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Individual Decisions, Games and Markets

1. Exam Winter Term 2017/18

Important Instructions

1. There are 90 points on this 90 minutes exam.
2. You are not allowed to use any course material (books, slides, lecture notes etc.).
3. Please answer the questions only on the paper that is handed out to you.
4. Please write your name and matriculation number on each sheet of paper, number the pages and leave a margin (2.5cm) on the right of each page.
5. Please write legibly and make sure that your answers are coherent and complete.
6. At the end of the exam, please return the exam, your answers and all paper that was handed out to you, including any notes that you do not want to be evaluated (please mark).

Good Luck!

Problem 1 (30 Points)

Ann has a budget $m = 25$ which she can spend on the consumption of two goods. The price of good 1 is $p_1 = 1$ and the price of good 2 is $p_2 = 2$. Ann's utility is

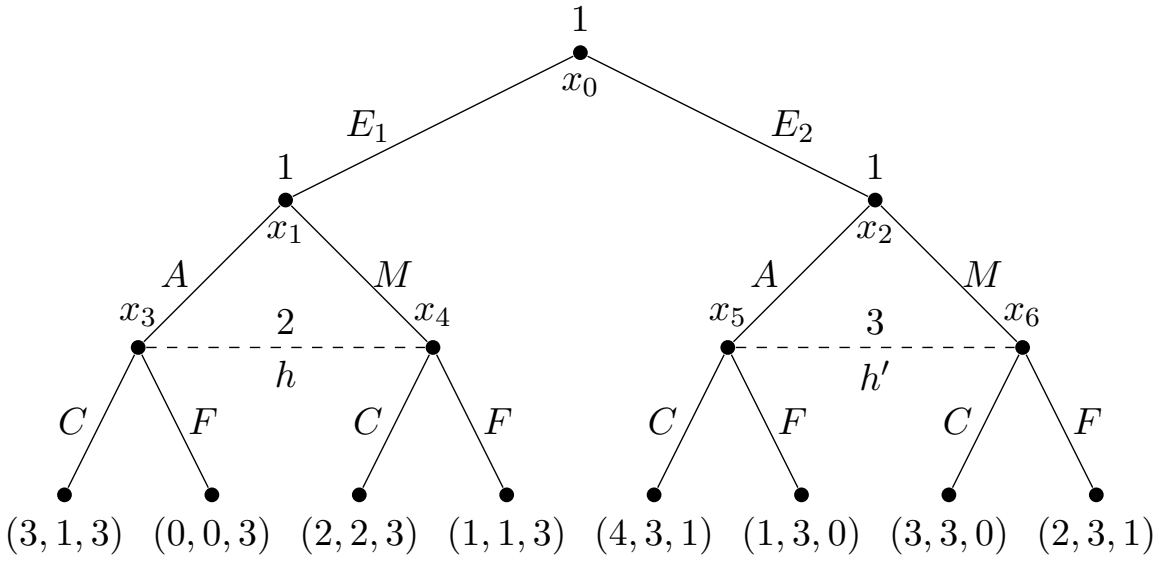
$$U(x_1, x_2) = \ln(x_1) + \ln(x_2)$$

when consuming $x_1 > 0$ units of good 1 and $x_2 > 0$ units of good 2. The two goods are rationed such that the total quantity $x_1 + x_2$ that Ann can buy is at most 10.

Use Kuhn-Tucker to solve for the quantities of both goods that maximize Ann's utility subject to the constraints that Ann's total expenditure on consumption must not exceed her budget and that the total quantity must not exceed 10. *Hint: You can assume that the first-order conditions are also sufficient for a solution.*

Problem 2 (30 Points)

Consider the market entry game in the following figure. Firm 1 can enter market 1 (E_1) or market 2 (E_2). If firm 1 enters market 1 it competes with the incumbent firm 2 and if it enters market 2 it competes with the incumbent firm 3. After market entry firm 1 chooses either an aggressive action (A) or a moderate action (M). The incumbent firms 2 and 3 observe the market entry, but do not observe the action firm 1 chooses after market entry. The incumbent firms choose whether to cooperate (C) or fight (F) after firm 1 has entered their market. The firms' profits are quoted at the terminal nodes in the following figure. Note that the first (second, third) number is firm 1's (2's, 3's) profit.



Determine a perfect Bayesian equilibrium, where firm 1 enters market 1 with probability 1.

Problem 3 (30 Points)

Consider a monopolist who produces $q \geq 0$ units of a good that he sells to consumers who differ with respect to their valuation of the good. The monopolist's cost for producing q units of the good is $\frac{q^2}{2}$. If the monopolist sells q units of the good at price p his profit is

$$\Pi(p, q) = p - \frac{q^2}{2}.$$

A consumer of type θ who buys q units of the good at price p has utility

$$U^\theta(p, q) = \theta q - p.$$

The consumer's type θ is an element of the interval $[1, 10]$. For all consumer types $\theta \in [1, 10]$ the reservation utility is $\underline{U} = 0$.

1. Assume that the monopolist observes the consumer's type. For all $\theta \in [1, 10]$ determine the profit maximizing quantity $q(\theta)$ and price $p(\theta)$ the monopolist offers to a consumer of type θ.

   (10 Points)

2. Assume now that the monopolist does not observe the consumer's type. He only knows that the type must be from the interval $[1, 10]$. Suppose the monopolist is naive. So he asks the consumer for her type and if the consumer reports to be type $\theta \in [1, 10]$, the monopolist offers the quantity $q(\theta)$ at price $p(\theta)$ you determined in question 1. If the consumer reports a type $\theta \notin [1, 10]$ the monopolist does not make any offer and there is no trade.

   For every type $\theta \in [1, 10]$ determine the report $\tilde{\theta}$ that maximizes the utility of a consumer of type θ.

   (20 Points)