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THE EMPIRICAL RELEVANCE OF
GOODWIN’S CYCLE MODEL FOR THE US ECONOMY
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Abstract:

The paper attempts to verify Richard Goodwin's (1967) endogenous business cycle theory which states that the driving forces behind fluctuations are class struggles between capitalists and workers about income distribution. Based on a Marxian profit-led model non-linear differential equations lead to endogenous cycles in the wage-share-employment-space which can be observed empirically. Applying a bivariate vector autoregressive model we analyze the relationship between real unit labor costs and the employment rate for the US economy over a period from 1948:1 to 2006:4. Granger-causality tests, orthogonalized impulse-response functions and forecast error variance decomposition are conducted for the raw data as well as the cyclical components of the Hodrick-Prescott and Baxter-King filter methods. We verify the profit-led character of the US goods market and find that income distribution is driven by labor market dynamics.
1 Introduction

In this paper we try to verify Goodwin’s (1967) baseline business cycle model empirically for the USA economy. It is an attempt – built on previous studies – to ask for the actual relevance of class struggle business cycle models. Even though the model is more than 40 years old, only few empirical studies exist and even fewer which apply modern econometric instruments. Here we want to test the central hypothesis of the model: there exists non-linear relationship between the employment rate and functional income distribution which causes fluctuations in output, the profit rate and accumulation. The interaction between the profit share and employment rate is seen as the dominant factor which drives the cycle endogenously. If this is the case, one should be able to verify it empirically. In order to test the hypotheses, the econometric analysis of the cyclical components of the wage share and employment rate is the center piece of this work. Using a vector autoregressive model (VAR) and two different filter techniques, we estimate a bivariate system containing real unit labor costs and the employment rate based on quarterly data from 1948:1 to 2006:4 for the USA. The dynamics and propagation mechanisms are analyzed by impulse-response functions and variance-decomposition technique for different identification schemes. The analysis focuses only on the USA for the following reasons: 1. The US economy is the most advanced capitalist economy and a reference model of liberal character, 2. For the US economy some studies already exist which simplifies the comparison with our results and 3. The data availability and quality is comparatively good.

In the next chapters we briefly describe the model and give an overview about the existent literature. After this the econometric approach is presented before the estimation results are interpreted.

2 The Goodwin Model

The Goodwin (1967) model is a Marxian inspired one and puts the struggle over income distribution at the center. Thus, the model attempts to analyze whether the circumstances on the labor mar-

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1 I would like to thank Marcel Garz for comments on an earlier version in German, Antonio Rodriguez Gil and Christian Schoder for valuable comments. Of course, I am responsible for all remaining errors. I would be very thankful for any comments on this paper.
ket send business cycle relevant impulses or not. It is not about the issues of functional income distribution and its determinants but rather the repercussions of class struggles via the labor market as a disciplinary institution on the profit rate and hence the cyclical fluctuations of the total economy.

It is claimed that a certain stylized fact between the wage share and employment rate exists which is nowadays known as the – even if modified in this model – Phillips curve relation. This relationship is central since it is assumed that it reflects the balance of power between capitalists and workers.

We should add that Goodwin does not claim to present a complete model. Rather it is a

...starkly schematized and hence quite unrealistic model of cycles in growth rates. (Goodwin 1967, 54).

and has to be seen as an idea worth thinking about.

**The Formal Derivation**

The Goodwin model assumes a closed economy without any government activity. The model is a deterministic one with dynamic properties. Only two production factors exist: labor and capital which produce only one good which can be used for consumption or investment. There is no idle capacity and there is no lack of demand and hence the goods market is continuously cleared. All savings are used as investments. Savings are the prerequisite for investments. There are no savings out of wage income but only out of profits. All variables are in real terms since prices are assumed as given.

Technical progress is exogenous and Harrod-neutral which means that the capital intensity is continuously increasing but the capital coefficient stays constant. Technical progress is thus labor saving.

Labor productivity, $y$, grows at a constant rate $\alpha$:

$$\frac{Y}{L} = y = y_0 e^{\alpha t}$$

(1)

where $Y$ denotes total output and $L$ the number of workers employed.
The labor supply, $\eta$, grows at rate $\beta$:
\[ \eta = \eta_0 e^{\beta t} . \]

The employment rate is defined as $\nu = \frac{L}{\eta}$. Goodwin assumes that the real wage, $\omega$, grows the faster the higher the employment rate, $\nu$. The worker’s bargaining power increases linearly with the employment rate. This function can be interpreted as a real wage Phillips curve relation:
\[ D \ln(\omega) = -\gamma + \rho \nu \quad \text{mit } \gamma, \nu > 0 . \]

The share of the total wages relative to total output is given by $u$:
\[ u = \frac{\omega L}{Y} = \frac{\omega}{y} \]
which equals real unit labor costs.

If equation (4) is rewritten in growth rates and the change in real wages is substituted by (3) and labor productivity by equation (1) then we obtain a dynamic function for the growth rate of the wage share:
\[ \frac{\dot{u}}{u} = D \ln u = -(\gamma + \alpha) + \rho \nu . \]

If the employment rate increases faster than labor productivity, this has negative implications on the profit share under the assumed bargaining relations in (3) and (4). The situation on the labor market thus affects immediately the income distribution between capitalists and workers.

In equilibrium it holds that \textit{profits}=\textit{savings}=\textit{investments}:
\[ S = Y - \omega L = (1 - \frac{\omega L}{Y}) Y = (1 - u) Y . \]

Savings are equal to profits since we assume that only capitalists save and all savings are immediately invested which, since we assume away depreciation, implies changes in the capital stock, $K$:

---

2 Actually, this implies a non-linear relation but for the sake of simplicity a linear function is assumed.
3 $D$ denotes the change in time (difference operator).
4 The 'point' denotes changes in the respective variable.
\[
S = I = K .
\]

In order to obtain the accumulation function we divide equation (6) through the capital stock:

\[
D \ln K = \frac{\dot{K}}{K} = \frac{(1-u)Y}{k} - \frac{1-u}{k} = r .
\]

where \( k = \frac{K}{Y} \) denotes the capital-coefficient (or capital-to-output ratio). Since we assume that \( k \) is constant over time, the capital stock increases as fast as output does.

