical results.** Figure 2 provides our main empirical results. For the baseline specification of \( Z_{it} \), panel (a) shows the percentage response of capital to a monetary policy shock for a firm with a maturing bond share one standard deviation above the average relative to the average firm. The standard deviation of the maturing bond share is 1.81% (see last row of Table 1). While the estimated differential effect is statistically insignificant initially it becomes highly significant starting 6 quarters after the shock.\(^6\) Between 6 and 12 quarters after the shock, the responses are statistically different from zero at the 5% significance level. Around 2 years after the shock, we can reject that the response is above a 0.1% and below a 0.3% capital reduction at the 5% level. These estimated magnitudes are also economically important. Considering the average maturing bond share of a firm with maturing bonds of 7.67%, our point estimates imply that, on average, capital of a firm with maturing bonds grows by about 0.9% less than capital of a firm without maturing bonds at a two-year horizon. With an investment-capital ratio of 10%, this translates into a differential investment response of 9%. Together with a negative average response of capital

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\(^6\)The small and insignificant estimates at short horizons may reflect time to build. Meier (2020) documents that the average time between order and delivery is 6 months. If we add planning time, time to build may extend to more than a year.
Figure 2: Differential investment response to MP shocks for high maturing bond share

(a) Baseline

(b) ‘Placebo test’ ($M_{it-1}$)

(c) Within-firm variation & controls

(d) Alternative maturing bond shares

Note: In panel (a), the solid line is the estimated differential percentage response of capital to a one-standard deviation contractionary monetary policy shock for firms with a one standard deviation higher share of maturing bonds, $M_{it}$, relative to the average firm (baseline specification). Panel (b) is based on replacing $M_{it}$ by $M_{it-1}$. In panel (c), we consider differences in $M_{it}$ from the firm-specific average and add firm-time varying controls. In panel (d), we consider $M_{it}$ constructed with alternative denominators. The shaded areas indicate 95% error bands clustered by firms and quarters.

to monetary policy shocks, see Figure 7 in the Appendix, our findings imply that firms with a larger fraction of maturing bonds are more responsive to monetary policy shocks.

Panel (b) of Figure 2 provides a ‘placebo test’ in which we interact the monetary policy shock with the maturing bond share in the preceding quarter, but otherwise keep with the baseline specification. To the extent that firms have repaid or refinanced this bond by the time the shock arrives, we should expect the monetary policy shock to have little differential effect on firms. Empirically, the estimated differential effect is statistically insignificant, except for the longer horizons.

In panel (c), we consider the two alternative specifications of $Z_{it}$ described above. When restricting the analysis to within-firm variation without adding controls, the estimated
magnitudes are of similar magnitude and statistical significance compare to the baseline specification in (a). If we further add control variables, we obtain larger (in absolute terms) and more significant coefficients. Overall, our baseline result is robust to these alternative specifications and the magnitudes of the estimated differential investment response rather increases.\footnote{In Figure 8 in the Appendix, we show the coefficients on the interaction of monetary policy shocks with leverage and distance to default. This is at the center of the empirical analysis in Ottonello and Winberry (2020). While they find that firms with low leverage or high distance to default are most responsive to monetary policy shocks, our estimates are mostly statistically insignificant. Our analysis differs in two dimensions. We use a sample of bond-issuing firms and include the maturing bond share and average maturity as controls.}

Finally, in panel (d) we consider alternative specifications of the maturing bond share \( M_{it} \). Instead of dividing the value of maturing bonds by total debt, we divide by the capital stock, sales, or total assets.\footnote{In close analogy to the maturing bonds share with total debt in the denominator, we divide by the average of capital, sales, and total assets over the preceding four quarters.} In all three cases, the responses at a two-year horizon remain significant at the 5\% level.

\textbf{2.4 Robustness}

\textbf{Dummy specification.} Our baseline specification includes a linear interaction between monetary policy shocks and the maturing bond share. Instead, we consider a specification in which monetary policy shocks are interacted with a dummy variable that is one if the maturing bond share is above a threshold. As thresholds, we consider 0\% and 15\%. Figure 9 in the Appendix shows that this leads to broadly similar conclusions.

\textbf{Alternative} \( M_{it} \). Figure 10 in the Appendix shows the results when using the full specifications which only lever within-firm variation and add controls, but using the alternative maturing bond shares with capital, sales, or assets in the denominator. Our results are broadly robust. If we construct \( M_{it} \) using in the denominator the previous period’s total debt (or capital, sales, assets) instead of the average value over the preceding four quarters, our estimates are dampened and the confidence bands are substantially wider, see Figure 11. This may reflect the large transitory volatility in the quarterly debt and asset series.

Our main results in Figure 2 are based on constructing \( M_{it} \) exclusively from non-callable and fixed-coupon bonds. Figure 12 provides estimates when \( M_{it} \) is based on callable or variable-coupon bonds only. Overall, the estimates are closer to zero and less significant.

\textbf{Great Recession and ZLB.} We review the role of alternative samples for the baseline result in panel (a) of Figure 13. Our finding changes by relatively little if we exclude the Great Recession or only focus on the pre-Great Recession period.

\textbf{Monetary policy shocks.} Our results are further robust to alternative monetary policy shock series. Panel (b) of Figure 13 shows we obtain fairly similar results if we use as monetary policy shocks the 3-month fed funds future surprises without applying the sign restrictions of Jarociński and Karadi (2020).
Responses of debt and sales. Figure 14 shows the differential responses of debt and sales. We find that both debt and sales decline by more for firms with a higher maturing bond share. Both of these findings are consistent with the capital response.

3 Model Setup

The previous section has established empirically that the investment response to monetary policy shocks is stronger for firms with a larger share of maturing debt. In order to understand the implications of this result for the aggregate effects of monetary policy, we develop a heterogeneous firm New Keynesian model with financial frictions and endogenous debt maturity. The model generates rich heterogeneity in firms’ choices of debt maturity.

At the heart of the model is a continuum of heterogeneous firms which produce output using capital and labor. Capital is financed through equity and nominal debt. Debt has a tax advantage relative to equity but introduces the risk of costly default. Firms can choose between short-term debt and long-term debt. Long-term debt saves roll-over costs but generates debt overhang in the future thereby increasing future leverage and default risk. This part of the model builds on Jungherr and Schott (2020a) with the additional features of costly equity issuance and nominal debt.

In addition to production firms, the economy consists of retail firms, capital producers, a representative household, and a government. Retail firms buy homogeneous production goods and turn them into differentiated retail goods. They are monopolistically competitive and set prices subject to Rotemberg adjustment costs. Capital producers convert final goods into capital. The representative household works, consumes final goods, and saves by buying equity and debt securities issued by production firms. The government collects a corporate income tax and pays out all proceeds as transfers to the representative household. In addition, the government conducts monetary policy by setting the nominal riskless interest rate.

3.1 Production firms: Setup

A production firm $i$ generates at time $t$ a quantity $y_{it}$ of homogeneous production goods using capital $k_{it}$ and labor $l_{it}$:

$$y_{it} = z_{it} \left( k_{it}^{\psi} l_{it}^{1-\psi} \right)^{\zeta}, \quad \text{with: } \zeta, \psi \in (0, 1)$$

(3.1)

Firm-level productivity $z_{it}$ is a persistent random variable which is realized at the end of period $t - 1$ with conditional probability distribution $\pi(z_{it} | z_{it-1})$. Earnings before interest and taxes are

$$p_t y_{it} - w_t l_{it} + (\varepsilon_{it} - \delta) Q_t k_{it} - f,$$

(3.2)

where $p_t$ is the price of the homogeneous production good (in terms of the time $t$ numéraire), $w_t$ is the real wage, $\varepsilon_{it}$ is a firm-specific shock to capital quality, $\delta$ is depreciation, $Q_t$ is the price of capital goods, and $f$ is a fixed cost of production. The capital quality shock $\varepsilon_{it}$ is i.i.d. with mean zero and continuous probability distribution $\varphi(\varepsilon)$. It is realized in period
controls the speed of decay. The quantity of nominal long-term bonds chosen by the firm is \( \gamma \). In this manner, payments decay geometrically over time. The maturity parameter \( \gamma \) determines the speed of decay. The quantity of nominal short-term bonds sold by firm \( i \) and due in period \( t \) is \( \bar{b}^S_{it} \).

In the following, we will use the real face value of nominal debt expressed in terms of the numéraire good: \( \bar{b}^S_{it} \equiv \bar{B}^S_{it}/P_{t-1} \) (with \( P_{t-1} \) denoting the price of the numéraire good in period \( t - 1 \)). If at the end of period \( t - 1 \) the firm sells short-term bonds of real face value \( \bar{b}^S_{it} \) at price \( p^S_{it-1} \), it raises the amount \( \bar{b}^S_{it} p^S_{it-1} \) on the bond market.

Definition. Short-term debt: A short-term bond issued at the end of period \( t - 1 \) is a promise to pay one unit of currency in period \( t \) together with a nominal coupon \( c \). The quantity of nominal short-term bonds chosen by the firm \( i \) and due in period \( t \) is \( \bar{B}^L_{it} \).

This computationally tractable specification of long-term debt goes back to Leland (1994). Let \( \bar{b}^L_{it} \) be the real face value of period \( t \) long-term debt: \( \bar{b}^L_{it} \equiv \bar{B}^L_{it}/P_{t-1} \), and let \( B_{it-1} \) denote the stock of previously issued nominal long-term bonds outstanding. If the firm sells a quantity \( \bar{B}^L_{it} - B_{it} \) of additional long-term bonds, it increases the real face value of long-term debt by:

\[
\frac{(\bar{B}^L_{it} - B_{it-1})}{P_{t-1}} = \left( \bar{b}^L_{it} - \frac{B_{it-1} P_{t-2}}{P_{t-1}} \right) = \left( \bar{b}^L_{it} - \frac{b_{it-1}}{\pi_{t-1}} \right),
\]

where \( b_{it-1} \equiv B_{it-1}/P_{t-2} \), and \( \pi_{t-1} \equiv P_{t-1}/P_{t-2} \). The corresponding amount raised on the bond market is:

\[
\left( \bar{b}^L_{it} - \frac{b_{it-1}}{\pi_{t-1}} \right) p^L_{it-1}.
\]

Definition. Debt issuance cost: The firm pays a quadratic issuance cost whenever it sells new short-term or long-term debt. Repurchasing outstanding long-term debt (by choosing \( \bar{b}^L_{it} < b_{it-1}/\pi_{t-1} \)) is costless. Total debt issuance costs \( H(\bar{b}^S_{it}, \bar{b}^L_{it}, b_{it-1}/\pi_{t-1}) \) are therefore

\[
H \left( \bar{b}^S_{it}, \bar{b}^L_{it}, \frac{b_{it-1}}{\pi_{t-1}} \right) = \eta \left( \bar{b}^S_{it} + \max \left( \bar{b}^L_{it} - \frac{b_{it-1}}{\pi_{t-1}}, 0 \right) \right)^2
\]

The issuance cost makes short-term debt unattractive because it needs to be constantly rolled over. Long-term debt matures slowly over time and therefore allows maintaining a given stock of debt at a lower level of bond issuance per period.9

---

9Debt issuance costs capture underwriting fees charged by investment banks to bond issuing firms. Altinkö z and Hansen (2000) provide empirical evidence that marginal debt issuance costs are increasing in debt issuance.
The firm finances its capital stock by receiving equity from shareholders and by selling new short- and long-term bonds. Let $q_{it-1}$ denote the real market value of firm assets in place at the end of period $t - 1$, and let $e_{it-1}$ denote net equity issuance, that is, the net cash flow from shareholders to the firm:

$$e_{it-1} = Q_{it-1} \hat{k}_{it} - q_{it-1} - \hat{b}_{it}^s \hat{p}_{it-1}^s - \left( \hat{b}_{it}^L - \frac{b_{it-1}}{\pi_{it-1}} \right) p_{it-1}^L$$

A negative value of $e_{it-1}$ indicates a net dividend payment from the firm to shareholders. While dividend payout is costless, issuing equity is costly.\(^{10}\)

**Definition. Equity issuance cost:** The firm pays a quadratic issuance cost whenever it sells new shares (as in Hennessy and Whited, 2007). Net dividend payout ($e_{it-1} < 0$) is costless. Equity issuance costs $G(e_{it-1})$ are therefore

$$G(e_{it-1}) = \nu \left( \max \{e_{it-1}, 0\} \right)^2$$

Firm earnings are taxed at rate $\tau$. Debt coupon payments are tax deductible. After production, taxation, and payment of debt, the new market value of firm assets in period $t$ is therefore

$$q_{it} = Q_t k_{it} - \frac{\hat{b}_{it}^S}{\pi_t} - \frac{\gamma \hat{b}_{it}^L}{\pi_t} + (1 - \tau) \left[ p_t y_{it} - w_t l_{it} + (\varepsilon_{it} - \delta) Q_t k_{it} - f - \frac{c(\hat{b}_{it}^S + \hat{b}_{it}^L)}{\pi_t} \right]$$

The real face value of nominal short-term debt (in terms of the time $t$ numéraire) depends on inflation $\pi_t$: $\tilde{B}_{it}^S / P_t = \hat{b}_{it}^S P_{i-1} / P_t = \hat{b}_{it}^S / \pi_t$. Similarly: $\tilde{B}_{it}^L / P_t = \hat{b}_{it}^L / \pi_t$. The fact that coupon payments are tax deductible lowers total tax payments by the amount $\tau c(\hat{b}_{it}^S + \hat{b}_{it}^L) / \pi_t$. This is the benefit of debt. The downside is that the firm cannot commit to repaying its debt.

**Definition. Limited liability:** Shareholders are protected by limited liability. They are free to default and hand over the firm’s assets to creditors for liquidation. Default is costly. Creditors recover only a fraction $1 - \xi$ of firm assets.

A defaulting firm exits the economy. In addition to this, there is exogenous exit. With probability $\kappa$, a non-defaulting firm exogenously leaves the economy. In this case, the exiting firm repurchases any outstanding stock of long-term debt at the market value ($b_{it} / \pi_t) p_{it}^L$. The remaining firm value $q_{it} - (b_{it} / \pi_t) p_{it}^L$ is paid out to shareholders.

The timing is summarized in Figure 3. The aggregate state of the economy is $S_{i-1}$. It includes last period’s inflation rate $\pi_{i-1}$ and will be specified below. At the end of period $t - 1$, a firm has an existing stock of nominal long-term debt $b_{i-1}$, and assets in place with market value $q_{i-1}$. Firm productivity $z_{it}$ is realized. The firm chooses capital $k_{it}$. Capital is financed by issuing equity and by selling short-term debt and additional long-term debt. In period $t$, the new aggregate state $S_t$ is realized and firm $i$ produces output $y_{it}$ with labor $l_{it}$. The idiosyncratic capital quality shock $\varepsilon_{it}$ is realized and the firm decides whether to default.

Exogenous exit occurs with probability $\kappa$. Continuing firms have an amount $b_{it} = (1 - \gamma) \hat{b}_{it}^L$ of long-term debt outstanding. The new market value of firm assets is $q_{it}$.

