

New Keynesian DSGE Models and the IS-LM Paradigm

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Abstract

New Keynesian DSGE models propose a dynamic and expectational version of the old IS-LM paradigm. Acknowledging that the Taylor rule as a substitute for the LM-curve has its merits we show that standard DSGE models do not model how the central bank achieves its targets. In filling this gap we make evident that models neglecting a store-of-value function of money but still assuming a Taylor rule are inconsistent. Our major point concerns the-so called new Keynesian IS-curve. We prove that DSGE models which typically rest on the assumption of representative agents are unable to derive the IS-curve. This implies that these models lack the capability to analyse the role of savings as a gap in aggregate demand. By assuming overlapping generations we make evident how this shortcoming can be avoided. We also show how OLG models add a richer dynamics to the standard DSGE approach.

1 Introduction

Until the 1980s the IS-LM model was used in Keynesian economics as a shortcut which allowed the reduction of underlying complex individual decision-making processes and their aggregation. In a nutshell the IS-LM-model was

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based on the following insights which characterize macroeconomic models irrespective of whether they are of neoclassical or Keynesian origin: In a closed economy a simultaneous equilibrium in the multitude of commodity markets boils down to the equality of aggregate saving and aggregate investment. Further, given equilibrium in the labour market, then due to Walras' Law, the equality of saving and investment implies equilibrium in the financial sector of the economy meaning that surplus units which spend less than their current incomes on goods transfer their savings to deficit units (the government or private investors) which want to spend more on goods than their current incomes allow them to do. These insights were combined with hypotheses which explain what has come to be named the IS-LM-paradigm: *First*, due to a store-of-value function of money, the aggregate goods market and the capital (bonds) market no longer mirror each other. Hence variations of the real interest rate will not suffice to equilibrate both markets, rather, we are in need of an additional variable. *Second*, if wages and prices are fixed, this role is assumed by real aggregate production, which has important consequences for the role of aggregate savings. Whereas in a neoclassical world high savings promote high growth rates of production, now high savings mean a low level of aggregate demand leading to a low level of production and employment. *Third*, given wage and price rigidity, then equilibrium in the goods market and equilibrium in the financial system can be represented by a corresponding equilibrium relation between the *nominal* interest rate and *real* aggregate income. This property made it possible to give the IS-LM model a simple graphical illustration which has governed macroeconomic textbooks for decades. *Fourth*, the Keynesian departure from a world in which the real interest rate ensures a simultaneous equilibrium in both the aggregate goods and bonds (capital) market thus leaving national income to be determined by components other than aggregate demand, also assigned a new role to demand management in particular in the hands of fiscal policy. With prices and wages taken to be fixed at some pre-determined level, demand management proved to be a suitable instrument for fiscal as well as monetary policy in order to fix real aggregate production and employment at some desired levels.

By the beginnings of the 1980s macroeconomic models without rigorous microfoundation (based on perfect rationality) had become obsolete at least in the so-called mainstream economics. Endogenous growth theories based on Ramsey (1928), Cass (1965), and Koopmans (1965), as well as real business cycle approaches based on Kydland and Prescott (1982), Long and Plosser (1983), Prescott (1986) and Black (1982), had a focus on macroeconomic implications of intertemporal decisions, whereas the then a-temporal New Keynesian macroeconomic models focused on real and nominal rigidities as the

outcome of optimizing behaviour (Blanchard and Kiyotaki, 1987; Mankiw and Romer, 1991 for a survey). The neglect of intertemporal decisions explains a shift of interest within the New Keynesian school from discussing the relationship between savings and investment towards the microeconomic foundation of nominal and real rigidities and thus of the Phillips curve. It was the integration of intertemporal decisions into so far a-temporal New Keynesian approaches which gave birth to New Keynesian DSGE models with the so-called New Keynesian IS-curve, the Taylor-rule as a substitute for the LM-curve, and the New Keynesian Phillips curve as their heartpieces.* Contrary to its traditional counterpart, the New Keynesian IS-curve is seen to be dynamic and expectational in nature[†] linking the present and the future and is thus considered to play an important role in determining an economy's dynamic behaviour outside its steady state equilibrium (Kerr and King, 1996; Goodfriend and King, 1997; Galí and Gertler, 1999). In particular McCallum, Nelson (1999) emphasize that the IS-LM-relations are indeed compatible with maximizing behaviours of rational agents. They conclude that the obtained pair of linear equations are analogous to the traditional IS and LM functions with the qualification that intertemporally optimizing households modify the traditional framework by a dynamic forward-looking aspect. However, as Romer (2000) and McCallum, Nelson (1999) rightly observe, any departure from a *static* fixed price, fixed wage setting, reduces the value of a *graphical* representation of the IS-LM model significantly for basically two reasons: First, whereas household optimization yields monetary demand as a function of the *nominal* interest rate, consumption follows as a function of the *real* interest rate.[‡] Second, the dependency of current consumption on future expected income renders a graphical comparative-static analysis of fiscal and monetary policy outside the long-run steady state as useless.

In the following we show that the New Keynesian DSGE model does not only complicate a graphical analysis within a IS-LM framework. On the contrary, we deny that New Keynesian DSGE models share the proposed similarities with the traditional IS-LM model at all. In particular we state that what is called the New Keynesian IS curve does in fact not represent the equality of saving and investment. Rather, it represents the relation-

*A comprehensive and rigorous analysis is provided by Woodford (2003)

[†]The exclusion of rational expectations has been acknowledged as a major weakness of the traditional IS-curve by King (1993).

[‡]Romer's proposal to replace the LM-curve by the Taylor-rule does in fact not show a way to make both consumption and money demand dependent on the same variables because the nominal interest rate as determined by a Taylor-rule depends on current inflation whereas the real interest rate is linked to future expected inflation.

ship between present and future (expected) aggregate consumption based on the individual Euler equations - given that the aggregate goods market is in equilibrium. Whereas the Keynesian IS-curve brings to the fore that well-specified combinations of the interest rate *and* aggregate production are needed in order to equate savings and investment, the aggregate Euler equation expresses that in equilibrium the ratio of the future (expected) and current *difference* between the equilibrium level of national income and investment (or public expenditures) depends on the representative household's discount factor, its relative risk-aversion and the real interest rate.

We could of course conclude that the New Keynesian IS-curve is just a misnomer without having substantial consequences. Accepting this view, however, is equivalent to accepting that indeed any explanation of how the equality of aggregate saving and investment is established in an economy is no longer of interest and that it is therefore more important to focus exclusively on the equilibrium growth rates in consumption or aggregate production (national income) as a function of the real interest rate. This indeed has been the primary focus of neoclassical stochastic growth models where $S = I$ holds quasi automatically because households always invest all of their savings in real capital either for a lack of alternatives or due to smooth adjustments of the real interest rates. In both cases high savings promote high economic growth rates.

However, once we are sceptical about the relevance of such a quasi automatic adjustment process, the way how the equality of investment and saving is achieved should very well be of interest. Such a scepticism may have its roots in the existence of a store-of-value function of money or alternatively in financial market imperfections rendering investment as a function of current cash flows instead of the real interest rate which may be the consequence of financial restrictions. In such a world a high level of savings as well as a low level of investment regularly imply a low level of *aggregate* production thus leaving an active role for demand management at least in the short to medium run. In order to examine its effects, a macroeconomic model should be chosen which allows a formal (not necessarily graphical) representation of the IS-curve thus making it possible to derive the impact of for example tax reductions on the absolute level of current production as well as on its time path. The most prominent example of the necessity of such a perspective is given by the current international imbalances of current accounts which the real interest rate has obviously been unable to remove.

Whereas we hold the view that macroeconomic models with a focus on business cycle aspects should indeed assign an important role to the interplay of aggregate savings and investment, we also acknowledge that the replacement of the LM-curve by the Taylor-rule is justified both by theoretical as

well as empirical reasons.[§] What we criticize, however, is that in a typical DSGE-model the question remains unanswered, *how* the central bank realizes its strategy and what this implies for the relationship between the aggregate goods market and the financial sector. In our paper we will fill this gap and in doing so we show that a frequently held position according to which the Taylor-rule can do without a store-of-value function of money, is not justified because this would give rise to rationing equilibria in the financial sector.

The remainder of the paper is structured as follows: In Section 2 we present a simple baseline DSGE model with two goals: First we make the difference between the IS-curve and the aggregate Euler equation explicit. Second, we show that the introduction of the Taylor-rule without establishing a necessary link to the capital and money market puts the economy at disequilibrium. Section 3 deals with the question whether and how the DSGE model can be modified such that the equilibrium of I and S form an integral part. We show why such a modification is hardly possible as long as we stick to the conception of the representative agent and point to heterogeneous agents as a way out. Taking overlapping generations as an example, we derive a dynamic IS-curve in Section 4. In doing so we start with the simple case of perfect foresight and logarithmic utility and then go on to assume uncertainty and a general CRRA utility function. Using log-linearizing and linearizing techniques around the steady state, this allows us to present a comparison with the aggregate Euler equation. Section 5 concludes.