The term \( \frac{1-u}{k} \) describes the profit rate, \( r \). In this system the savings rate, accumulation rate and profit rate are equal in equilibrium:

\[
D \ln K = D \ln Y = \frac{1-u}{k} = r .
\]

The inverse function of (1) determines the growth rate of labor demand, \( L \):

\[
D \ln L = D \ln Y - \alpha = \frac{1-u}{k} - \alpha .
\]

The growth rate of \( L \) is only positive if output grows faster than labor productivity or if the profit rate \( \frac{1-u}{k} = r \) is higher than technological progress \( \alpha \).

The growth rate of the employment rate is given by \( D \ln \nu = D \ln L - D \ln \eta \). Because labor supply grows at rate \( \beta \) and if we substitute \( D \ln L \) by (10) we get:

\[
\frac{\dot{\nu}}{\nu} = D \ln \nu = \frac{1-u}{k} - (\alpha + \beta) .
\]

From equations (5) and (11) one can derive a differential equation system of the following form:

\[
\dot{u} = [-(y+\alpha) + \rho \nu] u
\]

\[
\dot{\nu} = [(\frac{1}{k} - (y+\alpha)) - \frac{1}{k} u] \nu .
\]

Both equations (12 and 13) are similar to those of Lotka (1956; 1925) and Volterra (1927;
who developed a so-called Predator-Prey model in which two populations exist, whereby one of
them is the food source of the other one. These populations are rivals but they also live in symbiosis. In Goodwin’s model the workers are the predators and the capitalists are the preys (Solow 1990, 36).

This system represents a central characteristic of capitalist economies for Goodwin:

> It has long seemed to me that Volterra’s problem of the symbiosis of two populations – partly complementary, partly hostile – is helpful in the understanding of the dynamical contradictions of capitalism, especially when stated in a more or less Marxian form (Goodwin 1967, 55).

The mathematical statements (12) and (13) will be the central equations for the econometric analysis.

3 A Literature Review

The results concerning the empirical studies of the Goodwin model are not unambiguous, as Mohun/Veneziani (2006) state. Also, there is no unique methodology of how to test the theoretical hypotheses empirically.

Mattfeldt (1999) analyses the total US economy. He uses annual data from the German Sachverständigenrat which cover a period from 1960 to 1994. The wage share is defined as the employment-adjusted wage share. He finds indication that the US economy – which is one with flexible labor market relations – follows Goodwin’s center model (Mattfeldt 1999, 163). A cross-spectrum analysis verifies the predicted lag structures of the baseline model: Changes in the wage share follow changes in the employment rate pro-cyclically which corresponds to the characteristics of predator-prey models. The analysis of the individual wage share components shows the relative importance of employment growth for the ‘path’ of the wage share in the USA. The calculation of the employment-rate-elasticity-of-wage-share\(^5\) yields mostly a negative sign which implies a kind of profit-led goods market which is in line with Goodwin’s argumentation.

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\(^{5}\) The elasticity is calculated as the growth rate of the employment rate in relation to the growth rate of the wage share of the previous year in order to consider the lag structures between the variables adequately.
Goldstein (1999) uses quarterly data for his research. He takes the unemployment rate (civilian unemployment rate) instead of the employment rate. The profit share is given as the quotient of before-tax profits with inventory valuation and capital consumption adjustments to national income (Goldstein 1999, 147). He estimates a bivariate VAR(1) system including the unemployment rate and the profit share. Besides the total sample from 1949:1 to 1995:4 he also estimates the following sub periods: 1949:1-1970:4, 1970:1-1985:4 and 1985:1-1995:4, whereas it remains unclear how this is justified. He finds, with the exception of the last sub sample period, strong indication for the profit-squeeze hypothesis which also underlies the Goodwin model: a high employment rate leads to a relative decrease of the profit share and profit rate, respectively. He does not find a significant relationship between the unemployment rate and the profit share for the period after 1985 (Goldstein 1999, 165). In an extended version Goldstein estimates a VAR(1) system with the unemployment rate, profit share and the logarithm of real investments (non-residential). For the periods between 1949:1-1970:4 and 1970:1-1985:1 he can still verify the finding of a profit-squeeze moment. For the period after 1985 there seems to be no significant relationship any more.

Harvie (2000) published a widely cited paper which is often used as a reference article for the econometric testing of the Goodwin model. His estimations are based on annual OECD data from 1959-1994. The wage share is defined as the fraction of the sum of wages (compensation of employees) to the sum of wages plus profit income (operating surplus). The employment rate is given by the quotient of total employment to total labor force. Real GDP per employee defines labor productivity. The capital stock of the total economy is considered. A scatter plot between the employment rate and wage share shows clear Goodwin cycles for the USA. However, Harvie considers the raw data and not any trend adjusted components what is to criticize given the short-run business cycle character of the underlying model. He estimates a (within a single equation framework) labor productivity, employment rate (with a deterministic linear trend) and real wage Phillips curve which depends on the employment rate and a one-period lagged real wage component. Harvie comes to the conclusion that the baseline model is not able to forecast the Goodwin trajectories for the USA as well as nine other economies adequately:

The fact that the discrepancies between \( u^* \) and \( u \) (the mean-A.T.) are systematic, except

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6 The number in brackets denotes the number of used lags.
7 Sometimes Goldstein refers to structural breaks (Goldstein 1999, 147 and 149).
for the case of employment rate in Germany, suggests that the model, despite its qualitative similarities to the empirical trajectories, is inadequate at the quantitative level. Given the skeletal nature of the theoretical model here being tested, this is hardly surprising. (Harvie 2000, 363).