\(^{10}\)Using data on underwriting spreads, Altnikolç and Hansen (2000) provide empirical evidence on increasing marginal equity issuance costs. Besides underwriting fees, the reduced form equity issuance cost $G(e_{it-1})$ may also capture costs from adverse selection on the stock market (cf. Myers and Majluf, 1984).
3.2 Production firms: Default decision

Production firms maximize shareholder value, that is, the expected real present value of net cash flows to shareholders. Consider the firm’s default decision in period $t$. The idiosyncratic state of the firm at the end of period $t$ will be summarized by the market value of assets in place $q_{it}$, the amount of outstanding long-term debt $b_{it}$, and firm-specific productivity $z_{it+1}$. The firm decides on default at time $t$ after production and after the realization of the firm-specific capital quality shock $\varepsilon_{it}$. The variables $q_{it}$ (which depends on $\varepsilon_{it}$) and $b_{it}$ are known at this point but next period’s firm productivity $z_{it+1}$ is not yet realized. If the firm does not default, expected shareholder value is therefore

$$(1 - \kappa) \hat{\mathbb{E}} V_t(q_{it}, b_{it}, z_{it+1}, S_t) + \kappa \left( q_{it} - \frac{b_{it}}{\pi_t} \hat{\mathbb{E}} p_{it}^L \right),$$

where the expectation $\hat{\mathbb{E}}$ is taken over future firm productivity $z_{it+1}$ conditional on $z_{it}$. With probability $1 - \kappa$, the firm continues to operate. Shareholder value is $V_t(q_{it}, b_{it}, z_{it+1}, S_t)$ in this case. Exogenous exit occurs with probability $\kappa$.

Limited liability protects shareholders from large negative realizations of $\varepsilon_{it}$. As we will see below, both $q_{it}$ and the firm-specific price of long-term debt $p_{it}^L$ are functions of $\varepsilon_{it}$. If (3.9) is strictly increasing in $\varepsilon_{it}$, there exists a unique threshold realization $\bar{\varepsilon}_{it}$ which sets expected shareholder value (3.9) to zero:

$$\bar{\varepsilon}_{it} : \quad (1 - \kappa) \hat{\mathbb{E}} V_t(q_{it}, b_{it}, z_{it+1}, S_t) + \kappa \left( q_{it} - \frac{b_{it}}{\pi_t} \hat{\mathbb{E}} p_{it}^L \right) = 0$$

(3.10)

If $\varepsilon_{it}$ is smaller than $\bar{\varepsilon}_{it}$, full repayment would result in negative expected shareholder value, whereas default provides an outside option of zero. In that case, the firm optimally defaults on its debt liabilities. Among other variables, the threshold value $\bar{\varepsilon}_{it}$ depends on $q_{it}$ and $b_{it}$. By choosing capital $k_{it}$ and debt $\tilde{b}_{it}^S$ and $\tilde{b}_{it}^L$ at the end of period $t - 1$, the firm affects the default threshold $\bar{\varepsilon}_{it}$ and thereby the probability of default in period $t$.

3.3 Production firms: Creditors’ problem

The optimal firm policy crucially depends on the two bond prices $p_{it-1}^S$ and $p_{it-1}^L$. Low bond prices imply high credit spreads which increase the firm’s cost of capital. Creditors
are perfectly competitive and break even even on expectation. They buy firm debt at the end of period \( t - 1 \). As all firm debt is held by the representative household, bonds are priced using the stochastic discount factor of the representative household \( \Lambda_{t-1,t} \). If the firm does not default in period \( t \), short-term creditors receive a real amount \((1 + c)\bar{b}_{it}^S/\pi_t\), and long-term creditors are paid \((\gamma + c)\bar{b}_{it}^L/\pi_t\). In case of default, the value of the firm’s assets is

\[
q_{it} \equiv Q_{it}k_{it} + (1 - \tau)\left[p_{it}y_{it} - w_{it}l_{it} + (\varepsilon_{it} - \delta)Q_{it}k_{it} - f\right]
\]  

(3.11)

At this point, creditors liquidate the firm’s assets and receive \((1 - \xi)q_{it}\). Short- and long-term debt have equal seniority. The break-even price of nominal short-term debt (in terms of the time \( t - 1 \) numéraire) is therefore

\[
p_{it}^S = \mathbb{E}_{t-1} \Lambda_{t-1,t} \left[1 - \Phi(\bar{\varepsilon}_{it})\right] \frac{1 + c}{\pi_t} \frac{1}{\bar{b}_{it}^S + \bar{b}_{it}^L} \int_{-\infty}^{\bar{\varepsilon}_{it}} q_{it}\varphi(\varepsilon) d\varepsilon ,
\]

(3.12)

where \(1 - \Phi(\bar{\varepsilon}_{it})\) is the probability that \(\varepsilon_{it} > \bar{\varepsilon}_{it}\). The price of short-term debt depends on firm behavior at time \( t \), in particular on the risk of default \(\Phi(\bar{\varepsilon}_{it})\). In contrast, the price of long-term debt \(p_{it}^L\) also depends on the future market value of long-term debt \(p_{it}^L = g_t(q_{it}, b_{it}, z_{it+1}, S_t)\):

\[
p_{it}^L = \mathbb{E}_{t-1} \Lambda_{t-1,t} \int_{\bar{\varepsilon}}^{\infty} \frac{\gamma + c + (1 - \gamma)g_t(q_{it}, b_{it}, z_{it+1}, S_t)}{\pi_t} \varphi(\varepsilon) d\varepsilon + \frac{1 - \xi}{\bar{b}_{it}^S + \bar{b}_{it}^L} \int_{-\infty}^{\bar{\varepsilon}} q_{it}\varphi(\varepsilon) d\varepsilon ,
\]

(3.13)

where the expectation \(\mathbb{E}\) is taken over the aggregate state \(S_t\) and future firm-level productivity \(z_{it+1}\). If the firm does not default in period \( t \), it repays a fraction \(\gamma\) of the outstanding debt plus the coupon \( c \). A fraction \(1 - \gamma\) of the debt remains outstanding. Because the future price of long-term debt \(p_{it}^L\) depends on future firm behavior, it is a function of the future state of the firm: \(p_{it}^L = g_t(q_{it}, b_{it}, z_{it+1}, S_t)\). The firm cannot directly control future firm behavior, but if can influence its future state through today’s choice of capital \(k_{it}\) and debt \(\bar{b}_{it}^S\) and \(\bar{b}_{it}^L\).

### 3.4 Production firms: Firm problem

We return to the firm’s problem and continue to proceed by backward induction. Before the realization of the firm-specific capital quality shock \(\varepsilon_{it}\) and the firm’s default decision, the aggregate state \(S_t\) is realized and the firm chooses labor demand \(l_{it}\):

\[
l_{it} = \frac{\zeta(1 - \psi)p_{it}y_{it}}{w_{it}} \quad \Leftrightarrow \quad l_{it} = \left(\frac{\zeta(1 - \psi)p_{it}z_{it}k_{it}^{\psi\xi}}{w_{it}}\right)^{1/(1 - \psi)}
\]

(3.14)

Before the aggregate state \(S_t\) is realized, at the end of period \( t - 1 \) the firm chooses next period’s capital stock and its preferred financing mix between equity and short- and long-term debt. Firm productivity \(z_{it}\) is known at this point. The firm anticipates that shareholder value in period \( t \) will be positive if \(\varepsilon_{it}\) is higher than the threshold value \(\bar{\varepsilon}_{it}\) and zero otherwise. Given the value of assets in place \(q_{it-1}\), existing debt \(b_{it-1}\), firm-level
productivity $z_t$, and the aggregate state $S_{t-1}$, the firm solves:

$$\max_{k_{it}, e_{it-1} \geq e} -e_{it-1} - G(e_{it-1}) - H\left(\tilde{b}^S_{it}, \tilde{b}^L_{it}, b_{it-1}/\pi_{t-1}\right) + \mathbb{E} \Lambda_{t-1,t} \int_{\bar{\varepsilon}}^{\infty} \left[(1 - \kappa) V_t(q_{it}, b_{it}, z_{it+1}, S_t) + \kappa \left(q_{it} - \frac{b_{it}}{\pi_{it}} p_{it}\right)\right] \varphi(\varepsilon) d\varepsilon$$  

(3.15)

$\Lambda_{t-1,t}$ is the stochastic discount factor of the representative household. The firm’s choice of $e_{it-1}$ is bounded from below: $e_{it-1} \geq \varepsilon$, with $\varepsilon < 0$. This constitutes an upper limit for dividend payments.\(^{11}\)

In equilibrium, a firm maximizes shareholder value (3.15) subject to creditors’ break-even conditions (3.12) and (3.13). Because we assume that the firm has no ability to commit to future actions, it must take its own future behavior as given and chooses today’s policy as a best response. In other words, the firm plays a game against its future selves. As in Klein et al. (2008), we restrict attention to the Markov perfect equilibrium, i.e. we consider policy rules which are functions of the payoff-relevant state variables. The time-consistent policy is a fixed point in which the future firm policy coincides with today’s firm policy.

The value $V_t(q_{it}, b_{it}, z_{it+1}, S_t)$ can be computed recursively. Time subscripts are dropped in the recursive formulation of the problem. Each period, the firm chooses a policy vector

\(^{11}\)If the stock of existing debt $b_{it-1}$ is sufficiently large, the firm may find it optimal to choose a corner solution and pay out the entire asset value of the firm as dividend: $e_{it-1} = -q_{it-1}$. In practice, it is illegal to pay dividends which substantially exceed firm earnings and deplete a firm’s stock of capital. We choose the value of the constraint $\varepsilon$ such that it rules out this corner solution but is not binding in equilibrium. The exact value of $\varepsilon$ does not affect equilibrium variables.
\[ \phi(q, b, z, S) = \{ k, e, \tilde{b}^S, \tilde{b}^L \} \text{ which solves} \]

\[ V(q, b, z, S) = \max_{\phi(q, b, z, S) = \{ k, e, \tilde{b}^S, \tilde{b}^L \}} -e - G(e) - H(\tilde{b}^S, \tilde{b}^L, b/\pi) \]

\[ + \mathbb{E} \Lambda \int_{\tilde{\varepsilon}}^{\infty} \left[ (1 - \kappa) V(q', b', z', S') + \kappa \left( q' - \frac{b'}{\pi} \right) p^L(q', b', z', S') \right] \varphi(\varepsilon) d\varepsilon \quad (3.16) \]

s.t.: \[ q' = Q'k - \frac{\tilde{b}^S}{\pi'} - \frac{\gamma \tilde{b}^L}{\pi'} + (1 - \tau) \left[ py - \omega l + (\varepsilon - \delta) Q'k - f - \frac{c(\tilde{b}^S + \tilde{b}^L)}{\pi'} \right] \]

\[ y = z \left( k^\psi l^{1-\psi} \right)^c \]

\[ l = \left( \frac{\zeta(1 - \psi)pzk^\psi c}{w} \right)^{\frac{1}{1-\zeta(1-\psi)}} \]

\[ \tilde{\varepsilon}: \quad (1 - \kappa) \mathbb{E} \int V(q', b', z', S') + \kappa \left( q' - \frac{b'}{\pi} \right) \mathbb{E} p^L(q', b', z', S') = 0 \]

\[ Qk = q + e + \tilde{b}^S p^S + \left( \tilde{b}^L - \frac{b}{\pi} \right) p^L \]

\[ b' = (1 - \gamma) \tilde{b}^L \]

\[ p^S = \mathbb{E} \Lambda \left[ \frac{1 - \Phi(\tilde{\varepsilon})}{\pi'} + \frac{1 - \xi}{\tilde{b}^S + \tilde{b}^L} \int_{-\infty}^{\tilde{\varepsilon}} q \varphi(\varepsilon) d\varepsilon \right] \]

\[ p^L = \mathbb{E} \Lambda \left[ \int_{\tilde{\varepsilon}}^{\infty} \frac{\gamma + c + (1 - \gamma) \Phi(q', b', z', S')}{\pi'} \varphi(\varepsilon) d\varepsilon + \frac{1 - \xi}{\tilde{b}^S + \tilde{b}^L} \int_{-\infty}^{\tilde{\varepsilon}} q \varphi(\varepsilon) d\varepsilon \right] \]

### 3.5 Production firms: Firm distribution

The continuum of production firms is characterized by the time-varying firm distribution \( \mu_{t-1}(q, b, z) \) which is part of the aggregate state of the economy \( S_{t-1} \). Firms exit the economy endogenously because of default and exogenously at rate \( \kappa \). The mass of firms always remains constant at one because in each period the mass of entrants \( M_t \) is equal to the time-varying mass of exiting firms. Entrants start without existing assets or debt and with initial productivity \( z = z^e \).

Let \( \tilde{b}^L_t = \tilde{b}^L(q, b, z, S) \) denote the firm’s choice of long-term debt as a function of its state, and let \( \tilde{\varepsilon}_t = \tilde{\varepsilon}(q, b, z, S, S') \) denote the firm’s default threshold (which also depends on the realization of aggregate state \( S' \)). The law of motion for the firm distribution is

\[ \mu_t(q', b', z') = \Gamma(\mu_{t-1}(q, b, z), S, S') = \]

\[ \int_0^\infty \int_0^\infty \int_{-\infty}^{\tilde{\varepsilon}} \int_{\tilde{\varepsilon}}^{\infty} \mu_{t-1}(q, b, z) \pi(z' | z) (1 - \kappa) I(q', b', q, b, z, S, S', \varepsilon) \varphi(\varepsilon) d\varepsilon dq db dz + M(q', b', z') , \]

(3.17)

where the indicator function \( I(q', b', q, b, z, S, S', \varepsilon) = 1 \) if the firm’s future market value of assets is equal to \( q' = q'(q, b, z, S, S', \varepsilon) \) and its future stock of existing debt is \( b' = \)
\( (1 - \gamma) \tilde{b}^L (b, z, S) \). The function \( \mathcal{M}(q', b', z') \) is equal to the mass of entrants \( M_t \) at \( q' = b' = 0 \) and \( z' = z^e \), and zero otherwise.

### 3.6 Retail firms and final goods sector

The remainder of the model setup closely follows Bernanke et al. (1999) and Ottonello and Winberry (2020). To keep the model tractable and transparent, we have separated firm heterogeneity from nominal frictions by modeling production firms as price takers. We now introduce nominal rigidities through a constant unit mass of retail firms. These retail firms buy undifferentiated goods from production firms, repackage them, and sell them as differentiated varieties to the final goods sector. The amount of retail goods \( \tilde{y}_{jt} \) produced by retailer \( j \in [0, 1] \) is

\[
\tilde{y}_{jt} = y_{jt},
\]

where \( y_{jt} \) is the quantity of undifferentiated production goods used by retailer \( j \). Period profits are

\[
\tilde{p}_{jt} \tilde{y}_{jt} - p_t y_{jt} - \frac{\lambda}{2} \left( \frac{\tilde{p}_{jt}}{\tilde{p}_{jt-1}} - 1 \right)^2 Y_t,
\]

where \( \tilde{p}_{jt} \) is the price of variety \( j \) and \( p_t \) is the price of undifferentiated production goods which is equal for all production firms. Rotemberg-style costs of price adjustment are expressed as a fraction of aggregate real output \( Y_t \).