2 A Baseline DSGE Model with Representative Agents

A baseline DSGE model typically models a stationary economy thus neglecting investment (Galí, 2008). Capital markets are assumed to be perfect thus allowing households to smooth consumption over time. Most importantly, macroeconomic behavioural functions are derived from an optimizing *representative* agent. As in a-temporal New Keynesian models there exists a multitude of heterogeneous goods which allow firms to set prices. By contrast, labour markets can be found to be modelled as perfectly competitive as well as imperfectly competitive. Increasingly, DSGE models account for real or nominal wage rigidities. Since we are primarily interested in the relationship between DSGE models and the IS-LM paradigm we take the labour market to be perfectly competitive. By contrast we assume monopolistic competition in the markets for heterogeneous goods since the implied price-setting

[§]An excellent discussion is provided in Woodford (2003).

behaviour of firms allows us to derive the New Keynesian Phillips curve. The following representation draws on Walsh (2003) with the exception that we introduce government expenditures as well as their financing.

2.1 Microeconomic Foundation

2.1.1 The Representative Household

We consider a representative infinitely lived household which maximizes its expected lifetime utility derived from consumption, leisure, and cash holdings.

$$E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[u(C_{t+i}) - \gamma(N_{t+i}) + v\left(\frac{M_{t+i}^n}{P_{t+i}}\right) \right] \right\} \Rightarrow \max \quad (1)$$

where M^n denotes nominal money balances with M as its real counterpart, N denotes labour supply, and C represents the household's preferences regarding heterogeneous commodities the number of which is typically assumed to fall into the interval $[0, 1]$:

$$C_t = \left[\int_0^1 C_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad (2)$$

$u(\cdot)$, $-\gamma(\cdot)$ and $\nu(\cdot)$ are assumed to be strictly concave with constant intertemporal elasticity of substitution and hence constant relative risk aversion. Following a common practice, t is used as a time index.

The household maximizes expected utility over its (infinite) lifetime subject to a sequence of period budget constraints

$$\int_0^1 C_{jt+i} P_{jt+i} dj + M_{t+i}^n + B_{t+i}^n = N_{t+i} W_{t+i}^n + \Omega_{t+i}^n - T_{t+i}^n + B_{t+i-1}^n i_{t+i-1} + M_{t+i-1}^n \quad (3)$$

and the solvency constraint

$$\lim_{T \rightarrow \infty} \left(\frac{B_T^n}{\prod_{t=1}^T (1 + i_t)} \right) \geq 0 \quad (4)$$

taking prices and the nominal wage as given. β denotes the discount factor, W^n the nominal wage rate, Ω_{t+i}^n nominal profits, B^n nominal bonds, with B and W as their real counterparts, i represents the nominal interest rate, T^n the nominal value of a lump-sum tax with T as its real counterpart...

The optimization problem is normally solved in two steps: First demand functions for individual commodities are derived by minimizing total consumption expenditures subject to (2) yielding as demand curves for individual commodities

$$C_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\theta} C_t \quad (5)$$

and as the aggregate price level (Walsh 2003, p. 233):

$$P_t = \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \quad (6)$$

As a second step we turn to solving the intertemporal optimization problem which requires the maximization of (1) subject to a sequence of temporal budget constraints

$$C_{t+i}P_{t+i} + M_{t+i}^n + B_{t+i}^n = N_{t+i}W_{t+i} + \Omega_{t+i}^n - T_{t+i}^n + B_{t+i-1}^n i_{t+i-1} + M_{t+i-1}^n \quad (7)$$

where due to (5) and (6) we obtain

$$\int_0^1 C_{jt} P_{jt} dj = C_t P_t \quad (8)$$

The first-order conditions deliver optimal relationships between current and future consumption, between consumption and cash holdings, as well as between consumption and the labour supply.

The optimal relationship between current and future consumption leads to the Euler equation (9) according to which the representative households either wishes to keep consumption unchanged over time if the expected real interest rate and the subjective discount rate coincide, or wants to tilt consumption if both rates diverge:

$$u'(C_t) = \beta E_t \left[u'(C_{t+1}) \frac{P_t}{P_{t+1}} (1 + i_t) \right] \quad (9)$$

or equivalently:

$$1 = E_t \left[\left(\beta \frac{u'(C_{t+1})}{u'(C_t)} \right) \frac{P_t}{P_{t+1}} (1 + i_t) \right]$$

with

$$\mathcal{M} = E_t \left[\left(\beta \frac{u'(C_{t+1})}{u'(C_t)} \right) \right] \quad (10)$$

denoting the stochastic discount factor.

The following equation (11) determines how optimal cash holdings are related to optimal consumption.

$$u'(C_t) = \beta E_t \left[u'(C_{t+1}) \frac{P_t}{P_{t+1}} \right] + v' \left(\frac{M_t^n}{P_t} \right) \quad (11)$$

We observe that each unit of income which is used in order to increase money holdings, leads to lower current consumption thus giving rise to a disutility. On the other hand, the implied increase in purchasing power allows the household to consume more in the next period. Since money holdings do not yield an income, and since we have assumed that the household can alternatively save in the form of riskless government bonds, the prospect of consuming more tomorrow would not suffice in order to legitimate positive money holdings. This explains why many DSGE models assume that money yields direct utility which renders bonds and money as *imperfect* substitutes because now each marginal unit of money holdings does not only increase future consumption but also yields direct utility. Hence the marginal utility of cash holdings comprises both an indirect component through increased future purchasing power and a direct component reflecting the utility of liquidity services. In the optimum any marginal increase in cash holdings leads to an increase in the direct and indirect utility of money which is perfectly balanced out by the marginal decrease in the current utility of consumption.

Equation (12) determines how the household splits each available unit of time between leisure and labour. A marginal decrease in leisure and thus increase in labour supply leads to a decrease in utility amounting $-\gamma'(N_t)$ but it also increases consumption possibilities and hence generates an additional utility of consumption amounting to $u'(c_t) \frac{W_t^n}{P_t}$. In the optimum any increase in utility due to more consumption is perfectly outweighed by an incumbent increase in the disutility of labour, hence:

$$u'(C_t) \frac{W_t^n}{P_t} = \gamma'(N_t) \quad (12)$$

It is important to note that equations (9), (11) and (12) do not represent a full-fledged solution of the optimization problem (Bagliano and Bertola 2007, p.8). Rather, they just explain how an optimizing household would use its available time for leisure and labour as alternatives and how this household would distribute a given income between current consumption, future consumption and cash holdings.

Explicit solutions for the first-order conditions (9), (11) and (12), are obtained by applying log-linearization techniques around their steady state

values yielding for the Euler equation:

$$\begin{aligned}\widehat{c}_t &= E_t \widehat{c}_{t+1} - \frac{1}{\sigma} \left(\widehat{i}_t - E_t \pi_{t+1} \right) \\ \sigma &= - \frac{u''(\overline{C})}{u'(\overline{C})} \overline{C}\end{aligned}\tag{13}$$

where by $\widehat{c}$, $\widehat{i}$, we denote percentage deviations of consumption, absolute deviations of the nominal interest rate from their steady state values, and where $\frac{1}{\sigma}$ represents the constant intertemporal elasticity of substitution. Note that in a baseline DSGE model the steady state value of inflation is typically assumed to be zero.

By the same procedure we obtain for the log-linearized optimal relationship between (current) consumption and real cash holdings ($\widehat{m}$):

$$\begin{aligned}\widehat{m}_t &= \frac{\sigma}{\mu} \widehat{c}_t - \frac{(1 - \bar{i})}{\mu} \widehat{i}_t \\ \mu &= - \frac{\nu''(\overline{M})}{\nu'(\overline{M})} \overline{M}\end{aligned}$$

and for the log-linearized optimal relationship between current consumption and optimal labour supply

$$- \sigma \widehat{c}_t + \widehat{w}_t = \psi \widehat{n}_t\tag{14}$$

$$\psi = \frac{\gamma''(\overline{N})}{\gamma'(\overline{N})} \overline{N} > 0\tag{15}$$

with $\widehat{w}_t$ ($\widehat{n}_t$) denoting percentage deviations of the real wage (labour supply) from its steady state value.