Flaschel et al. (2005) estimate an augmented Goodwin model for the long-run (\(\geq 40\) years) using quarterly data (1955:1-2004:4) for the USA. On the basis of a price and nominal wage Phillips curve and a type of interest rate reaction function (modified Taylor rule) they verify Goodwin’s hypotheses. Functional income distribution is determined by the dynamics on the labor market and the goods market follows a classical profit-led regime:

\[
\text{In the estimated situation the labor market dominates the law of motion of the wage share (which is therefore labor market led) and there is a negative impact effect of the wage share on the goods market dynamics (which are therefore profit led, as in the simple Goodwin model of the growth cycle (Flaschel et al. 2005, 76).}
\]

Mohun and Veneziani (2006) offer a detailed discussion about the correct definition of the distribution variable for empirical studies of the Goodwin model. They plead for an analysis only of the private sector since most of public sector’s products are not considered for sale and its planning does not follow profit-oriented aspects. They limit their analysis on the private sector. Mohun and Veneziani analyze trend and cyclical components of the profit share, profit rate and capital productivity applying the Hodrick-Prescott Filter (HP-Filter) for annual data from 1948-2002. They identify a structural break in the trend relationship between the wage share and the employment rate. The authors also find systematic cyclical patterns. However, the position and length of the cycles differ historically:

\[
\text{All of the cycles are clockwise in direction, as the underlying causal argument would predict. But each cycle is different in position, amplitude and duration, so that the economic relationships generating detrended cycles do so in a way that is both systemic.}
\]

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8 Additionally, Harvie tests an extended version proposed by Desai (1984) and comes to the result that the model’s baseline assumptions of a constant capital-to-output ratio, perfect foresight of the workers and the non-consideration of price dynamics are statistically not tenable.
(cycles exist) and historically contingent (no two cycles are the same) (Mohun & Veneziani 2006, 15).

Unfortunately, no econometric methods are applied (except the filtering technique). Instead, they interpret the phase diagrams and find strong support for a short-run cyclical relationship between income distribution and the employment rate. The long-run relationship (between the trend components) is not clear cut. Dependent on the used data set only weak indication exists for Goodwin cycles (Mohun & Veneziani 2006, 24).

Barbosa-Filho and Taylor (2006) consider a model based on Kalecki, Steindl and Goodwin. Its dynamics imply a clockwise orbit-like relationship between the degree of capacity utilization and wage share. This idea is closely linked to Goodwin’s baseline model. Their empirical study leads to the conclusion that the US economy is profit-led since the slope of the orbit within the wage-share-capacity space is negative, as described in figure 1. The authors use quarterly data from 1948:1 to 2002:4. The distributional variable is obtained only for the private sector. They argue that this time series is stationary and because supplemental incomes and income from public employment are not considered there is no trend in the data. Also, no price/quantity data are available for the non-private sector or they are not of the demanded quality (Barbosa-Filho & Taylor 2006, 400). The wage share is defined as an index (1992=100), taken from the Bureau of Labor Statistics, and is calculated by the nominal hourly wage deflated by the price level of the private sector divided by output per hour. This definition equals the real unit labor costs on hourly basis. The capacity utilization is obtained by filtering the real GDP (source: US Bureau of Economic Analysis) and taking the cyclical component of the HP-Filter (lambda = 1600). Two VAR(2) systems are estimated. First, a demand system is analyzed which considers the interaction between the wage share and the demand components (in real terms) of consumption, investment, net exports and government expenditures. Second, a distribution system is estimated which looks at the effects of the capacity utilization on the wage share. The regression results lead to the insight that an increase of the wage share has negative impact on the utilization rate – also here we find hints that the US economy follows a profit-led de-

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9 It remains unclear to me whether Barbosa-Filho/Taylor use transformed data. Stockhammer/Stehrer (2009, 22) argue that they regress the cyclical components of the HP-Filter on each other: „The effects for individual components of demand are decomposed from the aggregate results (rather than estimated as behavioral equations). They use quarterly data and use the cyclical component of the HP filter.“ I did not find any hints in the text.
mand regime. Furthermore, the wage share reacts positively to a capacity utilization shock what supports Goodwin’s profit-squeeze hypothesis (Barbosa-Filho & Taylor 2006, 408).

Stockhammer and Stehrer (2009) contrast Goodwin’s (1967) model with the Bhaduri/Marglin (1990) model and analyze their demand functions. While the Goodwin model proposes a profit-led accumulation regime, current Kaleckian models are open regarding the accumulation regime: under certain parameter constellations on the goods market both profit- as well as wage-led regimes are possible. Both approaches underlies that higher unit labor costs affect investments negatively. But the Kaleckian Bhaduri/Marglin model also considers the capacity effect of higher consumption demand on investments and thus makes a wage-led regime theoretically possible if the capacity effect more than compensates the cost effect. Different assumptions are taken regarding the relative size of each effect. Stockhammer/Stehrer only look on the demand function but not on the distributional sphere. The behavioral relations are estimated within a single equation approach – interactions between the functions are thus not considered. Dynamic difference models – only if possible error correction models – are considered. A special focus lies on the lag structure. The authors test the sensitivity of the results for different time lag specifications. Quarterly OECD data from 1970:1 to 2007:2 are used for the USA and 11 other countries. A Granger-causality test between the real wage, investments and consumption shows that the real wage (taken as a proxy for income distribution) is statistically rather determined by the expenditure variables. For the USA no indications for a profit-led economy are found. It can be criticized that only the demand side is taken into account whereas the interaction, which is so crucial for both underlying models, between the distributional and demand sphere is not considered. This puts a one-sided constraint on the analysis and makes the proper interpretation of the results more difficult.

