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Social Choice and Welfare

1. Exam Winter Term 2013/14

Important Instructions

1. There are 90 points on this 90 minutes exam.
2. You are not allowed to use any course material (books, slides, lecture notes etc.).
3. Please answer the questions only on the paper that is handed out to you.
4. Please write your name on each sheet of paper, number the pages and leave a margin (2.5cm) on the right of each page.
5. Please write legibly and make sure that your answers are coherent and complete.

Good Luck!

Problem 1**(28 Points)**

A university ranks applicants for a master's program in journalism according to two criteria: the applicant's grade point average (GPA) of the bachelor degree and the applicant's work experience as a journalist. Assume that the GPA is measured on a scale from 0 (lowest) to 5 (top) and that work experience is measured in months, where more months mean more work experience.

Let A and B be two applicants. Then the university weakly prefers A over B (ARB), if one of the following two conditions is satisfied:

- (i) A has a higher GPA than B .
- (ii) A and B have the same GPA and A 's work experience is at least as large as B 's work experience.

1. Is the university's weak preference relation over applicants complete? Give a reason for your answer.

(8 Points)

2. Is the university's weak preference relation over applicants transitive? Give a reason for your answer.

(14 Points)

3. Suppose there are three applicants, A , B , and C , whose GPA and work experience (in months) is given in the following table:

	A	B	C
GPA	3	4	3
work experience	24	0	12

Determine the university's preference (strict preference or indifference) over every pair of applicants.

(6 Points)

Problem 2**(34 Points)**

Let there be $m \geq 3$ alternatives and $n \geq 2$ voters. Suppose that every voter i has a strict preference ordering P_i over the m alternatives. Then consider the following social aggregation rule: Every voter casts a vote both for his first *and* second best alternative. Then, the social preference relation R is such that for every pair of alternatives (x, y) ,

$$xRy$$

if and only if the alternative x received at least as many votes as the alternative y . Observe that every voter's votes for his/her first and second best alternative are treated equally.

1. Is this aggregation rule a social welfare function? Give a reason for your answer.

(10 Points)

2. Which conditions in Arrow's impossibility theorem are satisfied by this rule and which are not satisfied? For each condition either argue why it is satisfied or provide an example where the condition is violated.

(24 Points)

Problem 3**(28 Points)**

Let there be two school boys. Boy 1 has a bike and can either go to school by bike (b) or walk (w). Boy 2 has a bike and a skateboard and hence can bike (b), take the skateboard (s) or walk (w) to school. Consider the alternatives ww, wb, ws, bw, bb, bs , where the first (second) letter specifies how the first (second) boy goes to school.

The boys' strict preferences over the six alternatives are

$$bb P_1 ww P_1 bw P_1 bs P_1 wb P_1 ws$$

$$bs P_2 ws P_2 bb P_2 wb P_2 ww P_2 bw.$$

1. For both boys determine whether he has conditional or unconditional preferences. Give a reason for your answers.

(8 Points)

2. For the preferences given above and a social choice function, that satisfies Gibbard's libertarian claim GL', determine which alternatives will NOT be chosen from the set $S = \{ww, wb, ws, bw, bb, bs\}$.

(8 Points)

3. Represent the collective choice problem as a non-cooperative game and solve for the pure strategy Nash equilibrium (equilibria).

(12 Points)