2.1.2 The Government and the Central Bank

In a DSGE baseline model public expenditures on goods ($G_t P_t$) and interest payments ($B_{t-1}^n i_t$) are financed by a lump-sum tax (T_t^n) and by the issue of bonds yielding as budget deficit:

$$\frac{B_t^{ns} - B_{t-1}^n}{P_t} = G_t - T_t + \frac{B_{t-1}^n}{P_t} i_t\tag{16}$$

where B_t^{ns} denotes the supply of government bonds. The government, too, has to obey a solvency constraint.

$$\begin{aligned}(G_0 - T_0) P_0 + \frac{(G_1 P_1 - T_1^n)}{1 + i_1} + \frac{(G_2 P_2 - T_2^n)}{(1 + i_1)(1 + i_2)} \dots &= 0 \\ \Leftrightarrow \lim_{T \rightarrow \infty} \left(\frac{B_T^n}{\prod_{s=0}^T (1 + i_s)} \right) &\geq 0\end{aligned}\tag{17}$$

We furthermore assume that the government purchases goods in each of the existing markets and that the optimal composition of aggregate expenditures is obtained by solving the following minimization problem:

$$\begin{aligned} & \min G_{jt} P_{jt} \\ & s.t. \\ & \left[\int_{j=0}^{\infty} G_{jt}^{\frac{\theta}{\theta-1}} \right]^{\frac{\theta-1}{\theta}} \end{aligned}$$

This yields as optimal public demand for commodity j :

$$G_{jt} = \left(\frac{P_{jt}}{P_t} \right) G_t \quad (18)$$

and will allow us to derive an explicit solution for firm j 's optimal commodity price as will be shown below.

Central bank behaviour is usually reduced to the Taylor rule according to which monetary policy successfully fixes the nominal interest rate at a target level which is determined by the steady state real interest rate, target inflation as well as deviations of current inflation from this target value, and current output from its steady state value. Setting target inflation equal to zero, this leads to

$$\begin{aligned} \hat{i}_t &= \rho_{\pi} \pi_t + \rho_y \hat{y}_t \\ \rho_{\pi} &> 0, \quad \rho_y > 0 \end{aligned} \quad (19)$$

with $\hat{y}_t$ representing percentage deviations of equilibrium output from its steady state value.

2.1.3 Firms

Each firm supplies a single commodity j in monopolistic competitive markets. In the simplest case the production function is linear with homogeneous labour as the single input:

$$Y_{jt} = Z_t N_t \quad (20)$$

with Z_t denoting a stochastic productivity shock which is the same for each firm j with $E_t[Z_t] = 1$. Monopolistic competition allows the firm to adjust the price of its commodity in each period in accordance with changing market conditions. However, the firm does not necessarily use this option. Whereas New Keynesian research of the 1980s had a keen interest in examining endogenous economic reasons for this phenomenon, DSGE models draw on Calvo (1983) where the decision on whether to adjust prices in each period or not, is modelled as a random event.[¶] Each firm maximizes its profit over

[¶]Roberts (1995) shows how the different approaches are related to each other.

an infinitely long planning horizon by choosing its optimal commodity price and in doing so takes into account that with probability ϖ this current price will be held constant in future periods whereas with probability $(1 - \varpi)$ it will be adjusted to new market conditions. In order to calculate the optimal price P_{jt} , we therefore have to solve the following optimization problem:

$$\max_{P_{jt}} E_t \left\{ \sum_{i=0}^{\infty} \varpi^i \Delta_{i,t+i} \left[\left(\frac{P_{jt}}{P_{t+i}} \right)^{1-\theta} - \left(\frac{P_{jt}}{P_{t+i}} \right)^{-\theta} MC_{t+i} \right] (C_{t+i} + G_{t+i}) \right\} \quad (21)$$

with MC_{t+i} standing for marginal cost as defined by

$$MC_{t+i} = \left(\frac{W_{t+i}}{P_{t+i}} \right) \frac{1}{Z_{t+i}} \quad (22)$$

and where usually $\Delta_{i,t+i}$ is replaced by the stochastic discount factor $\mathcal{M}$ as defined by (10). This allows us to rewrite the optimization problem to become

$$\max E_t \left\{ \sum_{i=0}^{\infty} \varpi^i \frac{\beta^i C_{t+i}^{-\sigma}}{C_t^{-\sigma}} \left[\left(\frac{P_{jt}}{P_{t+i}} \right)^{1-\theta} - \left(\frac{P_{jt}}{P_{t+i}} \right)^{-\theta} MC_{t+i} \right] (C_{t+i} + G_{t+i}) \right\} \quad (23)$$

We obtain as optimal relative price (Walsh 2003, p. 236)

$$\frac{P_{jt}}{P_t} = \underbrace{\left(\frac{\theta}{\theta - 1} \right)}_{\chi} \frac{E_t \sum_{i=0}^{\infty} \varpi^i \beta^i \left(\frac{P_{t+i}}{P_t} \right)^{\theta} MC_{t+i} (C_{t+i}^{1-\sigma} + G_{t+i})}{\sum_{i=0}^{\infty} \varpi^i \beta^i \left(\frac{P_{t+i}}{P_t} \right)^{\theta-1} (C_{t+i}^{1-\sigma} + G_{t+i})} = Q_t \quad (24)$$

where χ represents the firm's market power as depending on the price elasticity of demand.

2.2 The Macroeconomic Model

2.2.1 The New Keynesian Phillips Curve

Aggregate production (employment) follows from production (employment) at the firm level according to

$$Y_t = \int_{j=0}^1 Y_{jt} dj \quad (25)$$

$$N_t = \int_{j=0}^1 N_{jt} dj \quad (26)$$

Note that the aggregate price index is defined as

$$P_t = \left[\int_0^1 P_{jt}^{1-\theta} \right]^{\frac{1}{1-\theta}} \Leftrightarrow P_t^{1-\theta} = \int_0^1 P_{jt}^{1-\theta}$$

In each period a percentage of $1 - \varpi$ firms adjust their prices whereas a percentage of ϖ firms do not adjust. Hence we obtain for $P_t^{1-\theta}$:

$$\begin{aligned} P_t^{1-\theta} &= (1 - \varpi) P_t^{*1-\theta} + \varpi P_{t-1}^{1-\theta} \Leftrightarrow \\ 1 &= (1 - \varpi) Q_t^{1-\theta} + \varpi \left(\frac{P_{t-1}}{P_t} \right)^{1-\theta} \end{aligned} \quad (27)$$

with P_t^* denoting the optimal price. Loglinearization of (27) yields

$$0 = (1 - \varpi) \widehat{q}_t - \varpi \pi_t \quad (28)$$

where we have set the steady state value of inflation equal to zero. Note furthermore that in the steady state all firms adjust their prices simultaneously yielding $P_{jt} = P_t$. Therefore $\bar{q} = 0$, which implies

$$Q_t = \chi \left(\frac{W_t}{P_t} \right) \frac{1}{Z_t} = 1 \quad (29)$$

and hence the steady state value of marginal cost amount to

$$\overline{mc} = \frac{1}{\chi} \quad (30)$$

Log-linearization of (24) together with (28) finally yields

$$\pi_t = \beta E_t \pi_{t+1} + \frac{1 - \varpi}{\varpi} (1 - \beta \varpi) \widehat{mc}_t \quad (31)$$

We observe that current inflation is determined by basically two components. According to the first component, current inflation is positively correlated with expected future inflation which makes a standard New Keynesian Phillips curve forward-looking. The second component adds deviations of marginal cost from their steady state value. Since

$$\widehat{mc}_t = \widehat{w} - \widehat{z}_t \quad (32)$$

current inflation is positively affected by both a real wage and a productivity shock which exceed their steady-state levels. Whereas productivity shocks are exogenous, the real wage is determined endogenously. In the simplest case, the labour market is perfectly competitive and always in equilibrium.

Since labour supply is given by (14) and aggregate labour demand follows from equation (20) which after log-linearization and aggregation amounts to

$$\hat{n}_t = \hat{y}_t - \hat{z}_t \quad (33)$$

labour market equilibrium implies

$$\hat{c}_t - \hat{z}_t = \frac{1}{\eta} (\hat{w}_t) - \frac{\sigma}{\eta} \hat{c}_t \quad (34)$$

delivering as the equilibrium real wage

$$\hat{w}_t = (\sigma + \eta) \hat{c}_t - \frac{\hat{z}_t}{\eta} \quad (35)$$

Replacing consumption by the equilibrium condition for the aggregate goods market, we have

$$\hat{c}_t = \hat{y}_t - \hat{g}_t \quad (36)$$

This allows us to rewrite (31) as follows:

$$\pi_t = \underbrace{\frac{1-\omega}{\omega} (1-\beta\varpi) (\sigma + \eta)}_{\kappa} (\hat{y}_t - \hat{g}_t) - \underbrace{\frac{1-\omega}{\omega} (1-\beta\varpi)}_{\kappa} \left(\frac{1+\eta}{\eta} \right) \hat{z}_t + \beta E_t \pi_{t+1} \quad (37)$$

In Walsh (2003) $\hat{y}_t$ is furthermore split up into deviations of current production from its competitive steady state value and from its monopolistic steady state value. We ignore this distinction since it does not affect our discussion of the New Keynesian IS-curve.