Retail goods are bought by a perfectly competitive final goods sector which produces final goods \( Y_t \) at constant returns to scale:

\[
Y_t = \left[ \int_0^1 \tilde{y}_{jt}^\rho \, dj \right]^{1/\rho}, \quad \text{where} \quad \rho > 1
\]

Profit maximization in the final goods sector yields a demand curve for variety \( j \):

\[
\tilde{y}_{jt} = \left( \frac{P_t}{\tilde{p}_{jt}} \right)^\rho Y_t, \quad \text{where:} \quad P_t = \left[ \int_0^1 \tilde{p}_{jt}^{1-\rho} \, dj \right]^{1/(1-\rho)}
\]

Imperfect substitutability among different varieties gives each retailer some amount of market power. Optimal dynamic price setting by retailer \( j \) gives the following first order condition for \( \tilde{p}_{jt} \):

\[
\tilde{y}_{jt} - \rho \left( \frac{\tilde{p}_{jt} - p_t}{\tilde{p}_{jt}} \right) \tilde{y}_{jt} - \lambda Y_t \frac{\tilde{p}_{jt}}{\tilde{p}_{jt-1}} \left( \frac{\tilde{p}_{jt}}{\tilde{p}_{jt-1}} - 1 \right) + \mathbb{E} \Lambda_{t,t+1} \lambda Y_{t+1} \frac{\tilde{p}_{jt+1}}{\tilde{p}_{jt}} \left( \frac{\tilde{p}_{jt+1}}{\tilde{p}_{jt}} - 1 \right) \frac{\tilde{p}_{jt+1}}{\tilde{p}_{jt}} = 0 \quad (3.22)
\]

From symmetry (\( \tilde{p}_{jt} = P_t \) and \( \tilde{y}_{jt} = Y_t \)), it follows:

\[
1 - \rho \left( \frac{P_t - p_t}{P_t} \right) - \lambda \frac{1}{P_{t-1}} \left( \frac{P_t}{P_{t-1}} - 1 \right) + \mathbb{E} \Lambda_{t,t+1} \lambda Y_{t+1} \left( \frac{P_{t+1}}{P_t} - 1 \right) \frac{P_{t+1}}{P_t} = 0 \quad (3.23)
\]
The final good is the numéraire: \( P_t = 1 \). Using \( \pi_t = P_t / P_{t-1} \), we derive as the New Keynesian Phillips Curve:

\[
1 - \rho (1 - p_t) - \lambda \pi_t (\pi_t - 1) + \mathbb{E} \Lambda_{t,t+1} \frac{Y_{t+1}}{Y_t} \pi_{t+1} (\pi_{t+1} - 1) = 0
\]  

This nonlinear equation relates retailers’ markup \( 1/p_t \) to contemporaneous inflation \( \pi_t \) as well as to expected future inflation \( \pi_{t+1} \) and expected real output growth \( Y_{t+1}/Y_t \). After a positive shock to aggregate demand, the relative price of undifferentiated production goods \( p_t \) increases and the markup \( 1/p_t \) falls. Retailers respond by raising prices which increases inflation through (3.24). A higher value of the price adjustment cost parameter \( \lambda \) dampens the contemporary response of inflation.

### 3.7 Capital producers

There is a representative capital good producer who adjusts the aggregate stock of capital from \((1 - \delta)K_{t-1}\) to \(K_t\) using an amount \(I_{t-1}\) of final goods with decreasing returns:

\[
K_t = \Phi \left( \frac{I_{t-1}}{K_{t-1}} \right) K_{t-1} + (1 - \delta)K_{t-1}, \quad \text{where: } \Phi \left( \frac{I_{t-1}}{K_{t-1}} \right) = \frac{\delta^{\frac{1}{\phi}}}{1 - \frac{1}{\phi}} \left( \frac{I_{t-1}}{K_{t-1}} \right)^{1-\frac{1}{\phi}} - \frac{\delta}{\phi - 1},
\]

and \( \phi > 1 \). \( I_{t-1} = \delta K_{t-1} \) implies \( K_t = K_{t-1} \) for any \( \phi \). Furthermore, \( K_t \to K_{t-1} \) as \( \phi \to 1 \) for any \( I_{t-1} \). Profit maximization pins down the price of capital goods:

\[
Q_{t-1} = \left( \frac{I_{t-1}}{K_{t-1}} \right)^{\frac{1}{\phi}}
\]

This implies for the steady state price of capital goods when \( I_{t-1} = \delta K_{t-1} \): \( Q_{t-1} = 1 \).

### 3.8 Government and monetary policy

The government collects a corporate income tax and pays out the proceeds to the representative household as a lump-sum transfer. In addition, the government conducts monetary policy by setting the nominal riskless interest rate \( r_{t}^{\text{nom}} \) according to the Taylor rule:

\[
\ln(1 + r_{t}^{\text{nom}}) = \ln \frac{1}{\beta} + \varphi \ln \pi_t + \varepsilon_t^m,
\]

where \( \beta \in (0,1) \) is the representative households’ discount rate of future utility. The parameter \( \varphi \) is the inflation weight of the reaction function, and the stochastic component \( \varepsilon_t^m \) is driven by monetary shocks \( \eta_t^m \) following:

\[
\varepsilon_t^m = \rho \varepsilon_{t-1}^m + \eta_t^m, \quad \text{with: } \eta_t^m \sim N(0, \sigma_m^2)
\]
3.9 Households

We close the model by introducing a representative household that owns all equity and debt claims issued by firms and receives all income in the economy (including profits by production firms, retail firms, and capital producers). Government revenue from taxation is paid out to the household as a lump-sum transfer. The household works and consumes final goods. The household saves by buying equity and debt securities issued by production firms.

Future utility is discounted at rate $\beta$. We assume additive-separable preferences over consumption $C_t$ and labor $L_t$. Period utility is

$$\ln(C_t) - \frac{L_t^{1+\theta}}{1+\theta}, \quad \text{with: } \theta > 0 \quad (3.29)$$

The stochastic discount factor of the representative household is accordingly:

$$\Lambda_{t-1,t} = \beta \frac{C_{t-1}}{C_t} \quad (3.30)$$

3.10 General equilibrium

Let $y(q, b, z, S, S')$ denote output of production firms as a function of their state. In equilibrium, the amount of final goods produced is

$$Y_t = \int_0^\infty \int_0^\infty \int_{-\infty}^\infty y(q, b, z, S, S') \mu(q, b, z) \, dq \, db \, dz \quad (3.31)$$

The aggregate quantity of final goods which is available for consumption and investment (as well as for debt and equity issuance costs) is reduced by the fixed cost of operation, default costs, and price adjustment costs:

$$Y_{t}^{\text{net}} \equiv \int_0^\infty \int_0^\infty \int_{-\infty}^\infty \left[ y(q, b, z, S, S') - f - \xi \int_{-\infty}^\infty q(q, b, z, S, S', \varepsilon) \varphi(\varepsilon) \, d\varepsilon \right] \mu(q, b, z) \, dq \, db \, dz$$

$$- \frac{\lambda}{2} (\pi_t - 1)^2 Y_t \quad (3.32)$$

At the end of period $t-1$, the aggregate state of the economy $S$ consists of the firm distribution $\mu_{t-1}(q, b, z)$, net output $Y_{t-1}^{\text{net}}$, the aggregate stock of capital $K_{t-1}$, inflation $\pi_{t-1}$, and the stochastic component of the Taylor rule $\varepsilon_{t-1}^{m}$: $S = \{\mu_{t-1}(q, b, z), Y_{t-1}^{\text{net}}, K_{t-1}, \pi_{t-1}, \varepsilon_{t-1}^{m}\}$.

**Definition.** Given the aggregate state $S = \{\mu_{t-1}(q, b, z), Y_{t-1}^{\text{net}}, K_{t-1}, \pi_{t-1}, \varepsilon_{t-1}^{m}\}$, the equilibrium consists of (i) a policy vector $\phi(q, b, z, S) = \{k, e, b^e, b^L\}$, a value function $V(q, b, z, S)$, and bond price functions $p^S$ and $p^L$; (ii) a distribution $\mu_t(q', b', z')$ and a mass of entrants $M_t$; (iii) a production goods price $p_t$, inflation $\pi_t$, and a price of capital goods $Q_{t-1}$; (iv) household consumption $C_{t-1}$ and aggregate labor supply $L_t$, and (v) a stochastic discount factor $\Lambda_{t-1,t}$, a nominal interest rate $r_{t-1}^{\text{nom}}$, and a wage $w_t$, such that for all periods $t$ and for all realizations of the monetary shock $\eta_{t}^{m}$:

1. $\phi(q, b, z, S), V(q, b, z, S), p^S, \text{ and } p^L$ solve the firm problem (3.16).
2. $\mu_t(q', b', z') = \Gamma(\mu_{t-1}(q, b, z), S, S')$ as in (3.17)
3. The Phillips curve (3.24), the capital goods price (3.26), and the Taylor rule (3.27) hold.

4. The representative household chooses $C_{t-1}$ and $L_t$ optimally:

$$1 = \mathbb{E} \Lambda_{t-1,t}(1 + \frac{r_{t-1}^{\text{nom}}}{\pi_t})$$ and $w_t = L_t^0 C_t$

5. The market for capital goods, the labor market, and the final goods market clear.

In equilibrium, the real interest rate is $1/\mathbb{E} \Lambda_{t-1,t} - 1$. The capital goods market clears if and only if

$$K_t = \int_0^\infty \int_0^\infty \int_{-\infty}^\infty k(q, b, z, S) \mu_{t-1}(q, b, z) \, dq \, db \, dz \quad (3.33)$$

Labor market clearing implies that

$$L_t = \int_0^\infty \int_0^\infty \int_{-\infty}^\infty l(q, b, z, S, S') \mu_{t-1}(q, b, z) \, dq \, db \, dz \quad (3.34)$$

The market for final goods clears if and only if:

$$Y_{t}^{\text{net}} = C_t + I_t$$

$$+ \int_0^\infty \int_0^\infty \int_{-\infty}^\infty \left[G(e(q', b', z', S')) + H(\tilde{b}^S(q', b', z', S'), \tilde{b}^L(q', b', z', S'), b'/\pi')\right] \mu_t(q', b', z') \, dq' \, db' \, dz'$$

(3.35)

where $C_t$ is household consumption and aggregate investment $I_t$ is given by (3.25):

$$I_t = K_t \left[\frac{\phi - 1}{\phi} \delta^{-\frac{1}{\phi}} \left(\frac{K_{t+1}}{K_t} - 1 + \delta \frac{\phi}{\phi - 1}\right)^{1-\phi}\right]^{\frac{\phi}{\phi - 1}}$$

(3.36)

4 **Characterization**

How do firms choose investment in this model economy? How do they choose their preferred financing mix between equity, short-term debt, and long-term debt? And how do these choices matter for the effects of monetary policy? In this section, we characterize equilibrium firm behavior using first order conditions, and we analyze the role of debt maturity for firms’ investment response to monetary policy.

4.1 **Production firms: First order conditions**

The firm problem (3.16) can be expressed in terms of only three choice variables: the scale of production $k$ and the amounts of short-term debt $\tilde{b}^S$ and long-term debt $\tilde{b}^L$. Accordingly, the equilibrium behavior of firms is characterized by three first order conditions. For simplicity, we discuss these optimality conditions assuming that there is no exogenous exit ($\kappa = 0$) and that the liquidation value of a firm is zero ($\xi = 1$). All derivations are deferred to Appendix C.\(^{12}\)

\(^{12}\)The first order conditions for the general case of $\kappa, \xi \in [0, 1]$ can also be found in Appendix C.
With $\xi = 1$, the only firm characteristic which the short-term bond price $p^S$ depends on is the default threshold $\bar{\varepsilon}$:

$$p^S = \mathbb{E} \Lambda \left[ 1 - \Phi(\bar{\varepsilon}) \right] \frac{1 + c}{\pi'} \tag{4.1}$$

Besides $\bar{\varepsilon}$, the long-term bond price $p^L$ also depends on the future state of the firm $(q', b', z')$:

$$p^L = \mathbb{E} \Lambda \left[ 1 - \Phi(\bar{\varepsilon}) \right] \frac{\gamma + c}{\pi'} + \frac{1 - \gamma}{\pi'} \int_{\bar{\varepsilon}}^{\infty} g(q', b', z', S') \varphi(\varepsilon) d\varepsilon \tag{4.2}$$

Ceteris paribus, both bond prices are strictly decreasing in the default threshold $\bar{\varepsilon}$ which depends on firm actions through (3.10). As shown in Appendix C, $\bar{\varepsilon}$ is a function of $q'$ and $b'$, and therefore depends on $k$, $\bar{b}^S$, and $\bar{b}^L$. By choosing its scale of production $k$, long-term debt $\bar{b}^L$, and short-term debt $\bar{b}^S$, the firm affects the default threshold $\bar{\varepsilon}$ and thereby the probability of default.

**Capital** As usual, choosing capital involves trading off resources today against resources tomorrow. One additional aspect in this model is the effect of capital on default risk and therefore bond prices. The firm’s first order condition with respect to capital demand $k$ is:

$$\left[ 1 + \frac{\partial G(e)}{\partial e} \right] - Q + \left( \bar{b}^L - \frac{b}{\pi} \right) \mathbb{E} \Lambda \frac{1 - \gamma}{\pi'} \int_{\bar{\varepsilon}}^{\infty} \frac{\partial q'}{\partial k} \frac{\partial g(q', b', z', S')}{\partial q'} \varphi(\varepsilon) d\varepsilon$$

$$+ \left[ 1 + \frac{\partial G(e)}{\partial e} \right] \mathbb{E} \frac{\partial \bar{\varepsilon}}{\partial k} \left[ \bar{b}^S \frac{\partial p^S}{\partial \bar{\varepsilon}} + \left( \bar{b}^L - \frac{b}{\pi} \right) \frac{\partial p^L}{\partial \bar{\varepsilon}} \right]$$

$$+ \mathbb{E} \Lambda \int_{\bar{\varepsilon}}^{\infty} \frac{\partial q'}{\partial e'} \varphi(\varepsilon) d\varepsilon = 0 \tag{4.3}$$

Consider the first line of (4.3). The cost of one unit of capital is equal to the price of capital goods $Q$. For a given quantity of short-term and long-term debt, an increase of capital is a net injection of equity into the firm. The associated cost is therefore $[1 + \partial G(e)/\partial e] Q$. A higher capital stock increases expected future firm assets $q'$ (because $\partial q'/\partial k > 0$ on expectation). If a higher value of $q'$ increases the future price of long-term debt (i.e. if $\partial g(q', b', z', S')/\partial q' > 0$), this raises already today the current price of long-term debt $p^L$ and thereby increases the firm’s bond market revenue from selling long-term debt (first line). In addition, if additional equity lowers the risk of default (i.e. if $\partial \bar{\varepsilon}/\partial k < 0$), this increases the current market price of short-term (because $\partial p^S/\partial \bar{\varepsilon}<0$) and long-term debt (because $\partial p^L/\partial \bar{\varepsilon}<0$, second line). The last benefit is the direct gain from higher expected future firm assets: $\partial q'/\partial k$ (third line). Because of diminishing returns in production, this benefit is decreasing in the level of capital.

An existing stock of debt $b/\pi$ may decrease investment. If $\partial g(q', b', z', S')/\partial q' > 0$ and $\partial \bar{\varepsilon}/\partial k < 0$, the firm’s marginal benefit of $k$ is falling in $b/\pi$ (as is clear from the first and the second line of (4.3)). This is because a higher market price of long-term debt benefits shareholders only to the extent that it increases the firm’s revenue from selling new long-term debt. The fact that lower default risk also increases the market value of existing debt is not internalized by the firm. This is the classic *debt overhang* effect described by Myers (1977).
Short-term debt  The firm’s choice between equity and debt is determined by the trade-off between the tax benefit of debt on the one hand and expected default costs and debt issuance costs on the other hand. The firm’s first order condition with respect to the optimal amount of short-term debt $\hat{b}^S$ is

$$- H(\hat{b}^S, \hat{b}^L, b/\pi)^c + \frac{\partial H(\hat{b}^S, \hat{b}^L, b/\pi)}{\partial b^S} + \frac{\partial G(e)}{\partial e} \left( (1 + c) \frac{\partial G(e)}{\partial e} - (1 + c - \tau c) \frac{\partial G(e^c)}{\partial e^c} \right) \varphi(\varepsilon) d\varepsilon$$

$$+ \left[ 1 + \frac{\partial G(e)}{\partial e} \right] \left( \hat{b}^L - \frac{b}{\pi} \right) - \gamma \int_{\varepsilon}^{\infty} \frac{\partial G(q', b', z', S')}{\partial q'} \varphi(\varepsilon) d\varepsilon$$

$$+ \left[ 1 + \frac{\partial G(e)}{\partial e} \right] \frac{\partial \bar{\varepsilon}}{\partial b^S} \left[ \hat{b}^S \frac{\partial p^S}{\partial \bar{\varepsilon}} + \left( \hat{b}^L - \frac{b}{\pi} \right) \frac{\partial p^L}{\partial \bar{\varepsilon}} \right] = 0$$

When the firm sells short-term debt, it incurs the debt issuance cost $H(\hat{b}^S, \hat{b}^L, b/\pi)$. One benefit of short-term debt is that future tax payments are reduced by the amount $\tau c / \pi'$. This benefit materializes only if default is avoided. An additional aspect is that short-term debt transfers resources from the future to the present. Because of the tax advantage of debt, this is beneficial unless the marginal value of future cash is significantly higher than the marginal value of cash today, i.e. unless $\partial G(e') / \partial e' \gg \partial G(e) / \partial e$ (first line). Short-term debt is due next period which lowers future firm assets $q'$ (because $\partial q'/\partial \hat{b}^S < 0$). If lower future assets $q'$ imply that the future price of long-term debt is reduced (i.e. if $\partial g(q', b', z', S')/\partial q' > 0$), this is a cost of issuing short-term debt because it decreases already today the current price of long-term debt $p^L$ and thereby reduces the firm’s bond market revenue (second line). Finally, issuing short-term debt always increases next period’s probability of default ($\partial \bar{\varepsilon}/\partial \hat{b}^S > 0$) which again lowers bond market revenue (because $\partial p^S/\partial \bar{\varepsilon} < 0$ and $\partial p^L/\partial \bar{\varepsilon} < 0$, third line).