All in all, the different results confirm that the US economy experiences profit-led characteristics on

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10 Stockhammer/Stehrer (2009) criticize the used methods by Barbosa-Filho/Taylor for three reasons: 1. The distributional effects are quiet small and are exaggerated by the accelerator mechanism, 2. The effects of the wage share on the demand components show different signs for different lag structures what they interpret as a misspecification of the model, and 3. The distributional effect on consumption is quite high and negative. Theoretically, a positive effect is expected. Stockhammer/Stehrer tried to replicate their results on the basis of quarterly OECD data. This replication shows that A) The regression on the cyclical components is accompanied by autocorrelation problems which bias the coefficients, B) Their results react sensible to different lag structures and C) They find hints that a VAR in differences is a more adequate specification (2009, 22pp.).

11 Whether the comparison of the models is adequate can be discussed. The underlying intention of the (fix price) Bhaduri-Marglin model is to describe growth while the Goodwin model focuses on the short- to medium term perspective.
the goods markets and that the income distribution is determined by the employment rate. Both observations confirm Goodwin’s hypotheses. Despite the different methods used, the obtained results are similar, what indicates certain robustness. Nonetheless, we want to consider a further method in order to test the hypotheses and to make robust conclusions about the relevance of the baseline Goodwin model using time series econometrics.

4 Data

The data selection is based on the work done by Flaschel et al. (2005). For the USA long time series with high frequencies (quarterly) are available. All series are provided by the Federal Reserve Economic Data database of the Federal Reserve Bank of St. Louis.\(^\text{12}\)

Except for the unemployment rate and the number of the working population all data are available as quarterly data. The frequency of the monthly series of the unemployment rate and the number of the working population are compacted by simply averaging them to quarterly data.

Table 1 gives an overview of the used time series. The employment rate is calculated by 100 minus the unemployment rate. The logarithm of real unit labor costs is calculated as the difference between the logarithm of real hourly wages and the logarithm of output per hour.

<table>
<thead>
<tr>
<th>Series</th>
<th>Abbreviation</th>
<th>Description of the data</th>
<th>Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment rate</td>
<td>unrate</td>
<td>Civilian Unemployment Rate</td>
<td></td>
</tr>
<tr>
<td>Employment rate</td>
<td>emprate</td>
<td>100-unrate</td>
<td></td>
</tr>
<tr>
<td>Real hourly wage</td>
<td>comprnfb</td>
<td>Nonfarm Business Sector: Real Compensation Per Hour</td>
<td>log(comprnfb)</td>
</tr>
<tr>
<td>Output per hour</td>
<td>ophnfb</td>
<td>Nonfarm Business Sector: Output Per Hour of All Persons</td>
<td>log(ophnfb)</td>
</tr>
<tr>
<td>Log real unit labor costs</td>
<td>rulc</td>
<td></td>
<td>log(comprnfb) - log(ophnfb)</td>
</tr>
</tbody>
</table>

*Table 1: Data description*

In figure 2 we plot the employment rate and real unit labor costs as well as their first differences over time. Table 2 shows the results for the stationarity test.\(^{13}\) Since we only consider the employment rate and the real unit labor costs in our econometric work, we do not show the results for the other variables here. Here, the ADF-GLS test proposed by (Elliott et al. 1996) is used.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lag(max=4)</th>
<th>Deterministic</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>emprate</td>
<td>3</td>
<td>c, t</td>
<td>-2.84</td>
<td>&lt; 10%</td>
</tr>
<tr>
<td>diff(emprate)</td>
<td>3</td>
<td>c</td>
<td>-8.13</td>
<td>0.00</td>
</tr>
<tr>
<td>rulc</td>
<td>2</td>
<td>c, t</td>
<td>-1.39</td>
<td>&gt; 10%</td>
</tr>
<tr>
<td>diff(rulc)</td>
<td>1</td>
<td>c</td>
<td>-2.85</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Note: c – constant, t – trend, diff – 1\(^{st}\) difference

Table 2: ADF-GLS Test

The employment rate is assumed to be I(0), which means that it satisfies the stationarity conditions. For the real unit labor costs only the first difference is assumed to be stationary.

5 Empirical Facts

Figure 3 gives an overview about the relationship between the employment rate and the real unit labor costs from 1948:1 to 2006:4. The paths of the ‘raw’ series are not that obvious since they contain a lot of noise. Until the 1980s there seems to be a kind of closed orbit on a relatively high level of the wage share. Since the 1990s the wage share has declined successively whereas the employment rate remained quite stable. Hence, the center of the cycle has ‘moved’ to the left. The cyclical components are estimated by the Hodrick-Prescott filter (HPF). To stress the dynamics of these components we also estimated its trend (‘double’ HPF). Both diagrams show the short-run dynamics and confirm the non-linear relationship. It can be argued that the connection between the employment rate and real unit labor costs is quite stable over time. In conclusion we argue that Goodwin’s hypotheses seem to be relevant at least at the qualitative level (Harvie 2000) for the USA.

