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Social Choice and Welfare

2. Exam Winter Term 2014/15

Important Instructions

1. There are 90 points on this 90 minutes exam.
2. You are not allowed to use any course material (books, slides, lecture notes etc.).
3. Please answer the questions only on the paper that is handed out to you.
4. Please write your name on each sheet of paper, number the pages and leave a margin (2.5cm) on the right of each page.
5. Please write legibly and make sure that your answers are coherent and complete.

Good Luck!

Problem 1 (14 Points)

The mayor of a city evaluates public construction projects according to two criteria: economic viability and sustainability. To this end, for each project and each criterion he first determines the score of the project on a scale from 1 (lowest) to 5 (highest). The mayor's preference relation over a set of projects then is follows: For any two projects A and B, the mayor weakly prefers A over B if the sum of the economic viability and sustainability scores of project A are at least as large as the sum of the economic viability and sustainability scores of project B.

1. Is the mayor's preference relation complete? Argue why or why not.

   (6 points)

2. Is the mayor's preference relation transitive? Argue why or why not.

   (8 points)

Problem 2 (12 Points)

Consider the set of alternatives $X = \{x, y, z\}$ and let C be a choice function on X with

$$C(\{x, y, z\}) = \{x, y\}.$$

Determine $C(\{x, y\})$, $C(\{x, z\})$ and $C(\{y, z\})$ for the case where C satisfies contraction and expansion consistency. In case you encounter several possibilities for how the choice sets could look like, write down all of them. Give a reason for your answer.

Problem 3 (36 Points)

Consider a society with two individuals who have weak preference orderings R_1 and R_2. For $i = 1, 2$, let P_i (I_i) be individual i's strict preference (indifference) relation corresponding to the weak preference ordering R_i.

The two individuals use the following rule to aggregate their individual preference orderings (R_1, R_2) into a social preference relation R: Let x and y be two alternatives. Then,

$$\begin{aligned} xRy \iff & \quad xP_1y \\ & \text{or} \quad xI_1y \text{ and } xR_2y. \end{aligned}$$

1. Is this aggregation rule an Arrovian social welfare function on an unrestricted domain of individual preference orderings if there are at least three alternatives? Argue why or why not.

   (18 Points)

2. For each of the conditions *Weak Pareto Principle*, *Independence of Irrelevant Alternatives* and *Non-Dictatorship* argue whether it is satisfied or not satisfied by this rule.

   (18 Points)

Problem 4 (28 Points)

Two ladies are dressing up for a ball. Lady 1 has a red (r), a blue (b) and a green (g) dress. Lady 2 has a red (r) and a blue (b) dress. Consider the alternatives rr, rb, br, bb, gr, gb, where the first (second) letter corresponds to lady 1's (lady 2's) dress color at the ball.

The ladies' strict preferences over the six alternatives are

$$rb\, P_1\, rr\, P_1\, bb\, P_1\, gb\, P_1\, br\, P_1\, gr \text{ for lady 1}$$
$$\text{and } bb\, P_2\, rr\, P_2\, rb\, P_2\, br\, P_2\, gb\, P_2\, gr \text{ for lady 2.}$$

1. For every lady determine whether she has conditional or unconditional preferences. Give a reason for your answer.

   (8 Points)

2. For the preferences given above and a social choice function, that satisfies Gibbard's libertarian claim GL', determine which alternatives will NOT be chosen from the set $S = \{rr, rb, br, bb, gr, gb\}$.

   (8 Points)

3. Represent the collective choice problem as a non-cooperative game and solve for the pure strategy Nash equilibrium (equilibria).

   (12 Points)