Debt overhang also affects the firm’s choice of debt. If $\partial g(q', b', z', S') / \partial q' > 0$, the optimal value of $\hat{b}^S$ is increasing in the stock of existing debt $b/\pi$ (as can be seen from the second and third line of (4.4)) because $\partial q'/\partial \hat{b}^S < 0$, $\partial \bar{\varepsilon}/\partial \hat{b}^S > 0$, and $\partial p^L/\partial \bar{\varepsilon} < 0$). As explained above, the firm does not internalize potential default costs which pertain to the holders of existing long-term debt. While the firm fully internalizes the tax benefits of additional debt, it only internalizes part of the associated costs. If $b/\pi$ is high, long-term debt issuance $\hat{b}^L - b/\pi$ is small. This reduces the part of expected default costs which is internalized by the firm through the bond market and implies a higher optimal value of short-term debt.\(^{13}\)

Long-term debt  As for the case of short-term debt, issuing long-term debt saves future taxes but is costly because of default risk and debt issuance costs. The firm’s first order

\(^{13}\)In the sovereign debt literature (e.g. Hatchondo et al., 2016), this incentive to increase indebtedness at the expense of existing creditors is also known as debt dilution.
condition with respect to \( \tilde{b}^L \) is:

\[
- \frac{\partial H(\tilde{b}^S, \tilde{b}^L, b/\pi)}{\partial \tilde{b}^L} + \frac{\Lambda}{\pi} \int_{\bar{\varepsilon}}^{\infty} \left[ \tau_c + (\gamma + c) \frac{\partial G(e)}{\partial e} - (\gamma + c - \tau_c) \frac{\partial G(e')}{\partial e'} \right] \phi(\varepsilon)d\varepsilon \\
+ \left[ 1 + \frac{\partial G(e)}{\partial e} \right] \frac{\Lambda}{\pi} \frac{1 - \gamma}{\pi^2} \int_{\bar{\varepsilon}}^{\infty} \left[ \frac{\partial b'}{\partial b} \frac{\partial g(q', b', z', S')}{\partial b'} + \frac{\partial q'}{\partial q} \frac{\partial g(q', b', z', S')}{\partial q'} \right] \phi(\varepsilon)d\varepsilon \\
+ \left[ 1 + \frac{\partial G(e)}{\partial e} \right] \frac{\Lambda}{\pi} \frac{1 - \gamma}{\pi^2} \int_{\bar{\varepsilon}}^{\infty} \left[ \frac{\partial b'}{\partial b} \frac{\partial g(q', b', z', S')}{\partial b'} + \frac{\partial q'}{\partial q} \frac{\partial g(q', b', z', S')}{\partial q'} \right] \phi(\varepsilon)d\varepsilon \\
= 0
\]

A number of elements in (4.5) are identical to the first order condition with respect to short-term debt \( \tilde{b}^S \) in (4.4). However, there also are a few important differences. Combining (4.5) with (4.4) yields a condition for firms’ optimal maturity choice (or of substituting short-term debt \( \tilde{b}^S \) by long-term debt \( \tilde{b}^L \)):

\[
- \frac{\partial H(\tilde{b}^S, \tilde{b}^L, b/\pi)}{\partial \tilde{b}^L} + \frac{\Lambda}{\pi} \int_{\bar{\varepsilon}}^{\infty} \left[ \tau_c + (\gamma + c) \frac{\partial G(e)}{\partial e} - (\gamma + c - \tau_c) \frac{\partial G(e')}{\partial e'} \right] \phi(\varepsilon)d\varepsilon \\
+ \left[ 1 + \frac{\partial G(e)}{\partial e} \right] \frac{\Lambda}{\pi} \frac{1 - \gamma}{\pi^2} \int_{\bar{\varepsilon}}^{\infty} \left[ \frac{\partial g(q', b', z', S')}{\partial b'} + \frac{\partial q'}{\partial q} \frac{\partial g(q', b', z', S')}{\partial q'} \right] \phi(\varepsilon)d\varepsilon \\
- \left[ 1 + \frac{\partial G(e)}{\partial e} \right] \frac{\Lambda}{\pi} \frac{1 - \gamma}{\pi^2} \int_{\bar{\varepsilon}}^{\infty} \left[ \frac{\partial b'}{\partial b} \frac{\partial g(q', b', z', S')}{\partial b'} + \frac{\partial q'}{\partial q} \frac{\partial g(q', b', z', S')}{\partial q'} \right] \phi(\varepsilon)d\varepsilon \\
= 0
\]

Short-term debt and long-term debt both transfer resources from the future to the present. A given unit of short-term debt is more effective in this regard because its market price \( p^S \) is higher than the price of long-term debt \( p^L \) if future default risk (beyond next period) is positive (i.e. if \( g(q', b', z', S') < 1 \)). If the marginal value of cash today is higher than the marginal value of cash tomorrow (i.e. if \( \partial G(e)/\partial e > \partial G(e')/\partial e' \)), this is a downside of borrowing at long maturities (first line, first term). On the other hand, by issuing long-term debt the firm saves future debt issuance costs because long-term debt matures more slowly over time than short-term debt. This is a benefit because \( \partial H/\partial (b'/\pi) \leq 0 \) (first line, second term).

Today’s price of long-term debt \( p^L \) is affected by the firm’s maturity choice through tomorrow’s price of long-term debt \( g(q', b', z', S') \). On the one hand, issuing long-term debt instead of short-term debt implies a higher future outstanding stock of debt \( b' = (1 - \gamma)b^L \) which lowers \( g(q', b', z', S') \) because of debt overhang. This is the most important downside of borrowing at long maturities. On the other hand, it also increases tomorrow’s cash on hand \( q' \) because long-term debt requires lower debt service per period than short-term debt. This can increase \( g(q', b', z', S') \). The joint effect on tomorrow’s (and therefore today’s) price of long-term debt is theoretically ambiguous (second line).

Today’s price of long-term debt \( p^L \) also responds because of changes in the threshold value \( \bar{\varepsilon} \) and therefore next period’s default risk. Issuing long-term debt instead of short-term
debt lowers next period’s default risk through two channels: (i) Because 
$g(q', b', z', S') < 1$, a given stock of long-term debt tomorrow is less of a debt burden for the firm than the same amount of short-term debt. The reason is that the firm discounts the future at a rate higher than Λ because of future default risk beyond next period. (ii) Future debt issuance costs are lowered because $\partial H/\partial (b'/\pi') \leq 0$. Both effects increase next period’s shareholder value, and thereby lower next period’s risk of default which increases the price of short-term debt and long-term debt already today (because $\partial p^S/\partial \bar{\varepsilon} < 0$ and $\partial p^L/\partial \bar{\varepsilon} < 0$, third line).

4.2 Debt maturity and the investment effect of monetary policy

Our empirical results show that firms with higher amounts of maturing debt respond more strongly to monetary shocks. In this section, we identify and analyze the different channels in our model through which debt maturity generates heterogeneity in firms’ investment response to monetary policy.

Consider a contractionary monetary shock, i.e. a surprise increase in the nominal riskless rate $r^\text{nom}$. This reduces aggregate demand. The representative household consumes less today relative to expected future consumption: The stochastic discount factor $\Lambda = \beta C'/C''$ falls on expectation and the real interest rate $1/E \Lambda - 1$ rises. Financing any given stock of firm capital through equity or debt has become more costly now. Ceteris paribus, this reduces the benefit of investment for all firms in the economy.

Debt maturity generates heterogeneity in firms’ investment response through the state variable $b$. Comparing two otherwise identical firms, the firm which chooses a higher share of long-term debt $\bar{b}^L$ will also have a higher amount of outstanding long-term debt tomorrow $b = (1 - \gamma)\bar{b}^L$ implying a lower share of maturing debt. The amount of outstanding long-term debt $b$ enters the firm problem (3.16) through the cash flow from shareholders to the firm, $e + G(e) + H(\bar{b}^S, \bar{b}^L, b/\pi)$, where equity issuance $e$ is tied to the firm’s choice of capital $k$, short-term debt $\bar{b}^S$, and long-term debt $\bar{b}^L$ through (3.6):

$$e = Qk - q - \bar{b}^S p^S - \left(\bar{b}^L - \frac{b}{\pi}\right) p^L \quad (4.7)$$

The role of $b$ in generating heterogeneity in firms’ investment response to changes in the real interest rate can be decomposed into two distinct channels.

1. Roll-over risk: Through (3.12) and (3.13), changes in the stochastic discount factor of the representative household $\Lambda$ affect the firm’s bond prices $p^S$ and $p^L$.

$$\frac{\partial p^S}{\partial \Lambda} = E \left[1 - \Phi(\bar{\varepsilon})\right] \frac{1 + c}{\pi'} \quad (4.8)$$

$$\frac{\partial p^L}{\partial \Lambda} = E \left[\left[1 - \Phi(\bar{\varepsilon})\right] \frac{\gamma + c}{\pi'} + \frac{1 - \gamma}{\pi'} \int_{\bar{\varepsilon}}^{\infty} g(q', b', z', S') \varphi(\varepsilon) d\varepsilon\right] \quad (4.9)$$

By affecting bond prices and bond market revenues, a change in $\Lambda$ affects the amount of equity which shareholders need to inject into the firm for given choices $k$, $\bar{b}^S$, and

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14 As during the discussion of firms’ first order conditions in Section 4.1, we continue to focus on the special case $\kappa = 0$ and $\xi = 1$.
\[ \partial e \partial \Lambda = -\tilde{b}_S \frac{\partial p^S}{\partial \Lambda} - (\tilde{b}_L - \frac{b}{\pi}) \frac{\partial p^L}{\partial \Lambda} < 0 \] (4.10)

From (4.10), it is clear that a higher amount of existing long-term debt \( b \) dampens the impact of \( \Lambda \) on \( e \):

\[ \frac{\partial^2 e}{\partial \Lambda \partial b} = \frac{1}{\pi} \frac{\partial p^L}{\partial \Lambda} > 0 \] (4.11)

To the extent that the amount of equity issuance \( e \) matters for the firm’s choice of \( k \) (e.g. because of increasing equity issuance costs \( G(e) \)), a higher amount of existing long-term debt \( b \) also dampens the impact of \( \Lambda \), and therefore the real rate, on firm capital \( k \).

A high amount of outstanding long-term debt \( b \) therefore dampens the impact of changes in the real interest rate on firm investment. Firms with higher levels of \( b \) need to issue less new debt per period in order to finance a given amount of leverage. They are therefore less exposed to surprise changes in the real rate. Short-term debt exposes firms to these interest rate fluctuations. Long-term debt provides insurance against roll-over risk.\(^{15}\)

2. **Debt overhang:** The firm’s bond prices \( p^S \) in (4.1) and \( p^L \) in (4.2) can move for two reasons: (1.) changes in \( \Lambda \) (determining the real interest rate of a riskless bond), and (2.) changes in default risk (determining credit spreads). Through a debt overhang channel, the stock of existing debt \( b \) determines how a firm’s default risk and credit spread respond to monetary shocks.

Consider the firm’s first order condition with respect to short-term debt \( \tilde{b}_S \) in (4.4).\(^{16}\) As explained above (third line of (4.4)), a cost to the firm of increasing \( \tilde{b}_S \) is that the firm’s threshold value \( \tilde{\varepsilon} \) (and therefore next period’s risk of default) increases:

\[ \frac{\partial \tilde{\varepsilon}}{\partial \tilde{b}_S} = \frac{1 + (1 - \tau)c}{\pi'(1 - \tau)Q'k} > 0 \] (4.12)

Higher \( \tilde{b}_S \) therefore reduces the firm’s bond market revenue because creditors anticipate higher default risk after the increase in \( \tilde{\varepsilon} \). Let this marginal cost to the firm in the third line of (4.4) be denoted by Cost:

\[ Cost \equiv -E \frac{\partial \tilde{\varepsilon}}{\partial \tilde{b}_S} \left[ \tilde{b}_S \frac{\partial p^S}{\partial \tilde{\varepsilon}} + (\tilde{b}_L - \frac{b}{\pi}) \frac{\partial p^L}{\partial \tilde{\varepsilon}} \right] \] (4.13)

where: \( \frac{\partial p^S}{\partial \tilde{\varepsilon}} = -\Lambda \varphi(\tilde{\varepsilon}) \frac{1 + c}{\pi'} \), and: \( \frac{\partial p^L}{\partial \tilde{\varepsilon}} = -\Lambda \varphi(\tilde{\varepsilon}) \frac{\gamma + c + (1 - \gamma)g(q', b', z', S')}{\pi'} \) (4.14)

\(^{15}\)In a HANK model with household heterogeneity, Auclert (2019) identifies how surprise changes in the real interest rate affect household net worth. Using his terminology in our heterogeneous firm model, firms with high \( b \) have relatively small borrowing needs at the time of the shock and therefore small negative “ unhedged interest rate exposure”.

\(^{16}\) A similar argument can be made using the firm’s first order condition with respect to long-term debt \( \tilde{b}_L \) in (4.5). Debt overhang affects firms’ choice of both short-term debt and long-term debt.
Each period, the firm chooses $\tilde{b}^S$ according to the first order condition (4.4) trading off the benefits of higher debt with the associated costs including Cost. From (4.13) it is clear that Cost is decreasing in the stock of existing debt $b$:

$$\frac{\partial \text{Cost}}{\partial b} = \mathbb{E} \frac{\partial \tilde{\varepsilon}}{\partial b^S} \frac{1}{\pi} \frac{\partial p^L}{\partial \tilde{\varepsilon}} < 0 \quad (4.15)$$

As explained above, a shareholder value-maximizing firm does not internalize potential default costs which pertain to the holders of existing long-term debt. While the firm fully internalizes the tax benefits of additional debt, it only internalizes part of the associated costs. Firms with a higher stock of outstanding debt $b$ face a smaller Cost of elevated default risk and therefore choose higher leverage and accept higher default risk.

