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Individual Decisions, Games and Markets

1. Exam Winter Term 2016/17

Important Instructions

1. There are 90 points on this 90 minutes exam.
2. You are not allowed to use any course material (books, slides, lecture notes etc.).
3. Please answer the questions only on the paper that is handed out to you.
4. Please write your name and matriculation number on each sheet of paper, number the pages and leave a margin (2.5cm) on the right of each page.
5. Please write legibly and make sure that your answers are coherent and complete.

Good Luck!

Problem 1 (12 Points)

John is a risk averse expected utility maximizer. His von Neumann-Morgenstern utility function u is twice continuously differentiable and satisfies $u'(x) > 0$ and $u''(x) < 0$ for all $x > 0$. John has an initial wealth of $w > 0$ Euro and he faces the risk of losing L Euro in an accident, where $0 < L < w$. The probability of an accident is μ, where $0 < \mu < 1$.

An insurance company offers insurance against the loss in case of an accident. One unit of insurance costs p Euro and pays 1 Euro in case of an accident. Thus, if John buys z units of insurance, his final wealth is $w - pz$ if there is no accident and $w - pz - L + z$ if he suffers an accident.

Suppose $p = \mu$. Prove that John must buy L units of insurance in order to maximize his expected utility.

Problem 2 **(40 Points)**

Consider a market where two firms are producing the same good. The inverse demand function is given by

$$P(x) = \max\{80 - x, 0\},$$

where $x \geq 0$ is the total quantity produced by the two firms. Firm 1's cost of producing $x_1 \geq 0$ units of the good is

$$C_1(x_1) = \frac{1}{2}{x_1}^2,$$

and firm 2's cost of producing $x_2 \geq 0$ units of the good is

$$C_2(x_2) = {x_2}^2.$$

The two firms choose their quantities sequentially. Firm 1 is the leader and chooses its quantity x_1 first. Firm 2 observes x_1 and then chooses its quantity x_2.

1. Determine the strategy sets of both firms.

   (6 Points)

2. Determine the subgame perfect Nash equilibrium of the dynamic game.

   (24 Points)

3. For $i = 1, 2$, let x_i^* be the quantity produced by firm i in the subgame perfect Nash equilibrium. Is (x_1^*, x_2^*) a Nash equilibrium in the static game, where the two firms choose their quantities simultaneously rather than sequentially? Prove your claim.

   (10 Points)

Problem 3 (38 Points)

An entrepreneur (principal) wants to contract a worker (agent). A contract specifies the hours x the agent works for the principal and the wage w the principal pays to the agent. The principal's profit function is given by

$$\pi(x, w) = 12x - w \quad \text{for all } x \geq 0, w \geq 0.$$

The agent can be of two types, $\bar{\theta} = 2$ or $\underline{\theta} = 1$. The utility function of an agent with type $\theta \in \{\bar{\theta}, \underline{\theta}\}$ is given by

$$U^{\theta}(x, w) = w - \theta x^2 \quad \text{for all } x \geq 0, w \geq 0.$$

The reservation utility of both types of agents is 0.

The principal does not observe the type of the agent, but only knows that the agent is of type $\underline{\theta} = 1$ and type $\bar{\theta} = 2$ with probability 0.5 each.

1. Write down the principal's optimization problem if he wants to contract with both types of agents under incomplete information.

   (10 Points)

2. Determine the optimal contract menu the principal offers if he wants to contract with both types of agents under incomplete information.

   Hint: You can assume that the participation constraint of type $\bar{\theta} = 2$ and the incentive compatibility constraint of type $\underline{\theta} = 1$ are binding while the other constraints are slack.

   (28 Points)