2.2.2 The New Keynesian IS-Curve and the Determination of the Nominal Interest Rate

Without investment, equilibrium in the aggregate goods market implies

$$Y_t = C_t + G_t \quad (38)$$

or in a log-linearized version:

$$\hat{y}_t = c_{\bar{y}} \hat{c}_t + g_{\bar{y}} \hat{g}_t \quad (39)$$

with

$$c_{\bar{y}} = \frac{\bar{C}}{\bar{Y}}; \quad g_{\bar{y}} = \frac{\bar{G}}{\bar{Y}}$$

where $\hat{c}_t$ is determined by the Euler equation (13). Recognizing that we have equilibrium in the goods market in each period, the Euler equation can be rewritten to become

$$\hat{y}_t = E_t \hat{y}_{t+1} + g_{\bar{y}} \hat{g}_t - g_{\bar{y}} E_t \hat{g}_{t+1} - \frac{c_{\bar{y}}}{\sigma} \left(\hat{i}_t - E_t \hat{\pi}_{t+1} \right) \quad (40)$$

Equation (40) represents a version of the so-called new Keynesian IS curve. It is dynamic (Galí, 2008) in the sense that it contains current and future deviations from the steady state, and it is expectational (Kerr and King, 1996) in the sense that it contains expected next period income and inflation. Of course, we could object that since investment is absent from the model, equilibrium in the commodity market does in fact never represent the coincidence of saving and investment. However, this argument overlooks that equilibrium in the goods market only requires that those goods which are not purchased by households are demanded by agents which draw on financing funds other than their current incomes. In our model this role is taken by the government. The equilibrium relationship between income and interest rate remains even if government expenditures do not respond to varying interest rates since private consumption assumes this role.

Due to the lagged price adjustment, the aggregate goods market is assumed to clear by the adjustment of aggregate production. Whereas in traditional IS-LM-models the nominal interest rate is seen to be determined by the equality of money supply and demand rendering the market for bonds as redundant, in a baseline DSGE model neither the money market nor the aggregate bonds market are presented explicitly.

2.2.3 Some Policy Implications

The above presented DSGE model can be reduced to basically three equations:

1. The so-called New Keynesian IS-curve (equation 40)
2. the Phillips curve (37)
3. the Taylor rule (19)

Together they explain whether and how the economy adjusts to a steady state characterized by zero inflation and a level of aggregate production which is exclusively determined by supply-side-components. This leaves an active role for monetary and fiscal policy only outside the steady state which then is restricted to stabilizing the adjustment process. Until recently DSGE

model-builders were almost exclusively concerned about an active role of monetary policy with due consequences for the value of the parameters ρ_π and ρ_y (Goodfriend, King 1997, Clarida, Galí, Gertler 1999, Walsh, 2003, Galí 2008), whereas fiscal policy has only gradually received attention (Linnemann and Schabert 2003, Coenen and Straub 2004, Galí, Salido, Vallés, Coenen and Straub 2005, Galí, Monacelli 2008). In the baseline model the role of fiscal policy is indeed limited to changing expenditures. By contrast variations of income taxes apparently are without any effect since they do not affect the Euler equation. Apart from these discrepancies between a standard IS-LM-model and the DSGE world, both models share the property that fiscal deficits apparently play no role. However, the reasons for this result is different in both models. In a traditional IS-LM model it is assumed that in the examined short run budget deficits do not change the supply of bonds thus leaving the interest rate unaltered. As will be explained below, in a DSGE model, fiscal deficits affect the supply for government bonds in each period and the market for government bonds is related to other markets via the aggregate budget constraint. Usually the ineffectiveness of fiscal deficits as well as income tax variations are explained by the Ricardian Equivalence. This argument, however, is flawed since any proof of the Ricardian Equivalence requires to take the intertemporal budget constraint into account which has usually been ignored so far.

3 Critical Reflections

In the following we examine critically the following properties of the DSGE approach: First we investigate the role of the Taylor rule as a substitute for the LM-curve. Second, we analyse whether the aggregate Euler equation can really be interpreted as a version of the Keynesian IS-curve. Related to this we will also discuss the role of Ricardian Equivalence.

3.1 The Taylor-Rule as a Substitute for the LM-Curve

Usually DSGE models assume that the central bank is able to determine the nominal interest rate. However, it is left open, how this is achieved. In a market economy interest rates are determined in financial markets by the interplay of demand and supply. Hence a complete description of how the Taylor rule affects the interest rate makes it necessary to model the impact of monetary policy on both the bonds market and the supply of money. That this is also a necessary task with respect to the aggregate equilibrium in the economy, becomes evident from the aggregate budget constraint of our

baseline DSGE model:

$$(N_t^d - N_t^s) W_t + (Y_t - C_t - G_t) P_t = (B_t^{nd} - B_t^{ns}) + M_t^{nd} - M_{t-1}^n$$

where B_t^{nd} represents the demand for government bonds by private households, B_t^{ns} the supply of bonds by the government and $M_t^{nd} - M_{t-1}^n$ household hoardings which in the absence of a money supply function represent disequilibrium in the market for money. Observe that equilibrium in the labour and goods market are still compatible with disequilibrium both in the bonds and money market. Without a Taylor-rule variations of the interest rate would clear the bonds market thus also clearing the money market which here would require that households reduce their hoardings to zero. If the central bank fixes the interest rate according to the Taylor-rule this could imply rationing in the bonds market which then would have due consequences for the aggregate goods market. In particular, taking the Euler equation into account it is by no means clear whether and how in this process hoardings reduce to zero. Hence the only way to reconcile the Taylor-rule with equilibrium in the financial sector obtained by suitable adjustments of the interest rate, is to assume that the central bank purchases government bonds such that the target interest rate is realized. Central bank purchases of bonds then are financed by the printing of money, implying that

$$M_t^{ns} - M_{t-1}^n = B_t^{nz} - B_{t-1}^{nz} \quad (41)$$

where B_t^{nz} represents government bonds in the hands of the central bank. In the case of monetary targeting, the central bank will fix the supply of money. If instead a Taylor rule is applied, then the central bank fixes the interest rate and will accommodate its demand for bonds and the supply of money appropriately. The aggregate budget constraint then changes to

$$(N_t^d - N_t^s) W_t + (Y_t - C_t - G_t) P_t = (B_t^n + B_t^{nz} - B_t^{ns}) + (M_t^{nd} - M_t^{ns}) \quad (42)$$

If the goods and the labour market are always in equilibrium, then again variations of the interest rate suffice to equilibrate both the bonds and the money market, where now the central bank will adjust its purchases of bonds such that the target interest rate will be realized. Observe that this strategy is compatible with aggregate equilibrium only if the households do not only hold cash balances for mere transaction purposes thus ending up with zero money holdings at the end of the period. Rather, money has to serve a store-of-value function implying that cash balances are carried over into future periods. Taking this into account, Walras' Law allows us to treat either the bond or the money market as redundant. In a log-linearized version, equilibrium

in the money market requires that percentage deviations of money supply and demand from their steady state values coincide, which is represented by equation (43).

$$\hat{m}^s = \hat{m}_t = \frac{\sigma}{\mu} \hat{c}_t - \frac{(1 - \bar{i})}{\mu} \hat{i}_t \quad (43)$$

Substituting $\hat{i}_t$ by the Taylor-rule (19) equation (43) then specifies the percentage deviation of the money supply necessary to realize the target nominal interest rate. To summarize, for reasons of model consistency the Taylor rule always has to be combined with the assumption that money serves as a store of value. This qualification obviously has not always been taken into account (Woodford 2003 as an example).