Using (4.12), Cost in (4.13) can be re-written as:

$$\text{Cost} = -\mathbb{E} \frac{1}{\pi'(1-\tau)c} \left[ \tilde{b}^S \frac{\partial p^S}{\partial \tilde{\varepsilon}} + \tilde{b}^L \frac{\partial p^L}{\partial \tilde{\varepsilon}} \right] + \mathbb{E} \frac{1}{\pi'(1-\tau)Q'k} \frac{b}{\pi} \frac{\partial p^L}{\partial \tilde{\varepsilon}} \quad (4.16)$$

From the last term in (4.16), we see that Cost is lower if the term $(b/\pi)/(Q'k)$ is larger (because $\partial p^L/\partial \tilde{\varepsilon} < 0$). It is the real value of outstanding debt $b/\pi$ relative to the market value of firm assets $Q'k$ which matters for the firm’s choice of leverage and default risk. As is clear from (4.12), an additional unit of debt $\tilde{b}^S$ increases the risk of default by less if the market value of firm assets $Q'k$ is larger. At the same time, the firm’s incentive to increase $\tilde{b}^S$ is stronger if the amount of outstanding debt $b/\pi$ is higher. If $b/\pi$ is large relative to $Q'k$, the effect of debt overhang on default risk is particularly large. We therefore refer to the term $(b/\pi)/(Q'k)$ as the ‘real burden of outstanding debt’.

Now consider an expansionary shock (e.g. a surprise reduction of the nominal riskless rate $r^\text{nom}$). The market value of firm assets $Q'k$ increases (because the firm chooses a higher scale of production $k$ and/or because the price of capital goods $Q'$ rises). As the derivative of (4.15) with respect to $Q'k$ shows, the increase of $Q'k$ dampens the effect of debt overhang on the firm’s choice of leverage and default risk (holding $\tilde{\varepsilon}$ and $g(q', b', z', S')$ constant):^{17}

$$\frac{\partial^2 \text{Cost}}{\partial b \partial Q'k} = \mathbb{E} \frac{1}{\pi'(1-\tau)Q'^2k^2} \frac{1}{\pi} \Lambda \varphi(\tilde{\varepsilon}) \gamma + c + (1-\gamma)g(q', b', z', S') > 0 \quad (4.17)$$

The fact that previously issued outstanding debt $b$ results in a smaller value of Cost ($\partial \text{Cost}/\partial b < 0$) is counteracted by the increase in the market value of firm assets $Q'k$ ($\partial^2 \text{Cost}/\partial b \partial Q'k > 0$). Of course, the opposite holds after a contractionary monetary shock: A negative shock lowers the market value of firm assets $Q'k$ and thereby

\^{17} Holding the firm’s future price of long-term debt $g(q', b', z', S')$ constant when $Q'k$ increases is without loss of generality as long as future assets $q'$ are sufficiently high. As shown below, the firm state $q$ affects firm behavior only below a certain threshold which separates equity-issuing low-$q$ firms from dividend-paying high-$q$ firms. In Appendix C.1.2, we study how the debt overhang channel is modified by the impact of $Q'k$ on $q'$ and $g(q', b', z', S')$. 

28
increases the ‘real burden of outstanding debt’ \((b/\pi)/(Q'k)\). The value of \(Cost\) in (4.16) falls (because \(\partial p^L/\partial \bar{\varepsilon} < 0\)). Instead of actively reducing the outsized stock of existing debt \(b\) issued before the drop in \(Q'k\), the firm accepts an elevated risk of default which reduces the bond prices \(p^S\) and \(p^L\). The associated rise of credit spreads translates into an increase of the firm’s cost of capital which can reduce investment by even more.

Through this \textit{debt overhang} channel, a high stock of existing debt \(b\) exposes firms to strong increases in default risk and credit spreads after a contractionary shock. By avoiding future \textit{debt overhang}, short-term debt reduces the associated volatility in credit spreads. Long-term debt provides insurance against \textit{roll-over risk} but exposes firms to increased volatility in credit spreads because of \textit{debt overhang}.\footnote{A detailed analytical and quantitative study of how debt overhang can amplify fundamental shocks through increased volatility of default risk and credit spreads is provided in \cite{JungherrSchott2020b}.}

Note that the \textit{debt overhang} channel does not necessarily rely on fluctuations in inflation \(\pi\) or asset prices \(Q'\). For the ‘real burden of outstanding debt’ \((b/\pi)/(Q'k)\) to increase it is sufficient that the firm’s real stock of capital \(k\) falls relative to the stock of outstanding debt \(b\). However, surprise reductions of inflation \(\pi\) or asset prices \(Q'\) amplify the cyclical effects of \textit{debt overhang}.

Besides increasing the real interest rate, a contractionary monetary shock also lowers aggregate demand and inflation \(\pi\). This drop in \(\pi\) increases the real value of outstanding nominal long-term debt \(b/\pi\) and thereby amplifies firms’ heterogeneous exposure to both \textit{roll-over risk} and \textit{debt overhang}. Through this mechanism, \textit{Fisherian debt deflation} accentuates cross-sectional differences in firms’ investment response to monetary policy.

## 5 Quantitative Analysis

The Markov perfect equilibrium in (3.16) can only be computed using numerical methods. Furthermore, the time-varying firm distribution \(\mu_{t-1}(q, b, z)\) is part of the aggregate state \(S_{t-1}\) of the economy. High dimensionality of the state space in combination with aggregate uncertainty prohibits the use of standard numerical methods. In this section, we lay out our computational approach, discuss the calibration strategy, and present quantitative results.

### 5.1 Solution method

Our computational approach follows \cite{Reiter2009}. We first use global methods to solve the steady state of the model with idiosyncratic firm-level uncertainty but without aggregate shocks. This gives us a fully non-linear solution of the steady state including the stationary firm distribution \(\mu(q,b,z)\). We then use first-order perturbation methods to approximate the dynamics of the model close to the steady state in response to aggregate shocks.

Part of the steady state with the stationary firm distribution \(\mu(q,b,z)\) is a global solution to the dynamic firm problem in (3.16) in the absence of fluctuations in the aggregate state \(S\). This solution is found using value function iteration and interpolation as in \cite{JungherrSchott2020a}. The key difficulty consists in finding the equilibrium price of risky long-term debt \(p^L\). Optimal firm behavior depends on \(p^L\) which itself depends on current and
future firm behavior. A firm that cannot commit to future actions must take into account how today’s choices will affect future firm behavior. We solve this fixed point problem by computing the solution to a finite-horizon problem. Starting from a final date, we iterate backward until all firm-level quantities and bond prices have converged. We then use the first-period equilibrium firm policy as the equilibrium policy of the infinite-horizon problem. This means that we iterate simultaneously on the value and the long-term bond price (as in Hatchondo and Martinez, 2009). The presence of the idiosyncratic i.i.d. capital quality shock $\varepsilon$ with continuous probability distribution $\varphi(\varepsilon)$ facilitates the computation of $p^L$ (cf. Chatterjee and Eyigungor, 2012).

Given a global solution to the dynamic firm problem in (3.16), firms’ equilibrium policies $\phi(q, b, z, S) = \{k, \tilde{e}, \tilde{b}^S, \tilde{b}^L\}$ together with the stochastic process characterizing idiosyncratic uncertainty generate the stationary firm distribution $\mu(q, b, z)$. We compute this steady state distribution on a discrete grid of finite dimensionality.

After we have found the steady state of the model, we compute local dynamics close to steady state in response to aggregate shocks. We apply a numerical first-order perturbation method along the lines of Schmitt-Grohé and Uribe (2004). This gives us a solution of the equilibrium with aggregate uncertainty and a time-varying firm distribution $\mu_{t-1}(q, b, z)$ which is non-linear in idiosyncratic shocks and linear in aggregate shocks.

5.2 Calibration

A number of parameters can be set externally using standard values from the literature on firm dynamics and the New Keynesian business cycle literature. Other parameters which are key for firm financing and investment in our model are internally calibrated as explained below.

5.2.1 Externally calibrated parameters

The model period is one quarter. We set $\beta = 0.99$ which implies a quarterly steady state nominal riskless interest rate $r^{nom} = 1.01\%$. The steady state real interest rate is equal to the nominal rate since $\pi_t = 1$ and inflation is zero in the absence of aggregate shocks. The debt coupon is fixed at $c = r^{nom}$ which implies that the steady state equilibrium price of a riskless short-term and long-term bond are both equal to one. The preference parameter $\theta$ is chosen to generate a Frisch elasticity of 2 as in Arellano et al. (2019).

The production technology parameters $\zeta$ and $\psi$ are taken from Bloom et al. (2018). The quarterly depreciation rate $\delta$ is 2.5%. We follow Gomes et al. (2016) in setting the tax rate $\tau = 0.4$ and the repayment rate of long-term debt $\gamma = 0.05$. This choice of $\gamma$ implies a Macaulay duration of $(1 + r^{nom})/(\gamma + r^{nom}) = 16.8$ quarters or 4.2 years. This is a conservative choice relative to the average duration of 6.5 years calculated by Gilchrist and Zakrajšek (2012) for a sample of US corporate bonds with remaining term to maturity above one year.19

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19For the tax rate, Hennessy and Whited (2005) suggest a value of 0.3. The parameter $\tau$ should therefore be thought of as capturing additional benefits of using debt rather than equity (e.g. limiting agency frictions between shareholders and firm managers as in Arellano et al., 2019).
Table 3: Externally calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>preference parameter</td>
<td>0.99</td>
</tr>
<tr>
<td>$c$</td>
<td>debt coupon</td>
<td>$1/\beta - 1$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>inverse Frisch elasticity</td>
<td>0.5</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>production technology</td>
<td>0.75</td>
</tr>
<tr>
<td>$\psi$</td>
<td>production technology</td>
<td>0.33</td>
</tr>
<tr>
<td>$\delta$</td>
<td>depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>repayment rate long-term debt</td>
<td>0.05</td>
</tr>
<tr>
<td>$\tau$</td>
<td>corporate income tax</td>
<td>0.4</td>
</tr>
<tr>
<td>$\rho$</td>
<td>demand elasticity retail goods</td>
<td>10</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>price adjustment cost parameter</td>
<td>90</td>
</tr>
<tr>
<td>$\phi$</td>
<td>capital goods technology</td>
<td>4</td>
</tr>
<tr>
<td>$\varphi_m$</td>
<td>Taylor rule</td>
<td>1.25</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>Taylor rule</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Following Kaplan et al. (2018), we set the elasticity of substitution for retail good varieties $\rho = 10$ implying a steady state markup of 11 percent. The price adjustment cost parameter $\lambda$, the parameter of the capital goods technology $\phi$, and the Taylor rule parameters $\varphi_m$ and $\rho_m$ are taken from Ottonello and Winberry (2020). Externally set parameters are summarized in Table 3.

5.2.2 Internally calibrated parameters

The probability distribution of the firm-specific capital quality shock $\varepsilon$ is assumed to be Normal with zero mean and standard deviation $\sigma_\varepsilon$. Firm-level productivity $z$ follows a productivity ladder with discrete support $\{Z_1, ..., Z_j, ..., Z_J\}$. Entrants start at the lowest productivity level $z_e = Z_1$. Incumbent firms with last period’s productivity level $z = Z_j$ climb up the productivity ladder with probability $1 - \rho_z$:

$$z' = \begin{cases} 
Z_j & \text{with probability } \rho_z \\
Z_{\min\{j+1,J\}} & \text{with probability } 1 - \rho_z
\end{cases} \quad (5.1)$$

Once a firm has reached the highest productivity level $Z_J$, it remains there until it defaults or exits the economy exogenously. The support of the natural logarithm of $z$ is $\pm \sigma_z$.

This productivity process has two desirable features. First, it captures the positive skewness of empirical firm growth. Large negative firm growth is rare in the data. Second, it facilitates the computation of the Markov perfect equilibrium. Negative productivity shocks decrease the value $V(q, b, z, S)$ while the existing stock of debt $b$ remains unchanged. If a shock is sufficiently large, the incentive to pay out firm assets to shareholders at the expense of existing creditors causes the constraint $e \geq \bar{\varepsilon}$ in (3.16) to bind for any value of $\bar{\varepsilon}$. The
Table 4: Internally calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_\epsilon$</td>
<td>0.66</td>
<td>Average firm leverage</td>
<td>34.4%</td>
<td>29.3%</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.90</td>
<td>Average credit spread on long-term debt</td>
<td>3.1%</td>
<td>3.3%</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.0045</td>
<td>Average share of maturing debt</td>
<td>35.5%</td>
<td>33.6%</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.0005</td>
<td>Average annual equity issuance / assets</td>
<td>11.4%</td>
<td>14.7%</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.983</td>
<td>Median of average capital growth (quarterly)</td>
<td>1.0%</td>
<td>1.2%</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.184</td>
<td>Median of s.d. of capital growth (quarterly)</td>
<td>8.3%</td>
<td>9.7%</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.0151</td>
<td>Total exit rate (quarterly)</td>
<td>2.2%</td>
<td>2.3%</td>
</tr>
<tr>
<td>$f$</td>
<td>0.2739</td>
<td>Steady state value of entry $V(0, 0, z^e, S)$</td>
<td>-</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: The data sample is 1995-2017. Firm-level data on leverage, the share of maturing debt, equity issuance, and capital growth rates is from Compustat. Firm-level credit spreads are computed using data from Compustat and FISD. The exit rate is from Ottonello and Winberry (2020).

The productivity ladder described above does not feature large negative jumps in $V(q, b, z, S)$ and thereby avoids this problem. The constraint $e \geq \epsilon$ is not binding in equilibrium.\(^{20}\)

The remaining eight parameters $\sigma_\epsilon, \xi, \eta, \nu, \rho_z, \sigma_z, \kappa, \text{ and } f$ are jointly chosen to match the moments in Table 4. While the model is highly non-linear and all parameters are identified jointly, we provide some intuition for the identification of the model parameters. Average firm leverage depends on the standard deviation of the capital quality shock, $\sigma_\epsilon$, because higher earnings volatility induces firms to reduce leverage in order to contain the risk of default. The average credit spread is directly affected by the default cost $\xi$. The debt issuance cost parameter $\eta$ is pinned down by the average share of debt with remaining maturity of less than one year because higher debt issuance costs make short-term debt less attractive and thereby reduce the equilibrium share of maturing debt. The equity issuance cost parameter $\nu$ targets the average size of annual equity issuance relative to firm assets.\(^{21}\) The parameters $\rho_z$ and $\sigma_z$ determine the mean and the standard deviation of firm-level capital growth rates. The probability of exogenous exit $\kappa$ affects the overall rate of exit (exogenous and endogenous through default). Finally, the fixed cost of operation $f$ is chosen such that the steady state value of entry $V(0, 0, z^e, S)$ is equal to zero.

The model matches the data very well. Averaging across firms and time, the leverage ratio is 35.7% and around one third of total firm debt has remaining maturity of less than a year. Even though $\nu < \eta$, equity issuance is more costly in the model than debt issuance: the average ratio of equity issuance costs to firm output is 0.21% (conditional on positive

\(^{20}\)An alternative would be to use a standard AR(1) firm productivity process. Because in this case the dividend payout constraint would bind for many firms, the particular form of this constraint would significantly affect model results.