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\(^{13}\) All econometric work is done using the open source program gretl; available at http://gretl.sourceforge.net.
6 The Econometric Approach

We estimate a reduced form version of equations (12) and (13) using a VAR approach. The here used bivariate VAR(p) model (including real unit labor costs and the employment rate) can be written as

\[ x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \ldots + \phi_p x_{t-p} + \epsilon_t, \quad t = 1, \ldots, T \]  

where \( x_t \) is an 2x1 vector of variables, \( \phi_i \) is an 2x2 matrix of unknown coefficients and it is assumed that

\[ E(\epsilon_t) = 0; \quad E(\epsilon_t \epsilon_s') = \begin{cases} \Sigma & \text{for } t = s \\ 0 & \text{for } t \neq s \end{cases} \]  

where the residuals might be contemporaneously correlated. The model can be expressed as an infinite-order vector moving average representation

\[ x_t = \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \ldots = \sum_{i=0}^{\infty} \theta_i \epsilon_{t-i} \]  

where \( \theta_0 = I_m \) and \( \theta_i = \phi_i' \), \( i = 1, 2, \ldots \)

To conduct some structural analysis following the structural equations in (12) and (13), we apply the Cholesky decomposition where the covariance matrix \( \Sigma_e \) is decomposed into two 2x2 lower triangular matrices, \( P \)

\[ \Sigma_e = PP' \]  

Thus, equation (16) can be rewritten as

\[ x_t = \sum_{i=0}^{\infty} (\theta_i P)(P^{-1} \epsilon_{t-i}) = \sum_{i=0}^{\infty} (\theta_i P) \eta_{t-i} \]  

where \( \eta_t = P^{-1} \epsilon_t \) are the orthogonalized innovations. Thus, the lower triangular matrix \( P \) imposes a kind of causality structure since it determines the instantaneous relationship between variables. Hence, the results are not independent from the ordering of the variables. We will come back to this later when we discuss our identification strategies and robustness tests.
A short additional comment on the expected results: It is expected that real unit labor cost shocks affect the employment rate negatively in the short-run before the dynamics reverse to become positive in the medium term. Also it is expected that a positive employment rate shock leads to an increase in the real unit labor costs in the short-run before the effect reverts to become negative, as argued by the model dynamics. The variance decomposition should yield that the relative importance of the employment rate for the development of unit labor costs increases over time after an employment rate shock has occurred. The same is expected for the relative importance of unit labor cost shocks for the employment rate.

**6.1 Granger-Causality and VAR Estimation**

A two dimensional VAR with the variables $d_{\text{rule}}$ (first difference of log real unit labor costs) and $\text{emplrate}$ (employment rate) represents the baseline model. The information criteria recommend an optimal lag length between 2 and 3. We assume a VAR(3), otherwise autocorrelation problems occur. The VAR(3) does not contain a deterministic trend.

The test on Granger causality (table 3) indicates that no unambiguous direction of causality exists. For both directions the hypothesis of no Granger causality can be rejected at the 1% level. However, the F-statistics for the hypothesis that the employment rate Granger causes the change in real unit labor costs is significantly higher.

<table>
<thead>
<tr>
<th>Causality</th>
<th>Lag</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{\text{rule}} \rightarrow \text{emplrate}$</td>
<td>2</td>
<td>0.0135</td>
</tr>
<tr>
<td>$\text{emplrate} \rightarrow d_{\text{rule}}$</td>
<td>2</td>
<td>0.0005</td>
</tr>
</tbody>
</table>

Table 3: Test on Granger causality

In order to analyze the dynamics of the system two methods are applied. The first one is the impulse-response function which computes the propagation over time of a shock on the variable of interest. The variance decomposition analyzes the relative impact of a shock in one variable on the total variance of the variable of interest – it measures the relative impact of a structural shock for

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14 The impulse-response functions should show a cyclical reaction on each shock which resemble the ones from the Goodwin model.
15 The results of the information criteria can be obtained from the author on request.
the explanation of the total variance of the dependent variable. In order to apply these methods, the VAR system has to be transformed from the reduced form into the structural one which can be interpreted theoretically. For this, we use the Cholesky decomposition as described before. Since the direction of causality is not unambiguous as seen, we analyze two different identification schemes simply by reordering the system.

6.2 Identification Scheme I

Vector $x_t$ describes the dependent variables of the system. Matrix $B$ shows the imposed structure of restrictions imposed on the reduced form residuals.\(^{16}\) This identification scheme is called $ID1$:

$$x_t = \begin{pmatrix} \text{emplrate} \\ \text{drulc} \end{pmatrix}; \quad B = \begin{pmatrix} * & 0 \\ * & * \end{pmatrix}. \quad (19)$$

We only allow for a contemporaneous impact of an employment shock on the change of real unit labor costs here.

Figure 4 depicts the impulse-response function over 32 periods with an additional 95% confidence interval. The employment rate increases significantly after an employment shock and reaches its peak approximately after one year, before the effect declines and gets back to its equilibrium value 15 periods later. The change in unit labor costs reacts negatively on a positive employment shock in the short-run, which is not very intuitive. After two periods the change in unit labor costs increases significantly until the 7th quarter. The accumulated changes of unit labor costs reacts permanently positive on a unique positive employment shock, as figure 5 shows. An increase of real unit labor costs on a positive employment shock is in line with Goodwin’s hypothesis. Even though, one would not expect a permanent increase of it. According to the model the increase should be only temporary since the counter-forces come into play and lead to a more or less constant income distribution over time.

A positive unit labor cost shock (wage shock) reduces the employment rate between the second and fifth quarter significantly.\(^{17}\) The reduction is relatively high but only temporary. The point estimator

\(^{16}\) Matrix $B$ actually represents the lower triangular matrix $P$ as described in section 6.

\(^{17}\) The upper confidence interval is close to zero. Different approaches to compute confidence intervals may lead to different results.
indicates a long-term reduction of the employment rate. This reaction is in line with the model hypothesis. The level of unit labor costs increases permanently after a wage shock what is also not in line with the model.

The variance decomposition (table 4) shows that the variances of the individual variables are mainly determined by their own shocks. According to the Goodwin model one would expect that the influence of unit labor costs should increase over time and become the dominant factor in determining the employment rate.\textsuperscript{18} On the other side, also the employment rate should become a dominant factor in determining income distribution in the longer run.