3.2 A new Keynesian IS-Curve?

In the following we show that equation (40) does not represent the equality of aggregate investment and aggregate saving. Indeed equation (40) has been derived from the Euler equation which generally explains “...the dynamics of marginal utility in any two successive periods.” (Bagliano and Bertola, 2007, p. 4) and which can be reformulated to express the *ratio of present and next period consumption* – or the growth rate of consumption in a continuous time version of the model – as a function of the real interest rate. Assuming equilibrium in the goods market, the aggregate Euler equation in a model without private investment then represents a relationship between the current and next period difference between the equilibrium value of aggregate income and public expenditures as a function of the real interest rate – or the growth rate of the difference between equilibrium income and public expenditures in a continuous time version of the model. This is not the same as what is generally understood by the IS-curve which represents the equality of investment and/or fiscal deficits and private saving. A dynamic version of the IS-curve hence would link the current $I = S$ -equilibrium to its future values. Having a closer look at the log-linearized version (40), which transforms ratios into differences, the aggregate Euler equation states that the expected dynamics of percentage deviations of aggregate income from the steady state is a function of the expected dynamics of percentage deviations of public expenditures, and percentage deviations of the real interest rate from their steady state values. Log-linearization hence facilitates the description of the dynamic properties but it does of course not change the character of the model. However, it cannot be ruled out that it may have promoted interpretations according to which the Euler equation and the IS-curve are the same. Examining (40) further, we also observe that

variations in the lump-sum tax do not affect the so-called new Keynesian IS-curve whereas a shift of its traditional counterpart would always follow. This, too, can be explained by the property of the so-called New Keynesian IS-curve to represent the optimal ratio of current and future consumption which of course should not be affected by variations in the lump-sum tax. Stated differently, (40) represents the growth rate of optimal consumption, given that the goods market is always in equilibrium, and evidently this growth rate cannot be influenced by tax policies which leave the first-order conditions unaffected. This also casts some doubts on whether the observed ineffectiveness of variations in a lump-sum tax can in fact be explained by the Ricardian Equivalence which says that households leave their consumption expenditures unchanged if the government changes the financing structure of given expenditures. This result is commonly explained with the dependence of household consumption expenditures on the discounted value of lifetime income which remains unaffected by variations in the lump-sum tax. As has been shown above, this dependence does not play a role in a baseline DSGE model because a complete solution of the household optimization problem is missing. Consequences of this ignorance are in particular critical if we give up assumptions which ensure the Ricardian Equivalence. A prominent example in this respect are differences between the private and public discount factor. In case of a complete solution of the household optimization problem, we would of course obtain the result that now households change their consumption expenditures following a variation of the lump-sum tax. In a DSGE model, however, this would not be possible.

3.3 Implications of the Representative Agent for the Derivation of the IS-Curve

Of course we could always respond to this criticism by computing a complete solution for the household optimization problem and using the result to derive a macroeconomic consumption or equivalently, saving function, which would then allow us to derive the IS-curve. In the following we show that this last step does not make sense as long as we stick to the representative agent. Since our argument does neither depend on the type of the utility function nor on further assumptions concerning the behaviour of prices, we build our proof on a greatly simplified model. We assume utility to be logarithmic, we ignore household decisions on optimal labour, and we ignore uncertainty and inflation. These assumptions allow us to derive a closed-form explicit solution, without having to draw on log-linearization procedures.

The representative household has to solve the following optimization

problem:

$$\begin{aligned} \max U &= \sum_{i=0}^{\infty} \beta^i (\ln C_{t+i} + \gamma \ln M_{t+i}) \\ \gamma &> 0 \end{aligned} \quad (44)$$

subject to

$$C_{t+i} + B_{t+i} + M_{t+i} = Y_{t+i} - T_{t+i} + B_{t+i-1} (1 + i_{t+i-1}) + M_{t+i-1} \quad (45)$$

$$\lim_{T \rightarrow \infty} \frac{B_{t+T}}{\prod_{i=1}^T (1 + i_{t+i})} \geq 0 \quad (46)$$

(45) and (46) can be summarized to yield the intertemporal budget constraint, where we have ignored exogenously given initial stocks of bonds and cash balances:

$$V_t = C_t + \frac{C_{t+1}}{(1 + i_t)} + \frac{C_{t+2}}{(1 + i_t)(1 + i_{t+1})} + \dots + \frac{C_{t+T}}{\prod_{i=1}^T (1 + i_{t+i})} \quad (47)$$

where V_t is defined as follows:

$$\begin{aligned} V_t \equiv & (Y_t - T_t) + \frac{Y_{t+1} - T_{t+1}}{(1 + i_t)} + \frac{Y_{t+2} - T_{t+2}}{(1 + i_t)(1 + i_{t+1})} + \\ & \frac{Y_{t+3} - T_{t+3}}{(1 + i_t)(1 + i_{t+1})(1 + i_{t+2})} + \dots - M_t \left(\frac{i_t}{1 + i_t} \right) - \\ & \frac{M_{t+1}}{(1 + i_t)} \left(\frac{i_{t+1}}{1 + i_{t+1}} \right) - \frac{M_{t+2}}{(1 + i_t)(1 + i_{t+1})} \left(\frac{i_{t+2}}{1 + i_{t+2}} \right) - \dots \end{aligned} \quad (48)$$

Substituting the optimality conditions

$$C_{t+1} = \left(\frac{1}{\beta(1 + i_t)} \right) C_t \quad (49)$$

and

$$M_t = \gamma \left(\frac{1 + i_t}{i_t} \right) C_t \quad (50)$$

into equation (48), we obtain as optimal consumption

$$C_t = \frac{1 - \beta}{1 + \gamma} V_t \equiv \phi \tilde{V}_t \quad (51)$$

$$\begin{aligned} \tilde{V}_t = & (Y_t - T_t) + \frac{Y_{t+1} - T_{t+1}}{(1 + i_t)} + \frac{Y_{t+2} - T_{t+2}}{(1 + i_t)(1 + i_{t+1})} \\ & + \frac{Y_{t+3} - T_{t+3}}{(1 + i_t)(1 + i_{t+1})(1 + i_{t+2})} + \dots \end{aligned} \quad (52)$$

and optimal cash holdings

$$M_t = \left(\frac{\gamma}{1 + \gamma} \right) \left(\frac{1 + i_t}{i_t} \right) (1 - \beta) \tilde{V}_t \quad (53)$$

Equilibrium between aggregate production and demand then implies

$$Y_t = \phi \tilde{V}_t + G_t \quad (54)$$

with $\tilde{V}_t$ representing the discounted value of all future net incomes. Equation (54) represents the equality of S and I and hence the IS-curve of the model. It is dynamic in the sense that it contains current and all future values of aggregate income. However, the crucial point is that contrary to a-temporal models equation (54) does not represent the aggregate goods market for all $t + i$. Differently put, next period production cannot be obtained by just leading (54) one period further. Rather, due to the Euler equation, next period equilibrium is represented by

$$Y_{t+1} = \beta (1 + i_t) \phi \tilde{V}_t + G_{t+1} \quad (55)$$

The Euler equation thus implies that for each period we obtain a different consumption function and hence a different equation for equilibrium between aggregate production and aggregate demand. Assuming that demand and supply are equal in each period, we obtain a sequence of aggregate goods market equilibria according to

$$Y_t = \phi \tilde{V}_t + G_t \quad (56)$$

$$Y_{t+1} = \beta (1 + i_t) \phi \tilde{V}_t + G_{t+1} \quad (57)$$

$$Y_{t+1} = \beta^2 (1 + i_t) (1 + i_{t+1}) \phi \tilde{V}_t + G_{t+2} \quad (58)$$

$$\dots \quad (59)$$

Equation (56) represents the equality of saving and investment in the current period which contrary to its a-temporal counterpart now delivers a functional relationship between a sequence of incomes $\{Y_t, Y_{t+1}, Y_{t+2}, \dots, Y_{t+T}\}$ and a sequence of interest rates $\{i_t, i_{t+1}, i_{t+2}, \dots, i_{t+T}\}$. Using vectors, this can be expressed as

$$\mathbf{Y} = f_t(\mathbf{i}) \quad (60)$$

with

$$\mathbf{Y} = (Y_t, Y_{t+1}, Y_{t+2}, \dots, Y_{t+T})$$

$$\mathbf{i} = (i_t, i_{t+1}, i_{t+2}, \dots, i_{t+T})$$

Note that the assumption of infinitely lived households implies that $T \rightarrow \infty$. Hence in order to study the steady state equilibrium properties of the model one would have to examine whether both sequences converge towards some fixed value. The most important point, however, is that the functional relationship between incomes and interest rates of equation (60) is only valid for the current period t . Due to the Euler equation, we obtain for $t + 1$:

$$\mathbf{Y} = f_{t+1}(\mathbf{i})$$

or generally for period $t + i$:

$$\mathbf{Y} = f_{t+i}(\mathbf{i}), \quad i \in [0, \infty) \quad (61)$$

with

$$\begin{aligned} f_t &\neq f_{t+1} \neq \dots f_{t+i} \neq \dots f_{t+T}, \text{ if} \\ \beta(1 + i_t) &\neq \beta(1 + i_{t+1}) \neq \dots \neq \beta(1 + i_{t+T}) \neq 1 \end{aligned}$$

which characterizes situations outside the steady state. Since, however, outside the steady state the interest rate undergoes changes over time, each $f_{t+i}, i > 1$ cannot be treated as a constant multiple of f_t . Our findings therefore indicate that by assuming a representative household, we obtain an infinite number of different IS-curves each pertaining to a particular period and each establishing a relationship between an infinite sequence of incomes and interest rates. Of course this only holds outside the steady state but DSGE models like all Keynesian models have a focus on situations outside the steady state thus accounting for the evidence that market economies do not naturally converge quickly to a long-run equilibrium but that some policy interventions might be necessary. A multiplicity of IS-curves, however, poses serious problems to both the analysis of the time path of incomes and interest rates as well as for policy responses.