\(^{21}\)At quarterly frequency, the empirical distribution of firm-level equity issuance is heavily skewed with rare but large positive spikes. To replicate this kind of pattern, a model with a combination of convex and non-convex equity issuance costs would be needed. We therefore use as empirical target a moving average over the past four firm quarters. Averaging across time (in the data and in the model) reduces the skewness of the resulting distribution of equity issuance and thereby facilitates the comparison between the data and the model.
Table 5: Unconditional distributions

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25</td>
<td>50</td>
</tr>
<tr>
<td><strong>Model</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leverage</td>
<td>31.9</td>
<td>13.4</td>
</tr>
<tr>
<td>Long-term debt share</td>
<td>63.8</td>
<td>56.4</td>
</tr>
<tr>
<td>Credit spread on long-term debt</td>
<td>2.3</td>
<td>1.6</td>
</tr>
<tr>
<td><strong>Data</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leverage</td>
<td>32.8</td>
<td>3.6</td>
</tr>
<tr>
<td>Long-term debt share</td>
<td>66.9</td>
<td>44.8</td>
</tr>
<tr>
<td>Credit spread on long-term debt</td>
<td>2.5</td>
<td>1.1</td>
</tr>
</tbody>
</table>

\[\text{Note: Model moments are computed from the stationary distribution of the model. Source for data moments: Compustat and FISD. See Jungherr and Schott (2020a) for details.}\]

issuance) while the corresponding number is only 0.05% for debt issuance costs. The average annual credit spread on long-term debt is 3.1%. While we do not target default rates, the average quarterly default rate is 0.7% in our model which lies between the value of 0.3% targeted by Gomes et al. (2016) and the business failure rate of 0.8% used in Bernanke et al. (1999).\footnote{Hovakimian et al. (2011) report a quarterly default rate of 1.0% (Moody’s expected default frequency across rated and unrated Compustat firms).}

5.3 Heterogeneous debt maturity in the cross-section

In order to obtain quantitatively realistic investment behavior, our model must generate realistic distributions of debt maturity and default risk at the firm level. Our calibration targeted the average values of leverage, long-term debt shares, and credit spreads. We now show that the model is also consistent with a number of untargeted empirical moments from the conditional and unconditional distributions of leverage, long-term debt shares, and credit spreads.

**Unconditional distributions.** Table 5 shows the unconditional distributions of key financial variables in the model and in the data. For leverage, long-term debt shares, and long-term bond spreads, we calculate the mean and the 25th, 50th, and 75th percentiles across firms. While the means were targeted in the calibration, Table 5 shows that the model also produces a significant amount of dispersion across firms. Although the interquartile ranges are somewhat smaller than in the data, the model generates important features of all three distributions. For example, the median values of leverage and credit spreads lie below their respective means while the opposite is true for the long-term debt share.
Figure 4: Firm variables conditional on size

(a) Leverage (in %)  (b) Long-term debt share (in %)
(c) Credit spread on long-term debt (in %)  (d) Age (in quarters)

Note: Size is lagged total assets. For each variable, median values are shown by size quintile. Data moments are shown together with 95% confidence intervals. See Jungherr and Schott (2020a) for details.

Size. Size is a key dimension of firm heterogeneity. Figure 4 shows how leverage, long-term debt shares, credit spreads, and firm age co-vary with firm size in the data and in the model. Size is measured as lagged total assets. We group firms into size quintiles and compute median values for each variable by size quintile. The data is shown as the light blue bars. The error bands represent 95% confidence intervals. The red bars show the corresponding values in the model.

Figure 4 shows that larger firms have significantly higher leverage and higher long-term debt shares in the data. While the median share of long-term debt is only about 40% for the lowest size quintile, this value rises to 90% for the largest firms. Larger firms pay lower credit spreads and are older than smaller firms.  

Most small firms in Compustat are unrated which means that we cannot assign a credit spread to them. This explains the large confidence interval for the bottom quintile in panel (c) of Figure 4. 

23
5.4 Aggregate response to monetary policy

Figure 5 shows impulse response functions of key aggregate variables following a contractionary monetary policy shock. The shock increases the nominal interest rate. Because prices are sticky, the real rate increases as well. This leads to a decline in aggregate investment and a fall in consumption demand. The lower demand for goods lowers inflation. The price of the production good $p$, the price of capital $Q$, and the real wage $w$ all fall following the shock.

Key financial variables are plotted in the bottom panels of Figure 5. Firms’ default rates increase following the monetary policy shock. Lower prices of production and capital
goods render firms less profitable and reduce shareholder value which makes default more attractive. The increase in default risk drives up credit spreads. The bottom-right panel shows that the spread of short-term debt increases by more than the long-term debt spread. This is because the price of short-term debt $p^S_t$ only depends on default risk at time $t + 1$ whereas the price of long-term debt $p^L_t$ depends on default risk in all future periods. Because the increase in default risk is transitory, the long-term spread responds by less than the short-term spread. In order to contain the increase in default risk and credit spreads, firms reduce leverage and total debt. Short-term debt is more responsive to the monetary policy shock than long-term debt.

5.5 Heterogeneous responses to monetary policy

In Figure 6, we generate the model equivalent of panel a) in Figure 2. To do so, we generate a large panel of firm histories from the quantitative model and then run the regression specified in (2.2) on this model-generated panel data. In Figure 6 we show the coefficient of the interaction term between debt maturity and the monetary policy shock.

The figure shows that firms with a larger share of maturing debt reduce investment by more in reaction to a contractionary monetary policy shock.

6 Conclusion

This paper shows that debt maturity matters for firm-level and aggregate responses to monetary policy. Combining firm and bond-level data, we find that firms with a higher share of
maturing debt have a significantly larger investment response to monetary policy shocks than other firms in the economy. We show that this differential investment response can be explained in a heterogeneous firm New Keynesian model with endogenous debt maturity. The model implies that the effectiveness of monetary policy depends on the maturity distribution of firm debt in the economy.
References


Appendix A  Data Construction

**Bond-level data** We obtain detailed bond-level data from Mergent FISD. The initial sample contains 442,431 bonds. We construct a sample of comparable bonds by dropping the following types of bonds: convertible (5,394), convertible on call (810), exchangeable (39,848), not denoted in US$ (6,160), (yankee) bonds issued by foreign entities (70,158), and bonds that mature less than one year after issuance (60,175). This leaves us with 320,437 bonds. Of these bonds, for most of our analysis we exclude bonds that are callable (210,389) or have a variable coupon (71,484). We then create a monthly panel of bonds, which tracks the outstanding amount, computed as number of bonds issued times principal amount, throughout the life of the bond. Mergent FISD records the date, amount, and reason of reductions in the amount outstanding that occur before maturity, e.g., due to a a call, reorganization, or default. We use those records to adjust the outstanding amount in our bond panel. When the bond matures at its scheduled maturity date, we record the remaining amount of the bond at maturity as maturing amount.

**Linking the bond panel to the firm panel** To match bonds to the creditor firm in every period of the bond’s lifetime, we proceed in three steps. First, we construct a mapping from gvkey to the historical firm cusip. The historical firm cusip is the firm cusip identifier valid in a given time period and gvkey is the Compustat firm identifier. We link the two identifiers by combining the Compustat–CRSP link table, which links gvkey and permno, with CRSP data, which links permno and historical cusip. The Compustat–CRSP link contains start and end dates for which the respective links are valid. We only use links which are classified as reliable, coded “C” or “P” flag in the link table. We join this link table with the CRSP data and keep records that fall within link validity. For some cusip we have a link to more than one gvkey (163,441 of 3,702,116 observations), which may arise due to the presence of subsidiary firms in CRSP. We drop ambiguous links if Compustat contains no sales for a gvkey (113,843 observations). For the remaining ambiguous links we keep the gvkey link to the firm with the largest sales.

Second, we cannot simply match the bond panel to the firm panel by using the historical cusip in both panels. In the bond panel, the historical firm cusip, encoded in the bond cusip, is the firm cusip at the time of bond issuance. In contrast, the firm panel records the historical firm cusip as the one valid in a given period, which may change over time. Reasons for changes in the historical cusip are changes in the firm name or the firm trading symbol. To match firm and bond panel, we use the so-called header firm cusip associated to the bond’s initial historical firm cusip. The header cusip is the latest observed cusip in a firm’s history. The mapping between header cusips and historical cusips over time is provided in CRSP data. We match the header cusip to both the firm and the bond panel. The link between bond and firm panel along the header cusip is ambiguous in a small number of cases (11,350 of 3,817,626 observations). We delete those bonds for which no link to gykey is available in the CompustatCRSP table (7,445 observations) and drop the bonds with remaining ambiguous links (3,904 observations). Given the header cusip of the bond issuer, we can attach the historical cusip series throughout the lifetime of the bond using the same mapping. This identifier correctly reflects bond creditorship conditional on the bond not being sold to another firm intermittently.

Third, we account for M&A events. The Thomson–Reuters SDC database records events at which firms, as identified by historical cusip, are merged or acquired by another firm, also identified by historical cusip. This allows us to change a bond’s firm identifier to the identifier of the acquiring...
firm. We prepare the SDC data as follows. While we take the reported effective date of M&A as the baseline change of bond ownership, we use the announcement date if the effective date is missing. We do not consider M&A events for which no date is reported, the M&A status is not reported as completed, the target firm is classified as a subsidiary, or if the acquiring firm does not buy the target firm fully. If an M&A event is associated to multiple buyers, we drop buyers that do not have associated gykey’s as per the Compustat-CRSP link table and drop remaining events of this sort entirely. With this data at hand, we merge M&A events to the bond panel. For bond-months at which the creditor was subject to an M&A event, we replace the historical firm cusip associated to the bond by the acquiring firm’s cusip from the M&A date going forward. Because the acquiring firm may have changed its cusip after the M&A event, we need to repeat the steps outlined above to find the actual evolution of historical cusip for the new creditor firm. Having done so, we search for additional M&A events that may have happened after the first M&A event, now with the first acquiring firm being the target firm. We repeat this procedure until we find no M&A events that would imply a change in the cusip identifier.

(Merged) firm-level data We use quarterly Compustat data on balance sheets of publicly listed US firms for the period 1995Q1 to 2017Q2. We drop firms in finance, insurance, and real estate (SIC codes 6000-699), public administration (9100-9729), and utilities (4900-4999). We drop firm-quarters with non-positive net plants, property, and equipment (PPE). We drop net PPE for a small number of spikes. These are quarter, in which the real absolute growth rate of PPE exceeds 50% and is followed by a reversal in the opposite direction of more than 50% in the following quarter. We fill one-quarter gaps in net PPE with linear interpolation.

We next match bonds to creditor firms’ balance sheets by using the link between gykey and the correct historical cusip for every quarter in which a bond is outstanding (as described above). We compute the sum of issued bonds, outstanding bonds, maturing bonds, the average volume-weighted maturity of outstanding bonds for each firm-quarter. We compute the maturing bond share as in equation (2.1) and trim it above 100%. Out of a total of 462,173 firm-quarter observations, we observe outstanding bonds for 56,257 , i.e., 12.1%. Firms with bonds are on average much larger than firms without bonds, and they account for 67.0% of total net PPE and 69.1% of total sales. Eventually, we focus on firms with at least 10 years of observations as in Ottonello and Winberry (2020). Excluding firms with less than 10 years of observations, we observe outstanding bonds for 47,971 out of the remaining 298,008 firm-quarter observations, i.e., 16.1%. In the restricted sample, firms with bonds account for 71.6% of total net PPE and 73.8% of total sales.

We construct capital stock series from PPE using a perpetual inventory method. Within firms, we identify spells for which net PPE is observed without gaps. For every spell, we initialize the capital stock with (deflated) gross PPE. For all following quarters within the spell, we define the capital stock as the previous value plus the change in nominal net PPE, deflated by the CPI. If not stated otherwise, all nominal variables are deflated to 2012 US$ by the CPI. Moreover, we use log total assets as a measure of firm size and the log difference in sales to measure sales growth. We also compute the distance to default as in Ottonello and Winberry (2020).
Appendix B  Additional empirical results

Figure 7: Average capital growth response

Note: The shaded areas indicate 95% error bands clustered by firms and quarters.

Figure 8: Interaction with financial constraint

(a) Leverage  (b) Distance to default

Note: The shaded areas indicate 95% error bands clustered by firms and quarters.
Figure 9: Non-linear effects

(a) Effect of $M_{it} > 0$

(b) Effect of $M_{it} \geq 15$

Note: The shaded areas indicate 95% error bands clustered by firms and quarters.
Figure 10: Robustness for alternative denominators

(a) Capital stock

(b) Sales

(c) Total assets

Note: The shaded areas indicate 95% error bands clustered by firms and quarters.
Figure 11: Results without four-quarter-averaging of denominators

Note: The shaded areas indicate 95% error bands clustered by firms and quarters.

Figure 12: Responses by bond characteristics

(a) Callable bonds

(b) Variable-coupon bonds

Note: The shaded areas indicate 95% error bands clustered by firms and quarters.
Figure 13: Alternative samples and monetary policy shocks

(a) Alternative samples

(b) Alternative monetary policy shocks

Note: The shaded areas indicate 95% error bands clustered by firms and quarters.

Figure 14: Heterogeneous responses of further outcome variables

(a) Debt

(b) Sales

Note: The shaded areas indicate 95% error bands clustered by firms and quarters.
Appendix C  Model appendix

In this section of the appendix, we derive the first order conditions from Section 4 (Appendix C.1.1) and provide analytical details on the interaction between monetary policy and debt overhang (Appendix C.1.2). We also identify model counterparts of the empirical moments targeted in Table 4 (Appendix C.2).

C.1 Characterization

In the following, we show that the firm problem (3.16) can be expressed in terms of three choice variables: the scale of production $k$, and the amounts of short-term debt $b^S$ and long-term debt $b^L$.

Equity issuance is given by (3.6):

$$e = Qk - q - \tilde{b}^Sp^S - \left(\tilde{b}^L - \frac{b}{\pi}\right)p^L$$  \hspace{1cm} (A.1)

Applying (A.1) to (3.16) yields as the firm objective:

$$V(q, b, z, S) = q - Qk + \tilde{b}^Sp^S + \left(\tilde{b}^L - \frac{b}{\pi}\right)p^L - G(e) - H\left(\tilde{b}^S, \tilde{b}^L, \frac{b}{\pi}\right)$$

$$+ \mathbb{E} \Lambda \int_{\bar{\varepsilon}}^{\infty} \left[(1 - \kappa) V(q', b', z', S') + \kappa \left(q' - \frac{b'}{\pi}, g(q', b', z', S')\right)\right] \varphi(\varepsilon) d\varepsilon$$  \hspace{1cm} (A.2)

Equity and debt issuance costs are:

$$G(e) = \nu \max\{e, 0\}^2, \text{ and: } H\left(\tilde{b}^S, \tilde{b}^L, \frac{b}{\pi}\right) = \eta\left(\tilde{b}^S + \max\left\{\tilde{b}^L - \frac{b}{\pi}, 0\right\}\right)^2$$  \hspace{1cm} (A.3)

Using (3.14), firm revenue net of labor costs can be written as:

$$py - w = Ak^\alpha, \text{ where:}$$

$$A \equiv (pz)\frac{\zeta(1 - \psi)}{\pi(1 - \psi)} \frac{(1 - \psi)}{w} \left[1 - \zeta(1 - \psi)\right], \text{ and: } \alpha \equiv \frac{\psi\zeta}{1 - \zeta(1 - \psi)}$$  \hspace{1cm} (A.4)

From the definition of $\bar{\varepsilon}$ in (3.10) it follows that $\bar{\varepsilon}$ is a function of $q'$ and $b' = (1 - \gamma)\tilde{b}^L$:

$$\bar{\varepsilon} : (1 - \kappa) \mathbb{E} V(q', b', z', S') + \kappa \left(q' - \frac{b'}{\pi}, \mathbb{E} g(q', b', z', S')\right) = 0,$$  \hspace{1cm} (A.6)

where $q'$ depends on the three choice variables $k$, $b^S$, and $b^L$:

$$q' = Qk - \frac{\tilde{b}^S}{\pi'} - \gamma \frac{\tilde{b}^L}{\pi'} + (1 - \tau) \left[Ak^\alpha + (\varepsilon - \delta)Q^k - f - \frac{c(\tilde{b}^S + \tilde{b}^L)}{\pi'}\right]$$  \hspace{1cm} (A.7)

The short-term bond price depends on $\bar{\varepsilon}$, $k$, $b^S$, and $b^L$:

$$p^S = \mathbb{E} \Lambda \left[1 - \Phi(\bar{\varepsilon})\frac{1 + c}{\pi'} + \frac{(1 - \xi)}{\tilde{b}^L + b^S} \int_{-\infty}^{\bar{\varepsilon}} \left(Q^k + (1 - \tau)(Ak^\alpha + (\varepsilon - \delta)Q^k - f)\right) \varphi(\varepsilon) d\varepsilon\right]$$  \hspace{1cm} (A.8)
Because $\bar{\varepsilon}$ (through (A.6)) and $q'$ (through (A.7)) are pinned down by the firm’s choice of $k$, $\tilde{b}^S$, and $\tilde{b}^L$, the short-term bond price likewise only depends on the three choice variables $k$, $\tilde{b}^S$, and $\tilde{b}^L$. The same reasoning applies to the price of long-term debt:

$$p^L = \mathbb{E} \Lambda \left[ \int_\varepsilon^\infty (1 - \gamma)g(q', b', z', S') (\gamma + c) \right] \varphi(\varepsilon) d\varepsilon$$

$$+ \frac{(1 - \xi)}{\bar{b}^L + \tilde{b}^S} \int_\varepsilon^\infty \left[ Q'k + (1 - \tau)(Ak^\alpha + (\delta - 3)Q'k - f) \right] \varphi(\varepsilon) d\varepsilon$$

It follows that the solution to (3.16) is found by choosing $k$, $\tilde{b}^S$, and $\tilde{b}^L$ to maximize (A.2) subject to the default threshold in (A.6), the definitions of equity issuance in (A.1) and of $q'$ in (A.7), and the bond prices in (A.8) and (A.9).