<table>
<thead>
<tr>
<th>Periods</th>
<th>Variable</th>
<th>Employment shock</th>
<th>Wage shock</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>emprate</td>
<td>1.00</td>
<td>0.00</td>
<td>0.28</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>0.96</td>
<td>0.04</td>
<td>1.39</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>0.96</td>
<td>0.04</td>
<td>1.48</td>
</tr>
<tr>
<td>32</td>
<td></td>
<td>0.96</td>
<td>0.04</td>
<td>1.48</td>
</tr>
<tr>
<td>0</td>
<td>d_rule</td>
<td>0.03</td>
<td>0.97</td>
<td>0.01</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>0.09</td>
<td>0.91</td>
<td>0.01</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>0.09</td>
<td>0.91</td>
<td>0.01</td>
</tr>
<tr>
<td>32</td>
<td></td>
<td>0.09</td>
<td>0.91</td>
<td>0.01</td>
</tr>
</tbody>
</table>

\textit{Table 4: Variance decomposition, ID1}

But the results show that in the short- and long-run the variance of the employment rate is only marginally explained by wage shocks (4%).\textsuperscript{19} Employment shocks only explain 3% of the variance of the changes of real unit labor costs in the short-run and 9% for longer horizons.

All in all, the impulse-response functions show the expected reaction on the individual shocks. But the variance-decomposition analysis questions the relative importance of the individual shocks for the fluctuation of the other variables. Their variance is mainly determined by own shocks and only

\textsuperscript{18} In another paper (unpublished yet) I estimated a 5 dimensional SVAR model for the USA (1948:1 – 2002:3) and found that real wage shocks only account for 14% after 32 quarters of the total unemployment variance, whereas real gross private domestic investments, capacity utilization, unemployment rate and labor productivity shocks account for 19%, 19%, 27% and 21% respectively after 32 quarters.

\textsuperscript{19} The estimates of the standard errors for the employment rate are high which indicates some uncertainty about the obtained result.
marginally by the other one. Next, we are going to test whether the results are independent of the chosen identification scheme and propose a second strategy.

### 6.3 Identification Scheme II

Since the results may depend on the used identification scheme, we analyze a second identification strategy:

$$x_t = \begin{pmatrix} emplrate \\ drulc \end{pmatrix} ; \quad B = \begin{pmatrix} * & * \\ 0 & * \end{pmatrix}.$$  \hspace{1cm} (20)

Now, only a shock in real unit labor costs has an immediate effect on the employment rate, but not the other way around.

The changed impulse-response functions are depicted in figure 6 and can be seen on the diagonal from the bottom left to top right. The other two graphs are the same as before. After an employment shock the change in unit labor costs increases significantly after the second period. This effect remains significant until the 6\textsuperscript{th} quarter before it converges back to its equilibrium value. The employment rate decreases immediately after a wage shock. This effect holds about two and a half years before it dies away. The employment rate decreases immediately now. This effect holds 12 periods on before it fades away. The accumulated effects on unit labor costs can be seen in figure 7. Also here wage as well as employment shocks have a significant and permanent effect what is again not as expected.

Table 7 shows the results for the variance decomposition. In contrast to ID1 the relative importance of wage shocks for the total variance of the employment rate has increased from 4% to 14% in the medium to long run. The relative importance for the variance of unit labor costs have only marginally changed. Qua identification scheme, employment shocks do not explain anything in the short-run. But over time the relative importance increases up to 6% and hence is as before.

<table>
<thead>
<tr>
<th>Periods</th>
<th>Variable</th>
<th>Employment shock</th>
<th>Wage shock</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>emplrate</td>
<td>0.96</td>
<td>0.04</td>
<td>0.28</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>0.87</td>
<td>0.13</td>
<td>1.40</td>
</tr>
</tbody>
</table>
### 6.4 Analysis of the Cyclical Components

At this point we want to analyze the cyclical components instead the ‘raw’ data of the employment rate and real unit labor costs, since some of the responses are not as expected in the medium and long-term. According to the model unit labor costs should not increase permanently after any temporary shock. The extraction of the cyclical component is done by the Hodrick-Prescott filter (Hodrick & Prescott 1997) and the Baxter-King band pass filter (Baxter & King 1995).

#### 6.4.1 HP-Filter

For quarterly data we use the standard lambda value of 1600. Our VAR system is still the same as illustrated in equation (19). Also here we apply both identification strategies. The optimal lag length is 2 according to the HQC and BIC criteria.\(^\text{20}\) A VAR(2) without a constant\(^\text{21}\) shows no serial correlation in the residuals.

The Granger causality test (see table 6) shows that the null that unit labor costs do not Granger cause the employment rate can only be rejected at the 5% level. On the other side there is a highly significant influence of the employment rate on unit labor costs.

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\(^{20}\) The detailed results can be requested from the author.

\(^{21}\) Since the filtered series fluctuate around zero as the expected value, no constant is needed.
Causality | Lag | p-value
---|---|---
hp_rulc → hp_emplrate | 2 | 0.05
hp_emplrate → hp_rulc | 2 | 0.00

Table 6: Test on Granger causality, HP-data

The impulse-response functions for ID1 are presented in figure 8. It can be seen that the dynamics are more intensive and rather fit to Goodwin’s model. As before, unit labor costs decrease immediately after a positive employment shock, what is still not intuitive. Between the fourth and ninth period the effect becomes significantly positive before it becomes significantly negative between the 13th and 15th period.\(^2\) These fluctuations of unit labor costs can be interpreted as follows: An increase in employment has a positive effect on the worker’s bargaining power and leads to an increase of the wage share. This leads to a reduction of the profit rate which implies a decrease in employment and hence unit labor costs. This process works within 4 years before the effect becomes zero. This argumentation is confirmed by the impulse-response function of a wage shock on employment: higher unit labor costs reduce employment significantly after 5 quarters. This in turn has positive effects on the profitability and hence investment demand which leads to an increase in labor demand again; the employment rate increases after the 14th period.