We also observe, however, that each two subsequent IS-equations are related by the Euler equation which explains why in a representative agent context the dynamics of the system is best described by the first-order conditions of the household optimization problem together with the Phillips curve. As has been indicated by the continuous-time versions of RBC models, the central role played by the Euler equation means that in these models the focus should then naturally been laid on analysing the *growth rate* of consumption. Of course by assuming some initial value it is always possible to calculate subsequent absolute values of consumption over time. However, this does not enable us to interpret the Euler equation as the IS-curve. It just tells us that given an initial value of consumption, then next period

consumption can be determined according to a rule which is given by the representative household's optimality conditions.

In the following we replace the representative agent model by heterogeneous agent approaches, taking overlapping generations as an example. We show that this assumption enables us to derive the IS-curve.

4 Derivation of the IS-curve in an OLG-Setting

Benassy (2007), too, uses an OLG-framework in order to overcome implausible results of the baseline DSGE model. In this respect he, too, uses a full-fledged solution of the household optimization problem. His focus is on the implied ineffectiveness of tax policy which he states to have overcome by taking advantage of the wealth effect. However, taking intertemporal optimization seriously implies that decisions on wealth depend on interest rates and other relative prices and therefore a wealth effect should be restricted to shocks and hence unexpected policy measures which have not been considered during the process of optimization. We therefore choose an alternative approach which explains consumption as well as the demand for financial assets and the supply of labour as exclusively determined by relative prices and exogenously determined income components. Our analysis basically serves the purpose to derive a dynamic (and expectational) IS-curve which holds for all periods and which represents an equilibrium in the aggregate goods market in the sense that any demand gap due to household savings is filled by income-independent components of aggregate demand. We start our analysis with a simple case in which information is perfect and inflation is absent. Together with the assumption of logarithmic utility, this will allow us to derive explicit functions for aggregate consumption, money demand and labour supply. We then generalize our approach by assuming a non-explicit CRRA utility function and by accounting for inflation and uncertainty. This generalization allows us to compare the thus derived IS-curve with the aggregate Euler equation derived in Section 2.

4.1 A Simple Case

We maintain the assumptions of the model used in Section 3.3 with the exception that we now consider an economy which in each period is inhabited by a young and an old generation. Each generation lives two periods. The young generation earns an income by supplying labour and receiving profits, but for simplicity we will not make the decision on the optimal supply of labour explicit. The young generation saves part of its available income and

thus accumulates wealth which is used for consumption in the old age. We ignore bequests and consequently assume that the young generation does not start with some initial wealth.

When young each individual maximizes its lifetime utility

$$U = \ln C_t^j + \gamma \ln M^j + \beta \ln C_{t+1}^o \rightarrow \max \quad (62)$$

subject to the period budget constraints

$$C_t^j + B_t^j + M^j = Y_t - T_t^j \quad (63)$$

$$C_{t+1}^o = B^j (1 + i_t) + M^j - T_{t+1}^o \quad (64)$$

where C^j (C_{t+1}^o) represents consumption when the generation is young (old). As first-order conditions we obtain the Euler equation

$$\beta (1 + i_t) C_t^j = C_{t+1}^o \quad (65)$$

and the optimal relationship between consumption and cash holdings:

$$M_t^j = \gamma \left(\frac{1 + i_t}{i_t} \right) C_t^j \quad (66)$$

Inserting (65) and (66) into the intertemporal budget constraint

$$Y_t - T_t^j - \frac{T_{t+1}^o}{1 + i_t} = M_t^j \left(\frac{i_t}{1 + i_t} \right) + C_t^j + \frac{C_{t+1}^o}{1 + i_t} \quad (67)$$

we obtain

$$C_t^j = \frac{1}{(1 + \gamma + \beta)} \left(Y_t - T_t^j - \frac{T_{t+1}^o}{1 + i_t} \right) \quad (68)$$

for consumption when young and

$$C_{t+1}^o = \frac{\beta (1 + i_t)}{(1 + \gamma + \beta)} \left(Y_t - T_t^j - \frac{T_{t+1}^o}{1 + i_t} \right) \quad (69)$$

for consumption when old. In each period aggregate consumption is the sum of consumption of the young as well as of the old generation, delivering for the current period t ,

$$C_t = C_t^j + C_t^o = \frac{\left(Y_t - T_t^j - \frac{T_{t+1}^o}{1 + i_t} \right)}{(1 + \gamma + \beta)} + \frac{\beta (1 + i_{t-1}) \left(Y_{t-1} - T_{t-1}^j - \frac{T_t^o}{1 + i_{t-1}} \right)}{(1 + \gamma + \beta)} \quad (70)$$

and for $t + 1$

$$C_{t+1} = \frac{\left(Y_{t+1} - T_{t+1}^j - \frac{T_{t+2}^o}{1+r_{t+1}}\right)}{(1 + \beta + \gamma)} + \frac{\beta(1 + i_t) \left(Y_t - T_t^j - \frac{T_{t+1}^o}{1+i_t}\right)}{(1 + \beta + \gamma)} \quad (71)$$

Importantly, now we obtain future consumption by just leading (70) one period further, and we observe that contrary to the representative agent model, (70) delivers a complete description of the time path of aggregate consumption for all t . Likewise we obtain for the equilibrium between aggregate demand and production in t and $t + 1$,

$$Y_t = \frac{\left(Y_t - T_t^j - \frac{T_{t+1}^o}{1+i_t}\right)}{(1 + \gamma + \beta)} + \frac{\beta(1 + i_{t-1}) \left(Y_{t-1} - T_{t-1}^j - \frac{T_t^o}{1+i_{t-1}}\right)}{(1 + \gamma + \beta)} + G_t \quad (72)$$

$$Y_{t+1} = \frac{\left(Y_{t+1} - T_{t+1}^j - \frac{T_{t+2}^o}{1+i_{t+1}}\right)}{(1 + \gamma + \beta)} + \frac{\beta(1 + i_t) \left(Y_t - T_t^j - \frac{T_{t+1}^o}{1+i_t}\right)}{(1 + \gamma + \beta)} + G_{t+1} \quad (73)$$

Note that again (73) results from leading (72) one period further implying that (72) provides a full description of current production and its time path as well. More importantly, as is the case for the traditional IS-curve, equation (72) represents equilibrium between aggregate production and demand as a relationship between alternative levels of production and the interest rate. Note, however, that this relationship is dynamic since the past and current level of production as well as the interest rate are involved. Since we have assumed that each generation lives for two periods only and that old households do not earn an income from supplying labour or holding shares, the IS-curve is backward looking with respect to aggregate production. If by contrast for example each generation lives three periods, the resulting IS-curve will also contain a forward-looking element. We observe furthermore that contrary to the traditional version of the IS-curve it is no longer ensured that the resulting correlation between aggregate production and the interest rate is negative. If investment is ignored, the slope of the IS-curve depends on the shape of household utility. In the case of logarithmic utility, a positive correlation between production and the current interest rate will follow because each increase in the interest rate reduces the present value of future tax obligations thus leading to a higher level of lifetime income which allows the young generation to consume more in the present period. A positive correlation between aggregate current production and the interest rate also follows for the previous period interest rate where now in addition to the income effect, the consumption-tilting motive allows the old generation to consume more. As a further result we observe that we are now able to

examine the impact of variations of income taxes which implies that we are now able to test for the validity of the Ricardian Equivalence.

4.2 A More General Case

In order to allow a comparison with the baseline DSGE-model we now consider a non-explicit CRRA utility function, assume inflation and rational expectations. Furthermore we model the young generation's decision on its supply of labour explicitly. For simplicity we restrict uncertainty to the time path of the future price level and thus inflation which also implies that bonds continue to yield a safe nominal return and that households have perfect information with respect to the amount of taxes which they have to pay in their old age. We start our analysis by deriving the first-order conditions which are basically the same as in Section 2. We then derive a full-fledged solution to the optimization problem by log-linearizing the first-order conditions and by linearizing the intertemporal household budget constraint around their steady state values. This will allow us to obtain percentage deviations of aggregate consumption around its steady state value.