### C.1.1 First order conditions

An interior solution to (3.16) is characterized by three first order conditions. The three choice variables affect the threshold value $\bar{\varepsilon}$ through (A.6) and (A.7). Let $\mathbb{E} \left[ \Delta \bar{\varepsilon} \right]$ denote the effect of an anticipated marginal increase in $\bar{\varepsilon}$ on the firm’s objective (A.2) for given values of $k$, $\tilde{b}^S$, and $\tilde{b}^L$:

$$\Delta \bar{\varepsilon} \equiv \left[ 1 + \frac{\partial G(\bar{\varepsilon})}{\partial \bar{\varepsilon}} \right] \left[ \frac{b^S \partial p^S}{\partial \bar{\varepsilon}} + \left( \tilde{b}^L - \frac{b}{\pi} \right) \frac{\partial p^L}{\partial \bar{\varepsilon}} \right]$$

where:

$$\frac{\partial p^S}{\partial \bar{\varepsilon}} = \mathbb{E} \Lambda \varphi(\varepsilon) \left[ - \frac{1 + c}{\pi'} + \frac{1 - \xi}{\bar{b}^S + \tilde{b}^L} \left( Q'k + (1 - \tau)(Ak^\alpha + (\delta - 3)Q'k - f) \right) \right]$$

and:

$$\frac{\partial p^L}{\partial \bar{\varepsilon}} = \mathbb{E} \Lambda \varphi(\varepsilon) \left[ - \frac{\gamma + c}{\pi'} + \frac{(1 - \gamma)g(q', b', z', S')}{\pi'} + \frac{1 - \xi}{\bar{b}^S + \tilde{b}^L} \left( Q'k + (1 - \tau)(Ak^\alpha + (\delta - 3)Q'k - f) \right) \right]$$

A higher threshold value $\bar{\varepsilon}$ implies higher default risk. For given values of $k$, $\tilde{b}^S$, and $\tilde{b}^L$, higher default risk unambiguously reduces shareholder value. Higher expected default costs reduce the market price of short-term debt ($\partial p^S/\partial \bar{\varepsilon} < 0$) and long-term debt ($\partial p^L/\partial \bar{\varepsilon} < 0$). This lowers the revenue which the firm raises on the bond market and therefore requires higher equity issuance (or lower dividend payout) for a given stock of capital.

An important variable is shareholder value after the realization of $\varepsilon$:

$$EV \equiv (1 - \kappa) V (q', b', z', S') + \kappa \left( q' - \frac{b'}{\pi} g(q', b', z', S') \right)$$

If future assets $q'$ change, this affects $EV$ according to:

$$\Delta EV \equiv \frac{\partial EV}{\partial q'} = (1 - \kappa) \left[ 1 + \frac{\partial G(q')}{\partial q'} \right] + \kappa \left( 1 - \frac{b'}{\pi} \frac{\partial g(q', b', z', S')}{\partial q'} \right)$$

An additional dollar tomorrow saves the marginal equity issuance cost $\partial G(q')/\partial q'$. This effect is missing in case of exogenous exit. Furthermore, higher firm assets may increase the market value of existing debt which firms need to pay to creditors in case of exogenous exit.
Capital  The firm’s first order condition with respect to capital $k$ is:

$$
\left[ 1 + \frac{\partial G(e)}{\partial e} \right] \left[ -Q + \tilde{b}^S \frac{\partial p^S}{\partial k} + \left( \tilde{b}^L - \frac{b}{\pi} \right) \frac{\partial p^L}{\partial k} \right] + \mathbb{E} \left[ \frac{\partial \bar{\varepsilon}}{\partial k} \Delta \bar{\varepsilon} + \Lambda \int_{\bar{\varepsilon}}^{\infty} \frac{\partial q'}{\partial k} \Delta EV \varphi(\varepsilon) d\varepsilon \right]
$$

$$
= \left[ 1 + \frac{\partial G(e)}{\partial e} \right] \left[ -Q + \left( \tilde{b}^S + \tilde{b}^L - \frac{b}{\pi} \right) \mathbb{E} \Lambda \frac{(1 - \xi)}{\tilde{b}^L + \tilde{b}^S} \int_{-\infty}^{\bar{\varepsilon}} \left[ Q' + (1 - \tau)(A\alpha_{a-1} + (\varepsilon - \delta)Q') \right] \varphi(\varepsilon) d\varepsilon 
$$

$$
+ \left( \tilde{b}^L - \frac{b}{\pi} \right) (1 - \gamma) \mathbb{E} \Lambda \int_{\bar{\varepsilon}}^{\infty} \left[ \frac{\partial q'}{\partial k} \frac{\partial g(q', b', z', S')}{\partial q'} \right] \varphi(\varepsilon) d\varepsilon 
$$

$$
+ \mathbb{E} \left[ \frac{\partial \bar{\varepsilon}}{\partial k} \Delta \bar{\varepsilon} + \Lambda \int_{\bar{\varepsilon}}^{\infty} \frac{\partial q'}{\partial k} \Delta EV \varphi(\varepsilon) d\varepsilon \right] = 0 ,
$$

(A.15)

where: $\frac{\partial q'}{\partial k} = Q' + (1 - \tau)[A\alpha_{a-1} + (\varepsilon - \delta)Q']$, and $\frac{\partial q'}{\partial \bar{\varepsilon}} = (1 - \tau)Q'k$, (A.16)

$$
\frac{\partial \bar{\varepsilon}}{\partial k} = - \frac{\partial q'}{\partial \bar{\varepsilon}} = - \frac{Q' + (1 - \tau)[A\alpha_{a-1} + (\varepsilon - \delta)Q']}{(1 - \tau)Q'k} ,
$$

(A.17)

and: $\frac{\partial G(e)}{\partial e} = 2\nu e = 2\nu \left[ Qk - q - \tilde{b}^S p^S - \left( \tilde{b}^L - \frac{b}{\pi} \right) p^L \right]$, if $e > 0$, and zero otherwise.

(A.18)

Short-term debt  The firm’s first order condition with respect to $\tilde{b}^S$ is:

$$
\left[ 1 + \frac{\partial G(e)}{\partial e} \right] \left[ p^S + \tilde{b}^S \frac{\partial p^S}{\partial \tilde{b}^S} + \left( \tilde{b}^L - \frac{b}{\pi} \right) \frac{\partial p^L}{\partial \tilde{b}^S} \right] - \frac{\partial H(\tilde{b}^S, \tilde{b}^L, b/\pi)}{\partial \tilde{b}^S} + \mathbb{E} \left[ \frac{\partial \bar{\varepsilon}}{\partial \tilde{b}^S} \Delta \bar{\varepsilon} + \Lambda \int_{\bar{\varepsilon}}^{\infty} \frac{\partial q'}{\partial \tilde{b}^S} \Delta EV \varphi(\varepsilon) d\varepsilon \right]
$$

$$
= \left[ 1 + \frac{\partial G(e)}{\partial e} \right] \mathbb{E} \Lambda \left[ \frac{1 + c}{\pi'} [1 - \Phi(\bar{\varepsilon})] + \frac{(1 - \xi)}{\tilde{b}^L + \tilde{b}^S} \int_{-\infty}^{\bar{\varepsilon}} \left[ Q'k + (1 - \tau)(A\alpha_{a} + (\varepsilon - \delta)Q'k - f) \right] \varphi(\varepsilon) d\varepsilon 
$$

$$
- \left( \tilde{b}^S + \tilde{b}^L - \frac{b}{\pi} \right) \frac{1 - \xi}{\tilde{b}^S + \tilde{b}^L} \int_{-\infty}^{\bar{\varepsilon}} \left[ Q'k + (1 - \tau)(A\alpha_{a} + (\varepsilon - \delta)Q'k - f) \right] \varphi(\varepsilon) d\varepsilon 
$$

$$
+ \left( \tilde{b}^L - \frac{b}{\pi} \right) \frac{1 - \gamma}{\pi'} \int_{\bar{\varepsilon}}^{\infty} \left[ \frac{\partial q'}{\partial \tilde{b}^S} \frac{\partial g(q', b', z', S')}{\partial q'} \right] \varphi(\varepsilon) d\varepsilon 
$$

$$
- \frac{\partial H(\tilde{b}^S, \tilde{b}^L, b/\pi)}{\partial \tilde{b}^S} + \mathbb{E} \left[ \frac{\partial \bar{\varepsilon}}{\partial \tilde{b}^S} \Delta \bar{\varepsilon} + \Lambda \int_{\bar{\varepsilon}}^{\infty} \frac{\partial q'}{\partial \tilde{b}^S} \Delta EV \varphi(\varepsilon) d\varepsilon \right] = 0 ,
$$

(A.19)

where: $\frac{\partial H(\tilde{b}^S, \tilde{b}^L, b/\pi)}{\partial \tilde{b}^S} = 2\eta \left( \tilde{b}^S + \max \left\{ \tilde{b}^L - \frac{b}{\pi}, 0 \right\} \right)$, (A.20)

$$
\frac{\partial q'}{\partial \tilde{b}^S} = - \frac{1 + (1 - \tau)e}{\pi'} ,
$$

and: $\frac{\partial \bar{\varepsilon}}{\partial \tilde{b}^S} = - \frac{\partial q'}{\partial \tilde{b}^S} \frac{\partial \Phi(\bar{\varepsilon})}{\partial \tilde{b}^S} = \frac{1 + (1 - \tau)e}{\pi'(1 - \tau)Q'k}$ (A.21)
This first order condition can be re-written as:

\[
\mathbb{E} \Lambda \left\{ [1 - \Phi(\bar{\varepsilon})] \frac{\tau_c}{\pi} + \frac{\partial G(e)}{\partial e} [1 - \Phi(\bar{\varepsilon})] \frac{1 + c}{\pi'} \right. \\
+ \left. \left[ 1 + \frac{\partial G(e)}{\partial e} \right] \frac{(1 - \xi)}{b^L + b^S} \int_{-\infty}^{\bar{\varepsilon}} \left[ Q'k + (1 - \tau)(Ak^\alpha + (\varepsilon - \delta)Q'k - f) \right] \varphi(\varepsilon) d\varepsilon \\
- \left( \frac{b^S + b^L - \frac{\bar{b}}{\pi}}{(b^S + b^L)^2} \right) \int_{-\infty}^{\bar{\varepsilon}} \left[ Q'k + (1 - \tau)(Ak^\alpha + (\varepsilon - \delta)Q'k - f) \right] \varphi(\varepsilon) d\varepsilon \\
+ \left( \frac{b^L - \frac{b}{\pi}}{(b^S + b^L)} \right) \int_{-\infty}^{\bar{\varepsilon}} \left[ \frac{-\kappa}{\pi'} \frac{\partial g(q', b', z', S')}{\partial q'} \right] \varphi(\varepsilon) d\varepsilon \right) \\
- \frac{\partial H(\bar{b}^S, \bar{b}^L, b/\pi)}{\partial b^S} \mathbb{E} \left[ \frac{\partial \bar{\varepsilon}}{\partial b^S} \Delta \bar{\varepsilon} + \Lambda \int_{-\infty}^{\bar{\varepsilon}} \frac{\partial \varphi(e)}{\partial b^S} \frac{\partial g(q', b', z', S')}{\partial q'} \varphi(\varepsilon) d\varepsilon \right] = 0
\]  
(A.22)

**Long-term debt**  The firm’s first order condition with respect to \( \bar{b}^L \) is:

\[
\mathbb{E} \int_{-\infty}^{\bar{\varepsilon}} \left[ (1 - \kappa) \frac{\partial b^L}{\partial b^L} \frac{\partial V(q', b', z', S')}{\partial b'} - \frac{\kappa}{\pi'} \frac{\partial b^L}{\partial b^L} g(q', b', z', S') \\
- \frac{\kappa}{\pi'} b^L \left[ \frac{\partial b^L}{\partial b^L} \frac{\partial g(q', b', z', S')}{\partial b'} + \frac{\partial q'}{\partial b^L} \frac{\partial g(q', b', z', S')}{\partial q'} \right] \right] \varphi(\varepsilon) d\varepsilon
\]

\[
= \left[ 1 + \frac{\partial G(e)}{\partial e} \right] \mathbb{E} \Lambda \left[ \int_{-\infty}^{\bar{\varepsilon}} \varphi(\varepsilon) d\varepsilon \right]
+ \left[ \frac{(1 - \xi)}{b^L + b^S} \int_{-\infty}^{\bar{\varepsilon}} \left[ Q'k + (1 - \tau)(Ak^\alpha + (\varepsilon - \delta)Q'k - f) \right] \varphi(\varepsilon) d\varepsilon \\
- \left( \frac{b^S + b^L - \frac{\bar{b}}{\pi}}{(b^S + b^L)^2} \right) \int_{-\infty}^{\bar{\varepsilon}} \left[ Q'k + (1 - \tau)(Ak^\alpha + (\varepsilon - \delta)Q'k - f) \right] \varphi(\varepsilon) d\varepsilon \\
+ \left( \frac{b^L - \frac{b}{\pi}}{(b^S + b^L)} \right) \int_{-\infty}^{\bar{\varepsilon}} \left[ \frac{-\kappa}{\pi'} \frac{\partial g(q', b', z', S')}{\partial q'} \right] \varphi(\varepsilon) d\varepsilon \\
- \frac{\partial H(\bar{b}^S, \bar{b}^L, b/\pi)}{\partial b^L} \mathbb{E} \left[ \frac{\partial \bar{\varepsilon}}{\partial b^L} \Delta \bar{\varepsilon} + \Lambda \int_{-\infty}^{\bar{\varepsilon}} \frac{\partial \varphi(e)}{\partial b^L} \frac{\partial g(q', b', z', S')}{\partial q'} \varphi(\varepsilon) d\varepsilon \right] \\
+ \mathbb{E} \Lambda \left[ (1 - \kappa) \frac{\partial b^L}{\partial b^L} \frac{\partial V(q', b', z', S')}{\partial b'} - \frac{\kappa}{\pi'} \frac{\partial b^L}{\partial b^L} g(q', b', z', S') \\
- \frac{\kappa}{\pi'} b^L \left[ \frac{\partial b^L}{\partial b^L} \frac{\partial g(q', b', z', S')}{\partial b'} + \frac{\partial q'}{\partial b^L} \frac{\partial g(q', b', z', S')}{\partial q'} \right] \right] \mathbb{E} \varphi(\varepsilon) d\varepsilon = 0
\]  
(A.23)
where: 
\[ \frac{\partial \bar{e}}{\partial \tilde{b}_L} = -\frac{\partial q'}{\partial \tilde{e}} - \frac{(1 - \kappa)}{\pi} \frac{\partial \hat{E}V(q', b', z', S')}{\partial \tilde{b}_L} \frac{\partial \hat{E}g(q', b', z', S')}{\partial \tilde{b}_L} \frac{\partial \hat{E} \Delta EV}{\partial \tilde{b}_L}, \] 