Figure 9 shows the impulse-response functions for ID2. The relation of unit labor costs on an employment shock is the same as before, with the exception that no immediate negative effect can be observed. The employment rate reacts immediately significantly negative on a wage shock. The effect holds on up to the 8th period. Between the 12th and 15th period a positive and significant impact of the employment rate can be observed. The dynamics are almost the same as for ID1.

<table>
<thead>
<tr>
<th>Periods</th>
<th>Variable</th>
<th>Employment shock</th>
<th>Wage shock</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>hp_emplrate</td>
<td>1.00</td>
<td>0.00</td>
<td>0.26</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>0.93</td>
<td>0.07</td>
<td>0.75</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>0.94</td>
<td>0.06</td>
<td>0.77</td>
</tr>
<tr>
<td>32</td>
<td></td>
<td>0.94</td>
<td>0.06</td>
<td>0.77</td>
</tr>
<tr>
<td>0</td>
<td>hp_rulc</td>
<td>0.04</td>
<td>0.96</td>
<td>0.01</td>
</tr>
</tbody>
</table>

\(^2\) The accumulation of the effects shows that in the medium to long-run the shock has no permanent impact any more on unit labor costs what is more in line with the Goodwin model.
The variance decomposition analysis for the corresponding identification scheme (see table 7 and 8 respectively) stress the fact that the influence of employment shocks on the cyclical component of the real unit labor costs is relevant. In both cases these shocks explain about 20% of the total variance in the medium- to long-run. The immediate effect is rather low; but this is intuitive according the assumptions of some sort of rigidities for example due to employment protection. On the other side, the importance of wage shocks on the employment rate depends on the chosen identification scheme. In the ID1 case only up to 6% of the employment variance are explained by this kind of shocks whereas in the ID2 case about 6% in the short-run and 13% in the long-run are accounted for this shock. As already explained, the results of ID2 are more intuitive. Also the variance decomposition analysis confirms to a certain degree the underlying hypotheses of the baseline model. Both variables, real unit labor costs and the employment rate, are linked together and drive each other. The Granger causality analysis leads to the presumption that the employment rate drives the functional income distribution and not the other way around.

Table 7: Variance decomposition, HP-Data, ID1

<table>
<thead>
<tr>
<th>Periods</th>
<th>Variable</th>
<th>Employment shock</th>
<th>Wage shock</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>hp_emplrate</td>
<td>0.95</td>
<td>0.05</td>
<td>0.26</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>0.87</td>
<td>0.13</td>
<td>0.75</td>
</tr>
<tr>
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<td></td>
<td>0.87</td>
<td>0.13</td>
<td>0.77</td>
</tr>
<tr>
<td>32</td>
<td></td>
<td>0.87</td>
<td>0.13</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Table 8: Variance decomposition, HP-Data, ID2
6.4.2 Baxter-King Filter

Here we are going to apply the Baxter-King filter method – a band pass filter which allows extracting defined frequencies. With the application of it we want to check whether our results are robust against the choice of a filter technique.\(^{23}\) The vector of dependent variables still contains \textit{emplrate} and \textit{rulc} – but now filtered by the Baxter-King approach (BK) – as in equation (19). The maximum lag length is 16 since the AIC criterion recommends it. The HQC criterion recommends 9 and the BIC criteria 6 lags. We estimate a VAR(9) because no autocorrelation can be found for this lag selection and a VAR(16) seems to be too large. The direction of causality using the Granger test (see table 9) is not unambiguous. In both cases the null can be rejected. However, the null that the employment rate does not affect unit labor costs can only be rejected at the 5\% level. This result contradicts to a certain degree former results where the F-statistics was normally higher for the test whether the employment rate Granger causes changes in the real unit labor costs.

<table>
<thead>
<tr>
<th>Causality</th>
<th>Lag</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>bk_rulc → bk_emplrate</td>
<td>9</td>
<td>0.0042</td>
</tr>
<tr>
<td>bk_emplrate → bk_rulc</td>
<td>9</td>
<td>0.0236</td>
</tr>
</tbody>
</table>

\textit{Table 9: Test on Granger causality, BK-Data}

The impulse-response functions of ID1 are depicted in figure 10. In comparison with the results of the HP data, here the length of up- and downturns are different. On the basis of the HP data the employment rate reacts 6 quarter significantly positive on an employment shock before equilibrium is reached again. Using BK data the effect takes 11 periods. But also here the employment rate reacts negatively after some time on a positive shock – the dynamics are overall as before. Unit labor costs do not react immediately negative on a positive employment shock what is as expected; and increase significantly between the 8\textsuperscript{th} and 14\textsuperscript{th} quarter before the effect fades away. The length of the upturn corresponds to the results for the HP data, even though unit labor costs respond later but longer to an employment shock. Surprisingly, we do not obtain a significant effect of a wage shock on the employment rate. Indeed, the point estimator reacts negatively but the effect is not significant

\(^{23}\) For both variables we select 12 periods as the lower bound and 32 periods as the upper bound. The selection is based on the assumption that the relevant business cycle frequency lies between 3 and 8 years. The adjustment value is 12 which is standard and not further elaborated.
at all. The response of unit labor costs on a wage shock is much more volatile now. The increase is significant up to the 11\textsuperscript{th} quarter before it becomes negative between the 14\textsuperscript{th} and 20\textsuperscript{th} period. This indicates the temporary persistence of unit labor costs and is in line with Goodwin’s assumed dynamics.