Derivation of First-Order-Conditions The representative young household maximizes the following CRRA utility function:

$$\max V = \left[u(C_t^j) + v\left(\frac{M_t^{nj}}{P_t}\right) - \eta(N_t^j) \right] + \beta E_t[u(C_{t+1}^o)] \quad (74)$$

subject to the budget constraints which we now express in real terms:

$$C_t^j = N_t^j W_t + \Omega_t - T_t^j - \frac{M_t^{nj}}{P_t} - \frac{B_t^{nj}}{P_t} \quad (75)$$

$$C_{t+1}^o = \frac{M_t^{nj}}{P_{t+1}} + \frac{B_t^{nj}}{P_{t+1}}(1 + i_t) - T_{t+1}^o \quad (76)$$

$$\iff C_{t+1}^o = \frac{M_t^j}{1 + \pi_{t+1}} + \frac{B_t^j}{1 + \pi_{t+1}}(1 + i_t) - T_{t+1}^o \quad (77)$$

where Ω_t again denote real firm profits which we assume to incur exclusively to young households. C_{t+1}^o , P_{t+1} and π_{t+1} represent random variables. B_t^n (B_t) stands again for nominal (real) bond holdings and M_t^n (M_t) denotes nominal (real) cash holdings.

Taking expectations of (77) we obtain

$$E_t C_{t+1}^o = \frac{M_t^j}{1 + E_t \pi_{t+1}} + \frac{B_t^j}{1 + E_t \pi_{t+1}}(1 + i_t) - T_{t+1}^o \quad (78)$$

Substituting (75) into (78) and recalling that

$$N_t^j W_t + \Omega_t = Y_t \quad (79)$$

we obtain the intertemporal budget constraint

$$(Y_t - T_t^j - C_t^j) \frac{(1 + i_t)}{1 + E_t \pi_{t+1}} - \frac{M_t^j i_t}{1 + E_t \pi_{t+1}} = E_t C_{t+1}^o \quad (80)$$

The approach yields the same first-order conditions as in Section 2:

$$u'(C_t^j) W_t = \eta'(N_t^j) \quad (81)$$

$$u'(C_t^j) \frac{1}{P_t} = v'(M_t^j) \frac{1}{P_t} + \beta E_t \left[u'(C_{t+1}^o) \frac{1}{P_{t+1}} \right] \quad (82)$$

$$u'(C_t^j) \frac{1}{P_t} = \beta E_t \left[u'(C_{t+1}^o) \frac{1 + i_t}{P_{t+1}} \right] \quad (83)$$

(82) and (83) imply

$$\frac{i_t}{1 + i_t} u'(C_t^j) = v'(M_t^j) \quad (84)$$

Log-Linearization and Linearization around Steady State Values

Note that in the steady state, too, the young generation saves for its old age and consumes the entire accumulated savings in the second period of its life. The steady state is characterized by the following properties

1. $\beta(1 + \bar{i}) = 1 \Rightarrow C_t^j = C_{t+1}^o = \bar{C}$
2. $\pi_t = \pi_{t+1} = \bar{\pi} = E_{t-1} \pi_t = E_t \pi_{t+1} = 0$
3. $Y_t = Y_{t+1} = \bar{Y}$
4. $\bar{Y} - \bar{T}^j - \bar{C} = \bar{M}^j + \bar{B}^j > 0$

Log-linearization of (81), (83) and (84) yield as percentage deviations from the steady state:

$$\hat{n}_t = -\frac{\sigma}{\psi} \hat{c}_t^j + \frac{1}{\psi} \hat{w}_t \quad (85)$$

$$E_t \hat{c}_{t+1}^o = \hat{c}_t^j + \frac{1}{\sigma} \left(\hat{i}_t - E_t \pi_{t+1} \right) \quad (86)$$

$$\hat{m}_t^j = \frac{\sigma}{\mu} \hat{c}_t^j - \frac{1 - \bar{i}}{\mu} \hat{i}_t \quad (87)$$

Next we linearize the intertemporal budget constraint around the steady state:

$$\begin{aligned} \overline{C} E_t \hat{c}_{t+1}^o &= \frac{1 + \bar{i}}{1 + \bar{\pi}} \left(\bar{Y} \hat{y}_t - \overline{C} \hat{c}_t^j - \overline{T}_t^j \hat{t}_t^j \right) + \\ &\quad \left(\frac{\overline{M}^j + \overline{B}^j}{1 + \bar{\pi}} \right) \hat{i}_t - \left(\frac{\overline{M}^j + \overline{B}^j}{(1 + \bar{\pi})^2} \right) (1 + \bar{i}) E_t \hat{\pi}_{t+1} - \\ &\quad \frac{\overline{M}^j \bar{i}}{1 + \bar{\pi}} \hat{m}_t^j - \frac{\overline{M}^j}{1 + \bar{\pi}} \hat{i}_t + \frac{\overline{M}^j \bar{i}}{(1 + \bar{\pi})^2} E_t \hat{\pi}_{t+1} - \overline{T}^o \hat{t}_{t+1}^o \end{aligned} \quad (88)$$

where we have substituted $\overline{N}^j \overline{W} + \overline{\Omega} - \overline{T}^j - \overline{C}$ by $\overline{M}^j + \overline{B}^j$. Setting $\bar{\pi} = 0$ and rearranging terms, we finally obtain

$$\begin{aligned} \overline{C} E_t \hat{c}_{t+1}^o &= (1 + \bar{i}) \left(\bar{Y} \hat{y}_t - \overline{C} \hat{c}_t^j - \overline{T}_t^j \hat{t}_t^j \right) + \\ &\quad \overline{B}^j \hat{i}_t - \left(\overline{M}^j + \overline{B}^j (1 + \bar{i}) \right) E_t \pi_{t+1} - \overline{M}^j \bar{i} \hat{m}_t^j - \overline{T}^o \hat{t}_{t+1}^o \end{aligned} \quad (89)$$

Substituting the first-order conditions into (89), we obtain

$$\begin{aligned} \overline{C} \left(\hat{c}_t^j + \frac{1}{\sigma} \left(\hat{i}_t - E_t \pi_{t+1} \right) \right) &= (1 + \bar{i}) \left(\bar{Y} \hat{y}_t - \overline{T}_t^j \hat{t}_t^j - \overline{C} \hat{c}_t^j \right) + \\ &\quad \overline{B}^j \hat{i}_t - \left(\overline{M}^j + \overline{B}^j (1 + \bar{i}) \right) E_t \pi_{t+1} - \overline{T}^o \hat{t}_{t+1}^o - \\ &\quad \overline{M}^j \bar{i} \frac{\sigma}{\mu} \hat{c}_t^j + \overline{M}^j \bar{i} \left(\frac{1 - \bar{i}}{\mu} \right) \hat{i}_t \end{aligned} \quad (90)$$

Rearranging terms, we obtain

$$\hat{c}_t^j = \frac{1}{\xi} \left[\bar{Y} (1 + \bar{i}) \hat{y}_t + \Theta_1 \hat{i}_t + \Theta_2 E_t \pi_{t+1} + \Theta_{3t} \right] \quad (91)$$

$$\begin{aligned} \xi &= \overline{C} (2 + \bar{i}) + \overline{M}^j \bar{i} \frac{\sigma}{\mu} > 0 \\ \Theta_1 &= \overline{B}^j + \overline{M}^j \bar{i} \left(\frac{1 - \bar{i}}{\mu} \right) - \overline{C} \frac{1}{\sigma} \geq 0 \\ \Theta_2 &= - \left(\overline{M}^j + \overline{B}^j (1 + \bar{i}) \right) + \overline{C} \frac{1}{\sigma} \geq 0 \\ \Theta_{3t} &= - \left(\overline{T}_t^j \hat{t}_t^j (1 + \bar{i}) + \overline{T}^o \hat{t}_{t+1}^o \right) < 0 \end{aligned}$$

Θ_1 describes the impact of the nominal interest rate and Θ_2 the impact of expected inflation on percentage deviations of young households' consumption from the steady state. Obviously both terms are not the same which

implies that $\hat{c}_t^j$ responds differently to changes in the nominal interest rate as compared to changes in expected inflation. This result can be explained by the store-of-value function of money which affects the intertemporal budget constraint through the opportunity cost of cash holdings $\frac{M_t^j i_t}{1+E\pi_{t+1}}$. The opportunity cost of cash holdings obviously react differently to variations of expected inflation and the nominal interest rate. This is even the case if we hold real balances constant. Recall that in the baseline DSGE model it is the real and not the nominal interest rate which determines consumption since here consumption is directly derived from the Euler equation thus only the relationship between present and future consumption is considered without taking cash holdings into account. As a further difference compared to the baseline DSGE model we observe that in our OLG approach it is by no means certain that an increase in the (nominal) interest rate reduces consumption. Recall that in the absence of inflation and under the assumption of logarithmic utility we even obtained an unambiguously positive impact of interest rate changes on consumption. Now we have to consider several opposing effects. As we see from Θ_1 , both an income effect and the effect of changing interest rates on the opportunity cost of cash holdings establish a positive correlation whereas a negative correlation follows from the substitution effect. The fall in opportunity cost of cash holdings can be explained by the negative correlation between nominal interest rates and optimal money holdings. In order for the substitution effect as specified by the term $\bar{C}^{\frac{1}{\sigma}}$ to outweigh the income effect, the intertemporal elasticity of substitution $\frac{1}{\sigma}$, has to satisfy the condition

$$\frac{1}{\sigma} > \frac{\bar{B}^j}{\bar{C}} + \frac{\bar{M}^j \bar{i} (1 - \bar{i})}{\bar{C} \mu} \quad (\text{i})$$

On a qualitative level, the same argument applies to the impact of expected inflation on young households' consumption but as we have already emphasized, quantitative effects are different. A higher rate of expected inflation leads to an increase in younger households' consumption only if the intertemporal elasticity of substitution meets the following condition

$$\frac{1}{\sigma} > \frac{\bar{M}^j}{\bar{C}} + \frac{\bar{B}^j}{\bar{C}} (1 + \bar{i})^{\parallel} \quad (\text{ii})$$

Finally we see from Θ_{3t} that now taxes have a direct impact on deviations of young households' consumption from the steady state, provided that taxes, too deviate from their steady state values.