(A.24)

\[ \frac{\partial q'}{\partial b_L} = -\gamma + (1 - \tau) c, \quad \frac{\partial q'}{\partial \tilde{e}} = (1 - \tau) Q', \quad \frac{\partial b'}{\partial \tilde{b}_L} = 1 - \gamma, \] 

(A.25)

and:
\[ \frac{\partial H(\tilde{b}_S, \tilde{b}_L, b/\pi)}{\partial \tilde{b}_L} = \frac{2}{\eta} \left( \tilde{b}_S + \tilde{b}_L - \frac{b}{\pi} \right), \text{if } \tilde{b}_L - \frac{b}{\pi} > 0, \text{and zero otherwise.} \] 

(A.26)

We derive the effect of a marginal increase in \( b' \) on \( V(q', b', z', S') \). From (A.2) it follows:
\[ \frac{\partial V(q, b, z, S)}{\partial b} = -\frac{1}{\pi} \frac{\partial H(\tilde{b}_S, \tilde{b}_L, b/\pi)}{\partial \tilde{b}_L} - \frac{p}{\pi} \left[ 1 + \frac{\partial G(e)}{\partial \tilde{e}} \right] \] 

(A.27)

This implies:
\[ \frac{\partial V(q', b', z', S')}{\partial b'} = -\frac{1}{\pi} \left( \frac{\partial H(\tilde{b}_S, \tilde{b}_L, b/\pi)}{\partial \tilde{b}_L} + g(q', b', z', S') \left[ 1 + \frac{\partial G(e')}{\partial \tilde{e'}} \right] \right), \] 

(A.28)

where:
\[ \frac{\partial H(\tilde{b}_S, \tilde{b}_L, b/\pi)}{\partial \tilde{b}_L} = -2\eta \left( \tilde{b}_S + \tilde{b}_L - \frac{b}{\pi} \right), \text{if } \tilde{b}_L - \frac{b}{\pi} > 0, \text{and zero otherwise.} \] 

(A.29)

\[ \frac{\partial G(e')}{\partial \tilde{e'}} = 2 \nu e' = 2 \nu \left[ Q' k' - q' - \tilde{b}_S p - \left( \tilde{b}_L - \frac{b}{\pi} \right) g(q', b', z', S') \right], \text{if } e' > 0, \text{and zero otherwise.} \] 

(A.30)
Special case  In the main text, we consider the special case that $\xi = 1$ and $\kappa = 0$. Using these assumptions, the first order condition with respect to $\tilde{b}^L$ (A.23) can be re-written as:

$$\mathbb{E} \Lambda \left\{ \frac{\partial G(e)}{\partial e} - \frac{\partial G(e)}{\partial e} \right\} + \left( \tilde{b}^L - \frac{b}{\pi} \right) \frac{1 - \gamma}{\pi'} \int_{\tilde{e}}^{\infty} \left[ \frac{\partial b'}{\partial b^L} \frac{\partial V(q', b', b, z, S')}{\partial b'} + \frac{\partial q'}{\partial b^L} \frac{\partial g(q', b', z', S')}{\partial q'} \right] \varphi(\varepsilon) d\varepsilon
$$

$$- \frac{\partial H(\tilde{b}^S, \tilde{b}^L, b/\pi)}{\partial b^L} + \mathbb{E} \left[ \Lambda \int_{\tilde{e}}^{\infty} \left[ \frac{\partial b'}{\partial b^L} \frac{\partial V(q', b', b, z, S')}{\partial b'} - \frac{\partial q'}{\partial b^L} \frac{\partial g(q', b', z', S')}{\partial q'} \right] \varphi(\varepsilon) d\varepsilon \right]
$$

$$- \left[ 1 + \frac{\partial G(e)}{\partial e} \right] \mathbb{E} \varphi(\tilde{e}) \left[ \frac{\gamma + (1 - \gamma) c}{\pi'} - \frac{\gamma + (1 - \gamma) c}{\pi'} \right]
$$

$$= \mathbb{E} \Lambda \left\{ \frac{\partial G(e)}{\partial e} - \frac{\partial G(e)}{\partial e} \right\} + \left( \tilde{b}^L - \frac{b}{\pi} \right) \frac{1 - \gamma}{\pi'} \int_{\tilde{e}}^{\infty} \left[ \frac{\partial b'}{\partial b^L} \frac{\partial V(q', b', b, z, S')}{\partial b'} + \frac{\partial q'}{\partial b^L} \frac{\partial g(q', b', z', S')}{\partial q'} \right] \varphi(\varepsilon) d\varepsilon
$$

$$- \frac{\partial H(\tilde{b}^S, \tilde{b}^L, b/\pi)}{\partial b^L} + \mathbb{E} \left[ \Lambda \int_{\tilde{e}}^{\infty} \left[ \frac{\partial b'}{\partial b^L} \frac{\partial V(q', b', b, z, S')}{\partial b'} - \frac{\partial q'}{\partial b^L} \frac{\partial g(q', b', z', S')}{\partial q'} \right] \varphi(\varepsilon) d\varepsilon \right]
$$

$$- \left[ 1 + \frac{\partial G(e)}{\partial e} \right] \mathbb{E} \varphi(\tilde{e}) \left[ \frac{\gamma + (1 - \gamma) c}{\pi'} - \frac{\gamma + (1 - \gamma) c}{\pi'} \right] = 0 \quad (A.31)
$$

In the last term, $q'$ is evaluated at $\varepsilon = \tilde{e}$. Combining the two first order conditions for short-term debt (A.22) and long-term debt (A.31) yields a condition for firms’ optimal maturity choice:

$$\left[ 1 + \frac{\partial G(e)}{\partial e} \right] \mathbb{E} \Lambda \left\{ 1 - \gamma \right\} \int_{\tilde{e}}^{\infty} \left[ \frac{\partial b'}{\partial b^L} \frac{\partial V(q', b', b, z, S')}{\partial b'} + \left( \frac{\partial q'}{\partial b^L} - \frac{\partial q'}{\partial b^S} \right) \frac{\partial g(q', b', z', S')}{\partial q'} \right] \varphi(\varepsilon) d\varepsilon
$$

$$+ \mathbb{E} \left[ \frac{\partial \tilde{e}}{\partial b^L} - \frac{\partial \tilde{e}}{\partial b^S} \right] \Delta \tilde{e} + \mathbb{E} \Lambda \int_{\tilde{e}}^{\infty} \frac{\partial b'}{\partial b^L} \frac{\partial V(q', b', b, z, S')}{\partial b'} \varphi(\varepsilon) d\varepsilon
$$

$$- \mathbb{E} \Lambda \frac{\gamma - 1}{\pi'} \int_{\tilde{e}}^{\infty} \left[ \frac{\partial G(e)}{\partial e} - \frac{\partial G(e)}{\partial e} \right] \varphi(\varepsilon) d\varepsilon = 0 \quad (A.32)
This implies:

\[
\left[ 1 + \frac{\partial G(e)}{\partial e} \right] \mathbb{E} \Lambda \left( \frac{1 - \gamma}{\pi'} \right) \int_{\xi}^{\infty} \left[ g(q', b', z', S') \right. \\
+ \left( \hat{b}L - \frac{b}{\pi} \right) \left. \left[ \frac{\partial b'}{\partial b} \frac{\partial g(q', b', z', S')}{\partial b'} + \left( \frac{\partial q'}{\partial b} - \frac{\partial q}{\partial b} \right) \frac{\partial g(q', b', z', S')}{\partial q} \right] \right] \varphi(\varepsilon) d\varepsilon \\
+ \left[ 1 + \frac{\partial G(e)}{\partial e} \right] \mathbb{E} \left[ \frac{\gamma - 1}{\pi'(1 - \tau)Q'k} \int_{\xi}^{\infty} \left[ \frac{\partial g(q', b', z', S')}{\partial e} \right] \right] \varphi(\varepsilon) d\varepsilon
\]

Using (A.28), the optimality condition for firms’ maturity choice (A.33) becomes:

\[
\mathbb{E} \Lambda \int_{\xi}^{\infty} \left[ - \frac{\partial H(\hat{b}L', \hat{b}', b'/\pi')} {\partial e} \right] \\
+ \left[ 1 + \frac{\partial G(e)}{\partial e} \right] \left( \hat{b}L - \frac{b}{\pi} \right) \left[ (1 - \gamma) \frac{\partial g(q', b', z', S')}{\partial b'} + \frac{1 - \gamma}{\pi'} \frac{\partial g(q', b', z', S')}{\partial q} \right] \varphi(\varepsilon) d\varepsilon \\
+ \mathbb{E} \Lambda \int_{\xi}^{\infty} \left[ 1 - g(q', b', z', S') \right] \left[ \frac{\partial g(e')}{\partial e'} - \frac{\partial g(e)}{\partial e} \right] \varphi(\varepsilon) d\varepsilon \\
- \mathbb{E} \frac{1}{(1 - \tau)Q'k} \Delta \varepsilon \frac{\hat{E} \left[ 1 - g(q', b', z', S') \right] + \frac{\partial g(e)}{\partial e}} {\hat{E} \left[ 1 + \frac{\partial g(e)}{\partial e} \right]} = 0
\]

C.1.2 Debt overhang and the investment effect of monetary policy

In the analysis of the debt overhang channel in Section 4.2 of the main text, we derive how an increase of \( Q'k \) affects the reduction of the firm’s cost \( C^S \) induced by the stock of existing debt \( b \) (4.17). This is done holding the future price of long-term debt \( g(q', b', z', S') \) constant. This is only correct if future assets \( q' \) are sufficiently high (because firm behavior is independent of \( q' \) above a certain threshold of assets). The generalized version of (4.17) reads as:

\[
\frac{\partial^2 C^S}{\partial b \partial Q'k} = \mathbb{E} \frac{1 + (1 - \tau)c}{\pi'(1 - \tau)Q^2k^2} \left( \frac{\partial g(q', b', z', S')}{\partial \varepsilon} \right) \Gamma + \frac{1 - \gamma}{\pi'} \frac{\partial g(q', b', z', S')}{\partial q} \\
- \mathbb{E} \frac{1 + (1 - \tau)c}{\pi'(1 - \tau)Q^2k} \left( \frac{\partial g(q', b', z', S')}{\partial e} \right) \frac{1 - \gamma}{\pi'} \frac{\partial q}{\partial Q'k} \frac{\partial g(q', b', z', S')}{\partial q}
\]

For our purposes, the main object of interest is the derivative of future assets \( q' \) with respect to \( Q'k \) conditional on \( \varepsilon = \bar{\varepsilon} \). Since \( \partial g(q', b', z', S')/\partial q' \geq 0 \), equation (A.35) has the same sign as (4.17) if \( \partial q'/\partial Q'k < 0 \) (conditional on \( \varepsilon = \bar{\varepsilon} \)). In equilibrium, this is likely to be the case since we know from the firm’s first order condition with respect to capital (A.15) that capital \( k \) is chosen such that \( \partial q'/\partial k \) is ‘close to \( Q' \) conditional on \( \varepsilon > \bar{\varepsilon} \).

C.2 Model moments

In this section, we provide details on the model counterparts of the empirical moments targeted in Table 4. The total amount of nominal firm debt \( D \) is the present value of future debt payments
discounted at the quarterly nominal riskless interest rate $r_{\text{nom}}$:

$$D = \frac{1 + c}{1 + r_{\text{nom}}} \tilde{B}^S + \frac{\gamma + c}{1 + r_{\text{nom}}} \tilde{B}^L + (1 - \gamma) \frac{\gamma + c}{(1 + r_{\text{nom}})^2} \tilde{B}^L + (1 - \gamma)^2 \frac{\gamma + c}{(1 + r_{\text{nom}})^3} \tilde{B}^L + ...$$

$$= \frac{1 + c}{1 + r_{\text{nom}}} \tilde{B}^S + \frac{\gamma + c}{1 + r_{\text{nom}}} \tilde{B}^L \sum_{j=0}^{\infty} \left( \frac{1 - \gamma}{1 + r_{\text{nom}}} \right)^j = \frac{1 + c}{1 + r_{\text{nom}}} \tilde{B}^S + \frac{\gamma + c}{\gamma + r_{\text{nom}}} \tilde{B}^L$$  \hspace*{1cm} (A.36)

Firm leverage is total debt over total assets: $D/k$. The share of maturing debt of a given firm is the present value of debt payments due (weakly) less than four quarters from today divided by the total amount of firm debt $D$:

$$\frac{1 + c}{1 + r_{\text{nom}}} \frac{\tilde{B}^S}{D} + \frac{\gamma + c}{1 + r_{\text{nom}}} \frac{\tilde{B}^L}{D} + \frac{(1 - \gamma)(\gamma + c)}{(1 + r_{\text{nom}})^2} \frac{\tilde{B}^L}{D} + \frac{(1 - \gamma)^2(\gamma + c)}{(1 + r_{\text{nom}})^3} \frac{\tilde{B}^L}{D} + \frac{(1 - \gamma)^3(\gamma + c)}{(1 + r_{\text{nom}})^4} \frac{\tilde{B}^L}{D}$$

(A.37)

The Macaulay duration is the weighted average term to maturity of the cash flows from a riskless nominal bond divided by the price:

$$\mu = \frac{1}{P_r} \sum_{j=1}^{\infty} j(1 - \gamma)^{j-1} \frac{c + \gamma}{(1 + r_{\text{nom}})^j} + \frac{c + \gamma}{P_r^L} \frac{1 + r_{\text{nom}}}{(\gamma + r_{\text{nom}})^2}$$ \hspace*{1cm} (A.38)

where $P_r^L$ is the price of a riskless nominal long-term bond:

$$P_r^L = \sum_{j=1}^{\infty} (1 - \gamma)^{j-1} \frac{c + \gamma}{(1 + r_{\text{nom}})^j} = \frac{c + \gamma}{r_{\text{nom}} + \gamma}$$ \hspace*{1cm} (A.39)

It follows for the Macaulay duration:

$$\mu = \frac{1 + r_{\text{nom}}}{\gamma + r_{\text{nom}}}$$ \hspace*{1cm} (A.40)

The short-term spread compares the annual gross return (in the absence of default) from buying a nominal short-term bond with the annualized quarterly nominal riskless rate $r_{\text{nom}}$:

$$\left( \frac{1 + c}{p^S} \right)^4 - (1 + r_{\text{nom}})^4$$ \hspace*{1cm} (A.41)

The long-term spread compares the annual gross return (in the absence of default and assuming $p^L$ is constant) from buying a long-term bond with the annualized quarterly riskless rate:

$$\left( \frac{\gamma + c + (1 - \gamma)p^L}{p^L} \right)^4 - (1 + r_{\text{nom}})^4 = \left( \frac{\gamma + c}{p^L} + 1 - \gamma \right)^4 - (1 + r_{\text{nom}})^4$$ \hspace*{1cm} (A.42)

Firm-level capital growth is measured as:

$$\ln(k_t) - \ln(k_{t-1})$$ \hspace*{1cm} (A.43)

Given the aggregate states $S$ and $S'$, the total exit rate per quarter (exogenous and endogenous through default) is

$$\int_{0}^{\infty} \int_{0}^{\infty} \int_{-\infty}^{\infty} \Phi(\bar{v}(q, b, z, S, S')) \mu(q, b, z) dq db dz + \kappa$$ \hspace*{1cm} (A.44)