Figure 11 depicts the impulse-response function of ID2. Except for the reaction of the employment rate to a wage shock nothing has changed wherefore we do not comment these results here. The employment rate responses negatively to a wage shock between the 5\textsuperscript{th} and 14\textsuperscript{th} period. The point estimator still shows the cyclical behavior of the variable after a shock.

The results for the variance decomposition are given in table 10 and 11, respectively. For both identification schemes the relative importance of wage shocks for the employment rate are approximately equal in the long run (10%). The short term reaction is different; while a wage shock explains only 3% after ten periods in the ID1 case, the same shock explains 9% in the ID2 case. For the ID2 case the highest influence is measured after 15 periods (13%) before the relative importance decreases to 10%. Thus, the highest influence is measured in the medium and not in the long term as in the ID1 case. The relative influence of wage shocks on the employment rate is relatively small (10%) as before.

<table>
<thead>
<tr>
<th>Periods</th>
<th>Variable</th>
<th>Employment shock</th>
<th>Wage shock</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
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<td>bk_emplrate</td>
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<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>0.97</td>
<td>0.03</td>
<td>0.38</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>0.92</td>
<td>0.08</td>
<td>0.55</td>
</tr>
<tr>
<td>32</td>
<td></td>
<td>0.91</td>
<td>0.09</td>
<td>0.56</td>
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<tr>
<td>0</td>
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<td>0.08</td>
<td>0.92</td>
<td>0.01</td>
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<td>0.17</td>
<td>0.83</td>
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<tr>
<td>32</td>
<td></td>
<td>0.19</td>
<td>0.81</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 10: Variance decomposition, BK-data, ID1

On the other side, we find in both cases hints that the employment rate has a substantial impact on
the income distribution in the medium to long run. In the ID1 case the long term influence is 19% whereas it reaches 23% in the ID2 case.

<table>
<thead>
<tr>
<th>Periods</th>
<th>Variable</th>
<th>Employment shock</th>
<th>Wage shock</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
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<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
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<td></td>
<td>0.91</td>
<td>0.09</td>
<td>0.38</td>
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<td>0.91</td>
<td>0.09</td>
<td>0.55</td>
</tr>
<tr>
<td>32</td>
<td></td>
<td>0.90</td>
<td>0.10</td>
<td>0.56</td>
</tr>
<tr>
<td>0</td>
<td>bk_rulc</td>
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<td>1</td>
<td>0.00</td>
</tr>
<tr>
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<td>0.15</td>
<td>0.85</td>
<td>0.01</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>0.20</td>
<td>0.80</td>
<td>0.01</td>
</tr>
<tr>
<td>32</td>
<td></td>
<td>0.23</td>
<td>0.77</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 11: Variance decomposition, BK-data, ID2

7 Conclusion

We tried to verify Goodwin’s proposed dynamic relationship between the employment rate and functional income distribution empirically for the USA. The literature review has shown that no unique method exists on how to tackle the question. The approaches differ regarding the used empirical and econometric instruments and data. Here, we estimated a bivariate VAR system including the employment rate and real unit labor costs. Apart from the estimation based on the ‘raw’ data set, we also estimated the model using the cyclical components of the variables of interest – since, as we argued, the Goodwin model is a business cycle model and the use of filter techniques should be justified.

Generally, former results can be confirmed by orthogonalized impulse-response functions (for different orderings of the variables): Functional income distribution is driven by labor market dynamics in the sense that increasing employment causes real wages to rise (labor-market-led). Furthermore, the inverse relation between real unit labor costs (proxy of the wage share) and the employment rate are confirmed, which corresponds to the hypothesis of a profit-led goods market regime.
The variance decomposition has shown that the employment rate is a substantial factor in explaining the total variance of the wage share. On the other side, the role of real unit labor costs for the employment dynamics is rather low. The variance of the employment rate is only marginally explained by real unit labor costs which relativizes Goodwin’s hypothesis regarding the role of real wage dynamics for the labor market.

Especially of interest are the impulse-response functions of the cyclical components. The dynamics have higher amplifications compared to the ‘raw’ data set and correspond to those as known from the baseline model. The non-linear relations are confirmed by wave-like responses.

For further research it would be of interest whether the results remain in a higher dimensional system with further real and monetary variables or whether they will be relativized by these additional factors. One could ask whether recent findings by the RBC literature regarding the role of expected shocks, proxied by stock indices\(^\text{24}\), question or even support our results.\(^\text{25}\) Also the role and relevance of monetary and fiscal policy needs further research.

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\(^{24}\) On this research see Beaudry & Portier (2006) and Beaudry & Lucke (2009).

\(^{25}\) The Bundesbank has shown in a recent study that there exists a long-run relationship between the development of stock indices and corporate profits (Deutsche Bundesbank 2009).
8 Literature


9 Appendix

Figure 1: Wage-share-capacity-utilization-cycle; red orbit: profit-led regime, black orbit: wage-led regime

Figure 2: Overview of used time series: emprate – employment rate, rulc – log hourly real unit labor costs, USA, 1948:1-2006:4
Figure 3: Goodwin cycle for the USA, 1948:1-2006:4. Cyclical component is estimated by the use of Hodrick-Prescott filter (lambda=1600). Double-Hodrick-Prescott filter: HP-Trend of the cyclical component (lambda=1600).
Figure 4: Impulse-response function, ID1

Figure 5: Accumulated Impulse-response function, ID1
Figure 6: Impulse-response function, ID2

Figure 7: Accumulated Impulse-response function of d_rulc, ID2
Figure 8: Impulse-response function, HP-Data, ID1

Figure 9: Impulse-response function, HP-Data, ID2
Figure 10: Impulse-response function, BK-Data, ID1

Figure 11: Impulse-response function, BK-Data, ID2