^{||}Observe that condition (i) und (ii) only coincide in the special case that $\bar{B}^j = -\bar{M}^j [1 - \bar{i} (1 - \bar{i}\mu)]$ implying that only in this special case the effect of nominal interest rates and expected inflation on consumption coincide.

Aggregate Consumption Percentage deviations of aggregate consumption from the steady state are given by

$$\hat{c}_t = \hat{c}_t^o + \hat{c}_t^j \quad (92)$$

In order to calculate $\hat{c}_t^o$, we first linearize (78) now relating to the previous period, around the steady state thus obtaining

$$\begin{aligned} \overline{C} E_{t-1} \hat{c}_t^o &= \overline{B}^j (1 + \bar{i}) \hat{b}_{t-1}^j + \overline{B}^j \hat{i}_{t-1} - \overline{B}^j (1 + \bar{i}) E_{t-1} \pi_t - \\ &\quad \overline{M}^j \hat{m}_{t-1}^j - \overline{M}^j E_{t-1} \pi_t - \overline{T}^o \hat{t}_t^o \end{aligned} \quad (93)$$

Expected old age consumption and realized old age consumption differ whenever expected and realized inflation fall apart. Accordingly we obtain for realized old age consumption in t :

$$\begin{aligned} \overline{C} \hat{c}_t^o &= \overline{B}^j (1 + \bar{i}) \hat{b}_{t-1}^j + \overline{B}^j \hat{i}_{t-1} - \overline{B}^j (1 + \bar{i}) \pi_t + \\ &\quad \overline{M}^j \hat{m}_{t-1}^j - \overline{M}^j \pi_t - \overline{T}^o \hat{t}_t^o \end{aligned} \quad (94)$$

Substituting (93) into (94) yields

$$\overline{C} \hat{c}_t^o = \overline{C} E_{t-1} \hat{c}_t^o - \left(\overline{B}^j (1 + \bar{i}) + \overline{M}^j \right) (\pi_t - E_{t-1} \pi_t) \quad (95)$$

After inserting (86) into (95) we get

$$\hat{c}_t^o = \hat{c}_{t-1}^j + \frac{1}{\sigma} \left(\hat{i}_{t-1} - E_{t-1} \pi_t \right) - \left(\overline{B}^j (1 + \bar{i}) + \overline{M}^j \right) (\pi_t - E_{t-1} \pi_t) \quad (96)$$

with

$$\hat{c}_{t-1}^j = \frac{1}{\xi} \left[\overline{Y} (1 + \bar{i}) \hat{y}_{t-1} + \Theta_1 \hat{i}_{t-1} + \Theta_2 E_{t-1} \pi_t + \Theta_{3t-1} \right] \quad (97)$$

Given $\hat{c}_{t-1}^j$, current old age consumption is positively correlated with the previous period real interest rate and negatively with a surplus of realized over expected current inflation rate. The first effect results from the first-order condition (86). The second effect can be explained by the impact of second-period income on old-age consumption. If current inflation is higher than expected, then the old household incurs a loss in real income and will have to reduce his consumption accordingly. Taking (97) into account we observe that for the old generation, too, it will generally not be real but nominal interest rates which affect consumption. Furthermore we observe that whenever inflationary expectations prove to be false, the realized inflation rate, too influences old age consumption thus establishing a negative correlation.

We obtain as aggregate consumption:

$$\begin{aligned} \hat{c}_t = & \frac{\bar{Y}(1+\bar{i})}{\xi}(\hat{y}_t + \hat{y}_{t-1}) + \frac{\Theta_1}{\xi}\hat{i}_t + \frac{\Theta_1 + \frac{\xi}{\sigma}}{\xi}\hat{i}_{t-1} + \frac{\Theta_2}{\xi}E_t\pi_{t+1} + \\ & \frac{\Theta_2 - \frac{\xi}{\sigma}}{\xi}E_{t-1}\pi_t - \left(\bar{B}^j(1+\bar{i}) + \bar{M}^j\right)(\pi_t - E_{t-1}\pi_t) + \frac{\Theta_{3t} + \Theta_{3t-1}}{\xi} \end{aligned} \quad (98)$$

The IS-Curve Since we have ignored investment, the IS-curve in our simple model represents the equality of private savings and public dissavings which is equivalent to stating equilibrium in the aggregate goods market as specified by

$$\begin{aligned} \hat{y}_t = & \frac{\bar{C}(1+\bar{i})}{\Phi}\hat{y}_{t-1} + \left(\frac{\bar{C}}{\bar{Y}}\right)\left(\frac{\Theta_1}{\Phi}\right)\hat{i}_t + \left(\frac{\bar{C}}{\bar{Y}}\right)\left(\frac{\Theta_1 + \frac{\xi}{\sigma}}{\Phi}\right)\hat{i}_{t-1} + \\ & \left(\frac{\bar{C}}{\bar{Y}}\right)\left(\frac{\Theta_2}{\Phi}\right)E_t\pi_{t+1} + \left(\frac{\bar{C}}{\bar{Y}}\right)\left(\frac{\Theta_2 - \frac{\xi}{\sigma}}{\Phi}\right)E_{t-1}\pi_t - \\ & \left(\frac{\bar{C}}{\bar{Y}}\right)\left[\frac{\left(\bar{B}^j(1+\bar{i}) + \bar{M}^j\right)}{\Phi}(\pi_t - E_{t-1}\pi_t) - \frac{\Theta_{3t} + \Theta_{3t-1}}{\Phi}\right] + \\ & \left(\frac{\xi}{\Phi}\right)\left(\frac{\bar{G}}{\bar{Y}}\right)\hat{g}_t \end{aligned} \quad (99)$$

where

$$\Phi = \bar{C} + \bar{M}^j\bar{i}\frac{\sigma}{\mu}$$

In comparison with the baseline DSGE-IS-curve we now observe a richer dynamics of both aggregate consumption and aggregate production since now not only the current but also the previous period interest rate and the previous period expected inflation rate play a role. It is noteworthy that we thus obtain an explanation for inflation persistence which does not originate from deviations from rationality but from the assumption that consumption is determined by the rational behaviour of (two) overlapping generations. Finally we observe an impact of an inflationary expectation error concerning the old age generation. As we have already clarified as a further difference to the baseline DSGE model, now the time path of aggregate production is not determined by real but nominal interest rates, which follows as a due consequence of the store-of-value-function of money. Finally and importantly it is not only public expenditures but also tax policy which affects the dynamics of aggregate production. In particular we are now able to discuss the importance of Ricardian Equivalence in the true sense of the word.

5 Conclusions

In this article we have shown that the so-called new Keynesian IS-curve does in fact not represent the equivalence of saving and investment but relates current to next period consumption. We have furthermore made evident that the derivation of an IS curve in a new Keynesian framework of intertemporally maximizing households which holds for each period, requires the assumption of heterogeneous agents. Taking a simple overlapping generations model as an example we have derived an IS-curve which offers a richer dynamics than the standard DSGE case. It allows us furthermore to study Ricardian Equivalence because our behavioural functions provide a full-fledged solution to the household optimization problem taking the intertemporal budget constraint into account. We also have found differences concerning the relationship between aggregate consumption, interest rates and (expected) inflation. First, due to the store-of-value function of money it is not the real but the nominal interest rate which affects consumption. Second, a negative correlation between nominal interest rates and consumption is by no means clear. The reason for this result is twofold: A rising nominal interest rate reduces optimal money holdings and thus allows for a higher level of consumption. Furthermore since in an OLG-model the young generation continues to save in the steady state, the income effect of a rising interest rate does no longer cancel out. The same ambiguity holds for the impact of expected inflation whereas the realized level of current inflation unambiguously reduces aggregate consumption in case of an inflationary expectation error since the old generation experiences a lower real income than was expected. We think that these conclusions assign an innovative property to the new Keynesian IS-curve which should be further explored considering extensions of our simple model.

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